PHYSIOLOGICAL OPTICS

Being An Essay Contributed to *The American Encyclopedia of Ophthalmology*

By

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FOREWORD

Through the courtesies of Dr. Casey A. Wood, Editor-in-chief of *The American Encyclopedia of Ophthalmology,* and the publisher, Dr. Geo. Henry Cleveland, of the Cleveland Press, the writer of this monograph has been able to secure a limited number of reprints of the original essay for the use of students and practitioners who may be interested in the field of physiologic optics.

The body of the text is in the form presented in Volume XIII of the Encyclopedia. Certain references to other volumes of the set have been changed in this reprint and a few additions deemed necessary by virtue of the isolation of this essay from its context have been placed in the Appendices.

The present volume is neither a mere compilation nor does it consist of abstract theoretical discussions, but is a collection of the old and new scientific facts that have bearing upon the practical work of the refractionist and eye-practitioner. I hope that the reader will pardon any indulgence in details relative to some of the topics presented, but I feel that far too little attention is paid, in many cases, to the fundamentals and essentials underlying the science of optics as applied to visual phenomena and that a closer and keener study of these vital details will help make the "practical" man of the right type.

I take pleasure in acknowledging my indebtedness to various authors and publishers who have put at my disposal, through the medium of the printed page, such materials as have helped greatly to make this volume what it is. I have drawn freely from the writings of von Helmholtz, Tscherning, Maddox, Stevens, Howe and Savage. In particular do I acknowledge the permission granted by the Editor and the Publisher of *The American Encyclopedia of Ophthalmology,* The Keystone Publishing Co. of Philadelphia, Dr. Savage, Dr. Howe, Dr. Stevens and Chas F. Prentice, M. E., to have reproduced, or to use the original cuts of, several diagrams and drawings from their books.

CHARLES SHEARD.

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Physiological Optics

TABLE OF CONTENTS

INTRODUCTION

PART ONE. OCULAR OPTICS

INTRODUCTION

The domain of science has its many branches and they are veritably members one of another. Chemistry, with its unions of atoms and physics, with its fundamental molecule and its later concept, the electron, have found a common ground and meeting place in physical chemistry. These interminglings and correlations of the sciences have always been productive and fruitful. Physiologic optics is no exception to this statement for it forms the bond of union between the science of life and the functions of the human regime and that inanimate and invisible something which we call light or, more broadly, radiant energy. These radiations, transmitted by an elastic medium, designated as the ether, in the form of transverse waves at a velocity of approximately 186,000 miles a second, are received by the human system and furnish it external sources of heat, aid in metabolic and chemical processes and by some subtle transmutation from physical to mental, furnish it, through the medium of the eye, with a perception of surroundings external to itself.

When we say that we see an object we scientifically mean that we receive from it radiations of such wave-lengths as to produce retinal stimulation. If it is a self-luminous body, such as the sun, we receive light from it *per se;* if it is not, the object merely passes on or reflects the light which it receives from something else. It is obvious that in neither case do we see the thing itself; we are conscious only of a certain sense impression derived from it. The nature of this sense impression, the manner and the way in which it is developed from a physical stimulation of the retina, whether the impulses be electrical, chemical or mechanical in nature or origin, and the transition steps from stimulation to mental interpretation, are problems which are engaging the attention of physiologists, anatomists and psychologists. Certain it is that we are able to trace out the physical, anatomical and physiological processes involved in placing upon the retina, in accurate focus, the image of an object, situated between the two extremes technically known as the punctum remotum and the punctum proximum. These processes, in fact, in their mechanism and results, constitute the essentials of physiologic optics: we have to deal with the human eye as an optical instrument made of living parts, and with a pair of eyes as two such instruments possessed of duction and version powers through the agency of the extra-ocular muscles working, under normal conditions, in harmony to afford binocular single vision. *Physiologic optics* is a science which touches the highest philosophic problems of the human mind on the one hand, and, on the other, keeps in most intimate contact with the practical work of the practitioner upon, and student of, the eye who, in his work of refraction, must be guided by its fundamental principles.

It was with propriety, then, that the versatile von Helmholtz divided his classic *Handbuch der Physiologischen Optik* into three general portions;—(1) the passage of light into the eye or the dioptrics of the eye, (2) the functions of the retina and (3) the interpretations and appreciation of the outer world through the sense of sight. We shall, in turn, follow some such general classification of the subject-matter to be presented; various opinions, theories and practices which have been accorded the partial or complete acceptance of the scientific world will be given and briefly discussed.

Let us devote our attention first of all to a consideration of *ocular dioptrics;* a topic, by the way, too often slighted by the student and practitioner either because of the lack of fundamental mathematical and optical training or passed over by them in the ever prevalent attempt to grasp in a mechanical way the so-called "practical" results and conclusions to the exclusion of a thorough understanding of the "why and wherefore."

PHYSIOLOGICAL OPTICS

PART ONE

OCULAR OPTICS

I. REFRACTION AT CURVED SURFACES WITH APPLICATIONS TO OCULAR CALCULATIONS

1. Many optical phenomena, among them those which have been found to have most extensive and practical applications, occur in conformity to the following fundamental laws:—

(1) The law of *rectilinear propagation* of light.

(2) The law of *reflection:* this states that when regular reflection takes place the angles of incidence and reflection are equal and both lie in the same plane.

(3) The law of *refraction:* a constant ratio exists between the sines of the angles of incidence and refraction; this ratio is governed solely by the relative optical density of the two media and is known as the index of refraction of the body, for any given or specified wave-length, with respect to the surrounding medium. This is mathematically expressed as

$$\frac{\sin i}{\sin r} = n$$

in which *"i"* and *"r"* represent the angles of incidence and refraction respectively and "n" the index of refraction.

(4) The law of the *independence* of the *different portions of a beam of light.*

These four fundamental experimental facts as to the direction of rays of light are comprehended in a general way in a single law often referred to as the principle of least path or *Fermat's principle of least time.* If a ray of light passes from a point *A* to a point *B* and suffers any number of reflections and refractions, then the sum of the products of the index of refraction (n) of each medium multiplied by the distance traversed (l) in it differs from a like sum for all other paths which are infinitely close to it by terms of the second or higher order; i. e., the variation in the total paths, optically considered, approaches zero. This principle finds frequent and important application in geometrical optics. The four fundamental laws above enumerated relate only to geometrical determinations of the propagation of light and constitute, therefore, a sufficient basis for geometrical optics. We shall assume their validity and proceed to use them in the deductions of ocular constants and data.

2. In order to derive the general relations existing between object distance, (f_1), image distance, (f_2), and the radius of curvature (r), let us take a transparent body, such as glass, having an index of refraction (n_2) and a curved surface *DA* with its center at *C*. Any ray of light, such as *OA* (Fig. 1), directed toward *C*, is normal at the point of incidence *A* and passes into the second medium without deviation. A ray, *OD*, which is not normal to the surface but which makes an angle of incidence $ODM = i$ with the normal *DC* (which is also the radius of curvature when the curve *DA* is a portion of a sphere) will, after refraction, proceed in accordance with the laws of refraction, in the direction *DI*, since a small portion of the curved surface at *D* may be considered plane. In geometrical optics there must be two known geometrical paths of light rays emanating from or toward a point before either the object or image can be located. The rays *OD*

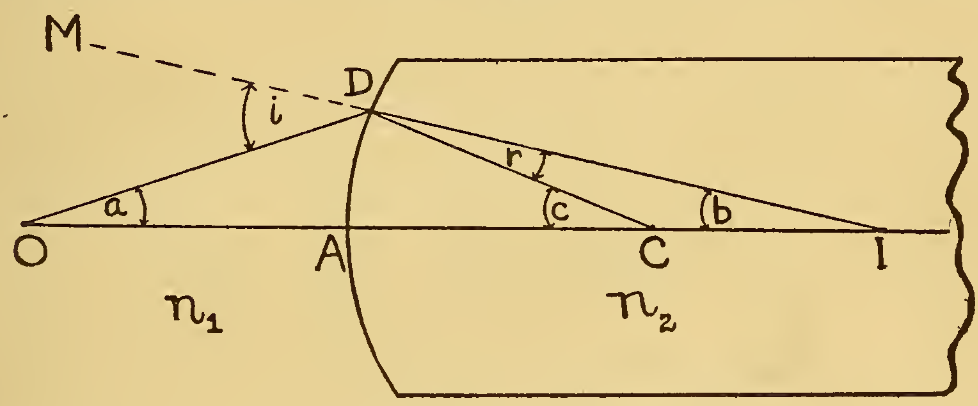


Fig. 1.—Illustrating the Refraction at a Curved Surface Separating Media of Different Optical Indices.

and *OA* intersecting at *O* give a geometrical object; the rays *DI* and *AI* upon intersection at *I* give an image point. The line *AOI*, which is perpendicular to the refracting surface and passes through the center of curvature *C* and the principal focus, is the principal axis. Let *O*, then, be any object point on the axis of a single refracting surface separating media of indices n_1 and n_2, and let *I* be its image formed by refraction at the surface *AD*. Let the angle of incidence $ODM = i$, the angle of refraction $IDC = r$ and the angles $DOA = a$, $DCA = c$ and $DIA = b$.

Then, (1) $n_1 \sin i = n_2 \sin r$. But $i = a + c$ and $r = c - b$.

Hence (2) $n_1 \sin (a + c) = n_2 \sin (c - b)$. If the incident light be considered small apertured and axial, then the angles *a*, *b* and *c* will be small and we can write with sufficient accuracy,

$$(3) \quad n_1 (\sin a + \sin b) = n_2 (\sin c - \sin b)$$

since *sin* $(a + b)$, for example, is trigonometrically equal to sin a · cos b + sin b · cos a, which expression, in turn, becomes equal to *sin a + sin b* when the angles under consideration are small, since the cosines of small angles approximate a value of unity.

Let OA = f_1; AI = f_2 and CA = r, the radius of curvature. Then

$$\sin a = \frac{DA}{OD} = \frac{DA}{OA} = \frac{DA}{f_1}$$

$$\sin b = \frac{DA}{DI} = \frac{DA}{AI} = \frac{DA}{f_2}$$

$$\sin c = \frac{DA}{DC} = \frac{DA}{r}$$

A substitution of these values in the preceding equation gives

$$(4) \quad n_1 \left(\frac{1}{f_1} + \frac{1}{r} \right) = n_2 \left(\frac{1}{r} - \frac{1}{f_2} \right)$$

or

$$(5) \quad \frac{n_1}{f_1} + \frac{n_2}{f_2} = \frac{n_2 - n_1}{r}$$

This is without doubt the most important and fundamental equation in optics from the ocular standpoint, as will be shown in its applications in these pages. It is to be noted that this equation definitely establishes a relationship between five optical quantities; it is the equation of *conjugate foci.* Likewise, it will be noted that f_1, f_2 and r are all treated as geometrically positive quantities: when object, image and center of curvature are situated as in the accompanying diagram or when object and image are interchanged in positions we shall assume this convention of signs. Frequently all quantities are considered positive when measured from left to right, and negative when measured from right to left, the pole *A* being the point from which all measurements are made. Other conventions as to algebraic signs are also in vogue and the reader is often bewildered and at a loss because of them and because of the lack of uniformity of symbolic optical nomenclature.

The refractive power of a curved surface depends, then, upon its curvature and the relative refractive index of the two bounding media. The focal length depends upon the refractive power.

3. In Fig. 2 (*a* and *b*), *RR* is the principal or refracting plane at which all refraction is assumed, to within second order effects, to occur. The point *C*, which is the center of curvature of the surface *DA*, is also the optical center (or single nodal point of this system) since any ray which passes through this point suffers no refraction or lateral deviation. In Fig. 2(a) the incident light is parallel to the

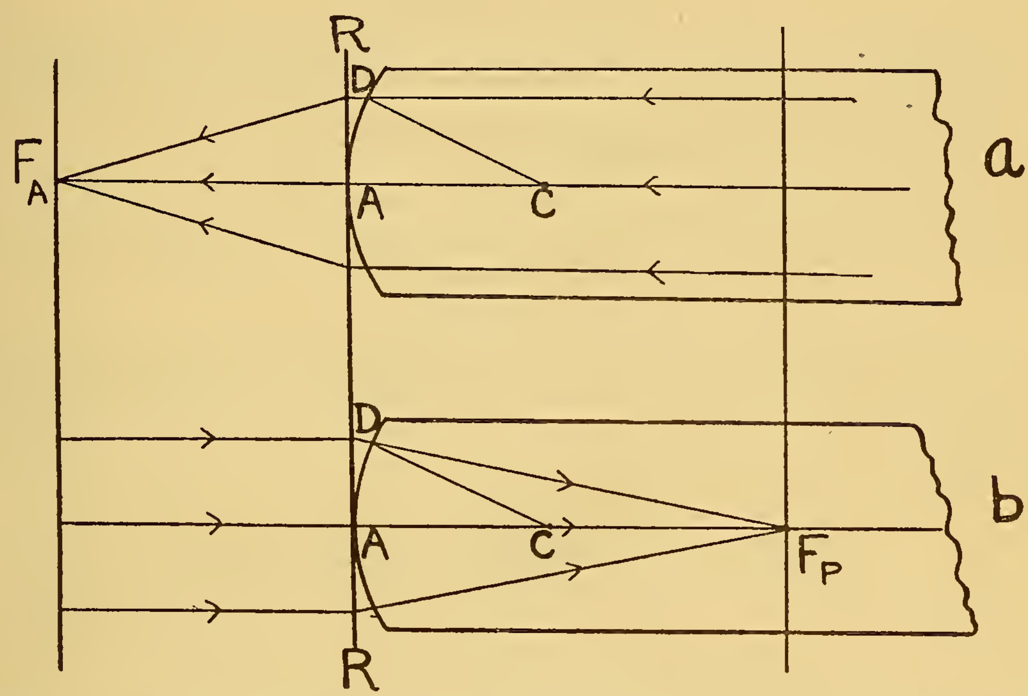


Fig. 2.—Illustrating the Refraction of Parallel Rays by a Curved Surface giving: (a) Anterior Focus, (b) Posterior Focus.

principal axis in the denser medium of index n_2 and upon emergence into the less dense medium, index n_1, is refracted so as to meet at the point F_A. F_AA is the *anterior focal distance.* In a similar manner, parallel light entering from the rarer medium will be focused in the denser medium at F_P. F_PA represents the *posterior focal distance.* F_A and F_P are the anterior and posterior focal points respectively.

If, then, the incident light in the rarer medium be regarded as parallel and hence coming from infinity, the term $\frac{n_1}{f_1}$ becomes zero and, therefore, the general law of conjugate foci which reads

$$\frac{n_1}{f_1} + \frac{n_2}{f_2} = \frac{n_2 - n_1}{r}$$

becomes

$$\frac{n_2}{f_2} = \frac{n_2 - n_1}{r}$$

and the *posterior principal focal length*

$$F_P = \frac{n_2 r}{n_2 - n_1} = \frac{n_1 r}{n_2 - n_1} + r.$$

In like manner, when parallel light is passing from the denser to the rarer medium, $\frac{n_2}{f_2}$ becomes zero and

$$F_A = \frac{n_1 r}{n_2 - n_1}$$

When one of the media is air, $n_1 = 1$ and the above equations simplify into

$$F_A = \frac{r}{n - 1}$$

$$\text{and} \quad F_P = \frac{nr}{n - 1} = \frac{r}{n - 1} + r$$

We note that $F_P = F_A + r = n F_A$ and draw from the preceding discussion the following conclusions:—

(a) The difference between the focal distances is equal to the radius of curvature.

(b) The distance of the center of curvature from the posterior focus is equal to the anterior distance and the distance of the center from the anterior focus is equal to the posterior focal distance.

(c) The ratio between the focal distances is equal to the ratio between the indices of the corresponding media.

4. To construct the image of a point removed from the principal axis, we can geometrically proceed as shown in Fig. 3.

(a) A ray, *MD*, parallel to the axis *NC:* it is refracted through the posterior focal point F_P.

(b) A ray, *MC*, proceeding toward the center of curvature or optical center *C:* it is not displaced upon refraction but passes straight through.

(c) A ray, MF_A, passing through the anterior focus: it is parallel, after refraction, to the axis.

From the similarity of the triangles MNF_A and F_ARX we have the ratio

$$\frac{NF_A}{F_AX} = \frac{MN}{RX} = \frac{MN}{M_1N_1}$$

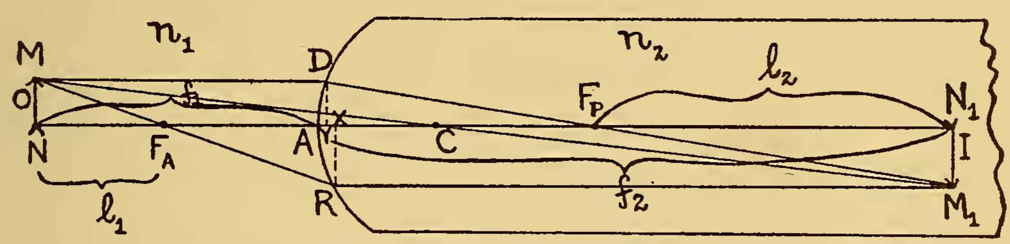


Fig. 3.—The Geometrical Construction of the Image of an Object as Produced by a Single Refracting Surface.

or $\frac{l_1}{F_A} = \frac{O}{I}$ and from the similarity of the triangles DYF_P and $F_PM_1N_1$ the ratio $\frac{F_P}{l_2} = \frac{O}{I}$. Hence we deduce the general formula

$$l_1 \, l_2 = F_A \, F_P$$

and by substitution therein of the values $l_1 = f_1 - F_A$ and $l_2 = f_2 - F_P$, we obtain the equation

$$\frac{F_A}{f_1} + \frac{F_P}{f_2} = 1$$

Other expressions for the magnification and size of the object or image when the size of one of these is known may be deduced from the similarity of triangles in Figure 3. Two magnification ratios, other than those just developed, are

$$(1)\ M = \frac{O}{I} = \frac{f_1 + r}{f_2 - r},$$

i. e., whatever may be the distance of the object, its size and that of the image are in the same ratio as their respective distances from the center of curvature, and

2

$$(2)\ M = \frac{O}{I} = \frac{f_1 n_2}{f_2 n_1}.$$

5. It will subsequently be shown that the complex optical system of the eye with its different refractive media having slightly different indices of refraction may be approximately represented by a single simple system consisting of a refractive medium of index, $n = 4/3$, bounded by a convex spherical surface, representing the cornea, having a radius of curvature of 5 millimeters. If we assume such a "reduced eye," we can calculate the positions of the anterior and posterior focal points, the sizes of retinal images, the apparent size and depth of the pupil and the amount of axial change corresponding to each diopter of axial ametropia.

When $n_1 = 1$, $n_2 = 4/3$ and $r = 5$ mms., an *emmetropic reduced eye,* with passive fixation at infinity, gives

$$F_A = \frac{r}{n_2 - n_1} = \frac{5}{4/3 - 1} = 15 \text{ mm.}$$

$$\text{and } F_P = \frac{n_2 r}{n_2 - n_1} = \frac{4/3 \times 5}{4/3 - 1} = 20 \text{ mm.}$$

The second or posterior principal focus therefore lies 20 mms. behind the refracting plane and the anterior focus is 15 mms. in front of it. This anterior focal distance of 13 to 15 millimeters is commonly assumed in practice and is of importance since any lens placed in the first focal plane of an eye produces no change in the size of the image formed unless accommodation occurs, but simply shifts the image backward or forward as the case may be.

6. In order to calculate the size of a retinal image under certain conditions of distance and size of object, let us determine the size of such an image for an emmetropic reduced eye viewing a letter *I* in the normal 20-foot line. This letter, which subtends at the nodal point of an assumed normal eye an angle of 5 minutes, is 0.35 inch or 8.7 mms. in height. Then

$$\frac{\text{Object size}}{\text{Image size}} = \frac{8.7 \text{ mms.}}{\text{Image}} = \frac{f_1 + r}{f_2 - r} = \frac{6100 + 5}{15}$$

or $6105 \times \text{Image} = 131$, or the size of the image is 0.023 mm. It will be noted, in passing, that to all practical intents and purposes the quantity $f_1 + r$ may be regarded as equal to f_1 whenever the distances of the object from the optical center are large in comparison with the value of the radius of curvature. Likewise, the numerical value of the retinal image as calculated above, 0.023 mm., is a constant quantity whenever the various letters of the visual acuity chart, calculated upon a 5′ basis, are viewed by an emmetropic eye at the distance specified upon the chart. The ratio of object size to size of retinal image for a normal eye exerting no accommodation is approximately 400 to 1.

7. The apparent size and position of the pupil of the eye may be calculated by a use of the law of refraction at curved surfaces and the magnification formulæ. By experimentation the average radius of curvature of the human cornea has been found to be about 7.8 mms. and its index of refraction approximately 1.33; the anterior surface of the crystalline lens is 3.54 mms. from the anterior surface of the cornea. For purposes of the present calculation let us assume $r = 8$ mms., $n_1 = 1.33$, $n_2 = 1$ and let the pupil of the eye serve as the object at 3.60 mms. behind the surface of the anterior surface of the cornea, and let it be 2 mms. in diameter; we desire to find its apparent size and position. In this case n_1 is the denser medium containing the object and the cornea is concave toward it. Hence

$$\frac{n_2}{f_2} + \frac{n_1}{f_1} = \frac{n_2 - n_1}{r}$$

becomes
$$\frac{1}{f_2} = \frac{1 - 1.33}{-8} - \frac{1.33}{3.6} = -\frac{1}{3.05}$$

Hence f_2, i. e., the image position, is virtual and 3.05 mms. behind the surface. Its size is

$$I = \frac{2 \times 3.05 \times 1.33}{3.6} = 2.25 \text{ mms.}$$

Hence the pupil appears larger and nearer the anterior corneal surface than it actually is.

8. The statement is frequently found in treatises on visual optics and ophthalmology that an axial change, whether it be an increase or a decrease in the depth of the eye from cornea to macula, of one millimeter approximately represents an ametropia of three diopters. This statement is sufficiently accurate for practical purposes; it can be shown, however, that this ratio of 1 mm. change in depth to 3 D of error is not exact, that its value varies slightly by different methods

of computation, that its value is not the same for myopic as for hyperopic changes of equal amounts and that this ratio does not remain constant as the axial dioptric error increases. The application of the general law of refraction at curved surfaces furnishes a clear and concise proof. Assume a condition of axial myopia of 1 D. The punctum remotum, f_1, or distance of distinct vision, will therefore be 100 cms. We desire to find its conjugate, f_2, which in this case is to lie upon the retina. Since $n_1 = 1$, $n_2 = 4/3$ and $r = 5$ mm. (employing the Donders' reduced eye) we have, upon substitution in the general equation of refraction at curved surfaces,

$$\frac{1}{1000} + \frac{4/3}{f_2} = \frac{4/3 - 1}{5}.$$

Simplifying and solving, we have

$$591\ f_2 = 1200$$
$$f_2 = 20.3 + \text{mms.}$$

By reference to the calculation made for an emmetropic reduced eye in one of the preceding paragraphs, the reader will find that the value of f_2 is 20 mms. The conclusion therefore follows that an axial change of 0.33 mm. represents an axial ametropic error of 1 diopter. In the particular case of myopia assumed for illustrative purposes, a — 1 D lens in the first focal plane of the eye would extend the punctum remotum from the one meter point back to infinity and place the image of an object situated at infinity upon the retina rather than allowing it to fall in the vitreous. It is, of course, to be clearly understood that such ametropic errors may be due to corneal, lenticular or indicial abnormalities: they have been treated and discussed thus far as "equivalent" axial errors only.

To illustrate the application of these principles to a condition of axial hypermetropia, let us assume a "short" eye 19 millimeters in depth. Taking the radius of curvature as 5 mms. and the index of refraction as $4/3$, it is necessary to calculate first of all the focal distance, f_1, conjugate to the known posterior focal distance $f_2 = 19$ mms. The expression is

$$\frac{1}{f_1} + \frac{4/3}{19} = \frac{4/3 - 1}{5}$$

$$\text{or } \frac{1}{f_1} = \frac{1}{15} - \frac{4}{57} = -\frac{3}{855}$$

Hence $f_1 = -288$ mms.

The negative sign is of significance, for it indicates that the anterior focal point is on the same side of the refracting surface as the posterior focal point, indicating, therefore, a virtual point back of the retina. This is in accord with the commonly known principle that only light converging toward the eye can be brought to a focus upon the retina of an hyperopic eye not exerting its accommodation. These conditions are diagrammed in Fig. 4. F_P and F_A are conjugate foci; $AF_P = 19$ mms. and F_A, by calculation, is -288 mms. This distance, 288 mms., in dioptric equivalent is practically 3.50 D; this eye has 19 mms. instead of 20 mms. as its posterior focal length; we therefore

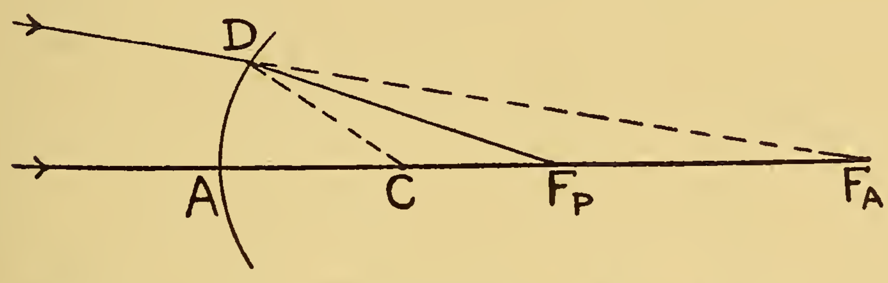


Fig. 4.—Illustrative of the Optical Conditions Existing in a Hyperopic Eye as to Conjugacy of Foci.

conclude that 1 mm. decrease in the depth of the normal eye, representing hypermetropia, indicates an axial error of 3.5 D.

The practitioner is herewith also furnished the basis for calculations which may be of diagnostic value in conjunction with ophthalmoscopic examinations. We do not refer to the use of the ophthalmoscope as an instrument for the measurement of refractive errors, but rather to its employment in determining differences in level between the edge or ring of the optic disc and the center of the cup in, for example, physiologic cup, glaucoma and choked disc. The difference in the lens quantity which the observer must turn up in his instrument in order to, let us say, clearly see in turn the edge and the center of the disc, gives a measurement of fair accuracy upon the relative elevation or depression; for instance, edge of disc -2 D, center -8 D, difference -6 D, indicating a maximum cup depth or difference in fundus level of about 2 mms.

II. THE GAUSS EQUATION AND ITS APPLICATION TO THE DIOPTRIC SYSTEM OF THE EYE

9. The theory of lenses is simple if the thickness is negligible. The axis of such a lens is designated as the straight line which joins the two centers of the surfaces, and the optic center is a point on this line such that any ray passing through it suffers no angular deviation

or lateral displacement. In the case of bi-convex and bi-concave lenses the optical center is at the geometrical center. The general formula for the focal length, $F_A = F_P$, of a thin lens in terms of the index (n) and the radii of curvature (r_1 and r_2) may be derived from a double application of the methods of procedure and mathematical processes considered under refraction at curved surfaces and is of the form

$$\frac{1}{F_A} = \frac{1}{F_P} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

If the lenses are not sufficiently thin so that their thicknesses may be neglected, nor placed so near together that their distances apart can be overlooked, the position and size of the image may still be found by construction or calculation; one must construct or calculate first of all the image formed by the first surface, which image in turn serves as the object for the second surface and so on. By the aid of the Gauss equation every optical system can be so simplified that all problems of conjugacy of foci and other optical data can be solved by formulæ applicable to single thin lenses. The system must, however, be centered, which is to say that all the centers of the surfaces must lie on the axis, the pencils of light passing through the various members of this system must be axial and small, hence aberration is neglected.

According to the Gaussian theory every optical system has six cardinal points, to wit:—

One anterior focus, F_A.
One posterior focus, F_B.
Two principal points, H_1 and H_2.
Two nodal points, N_1 and N_2.

To illustrate the course of light in a compound optical system and to aid in the definitions of principal and nodal points, we cannot do better than to take the case of the eye itself and presume at this point the correctness of some of the conclusions to which we shall ultimately arrive. The dioptric system of the eye consists, as shown in Fig. 5, of three refracting systems; the cornea S_1, the anterior surface of the lens S_2, the posterior surface of the lens S_3, separating four media of indices n_1 (air), n_2 (aqueous), n_3 (lens) and n_4 (vitreous).

The anterior focal distance, $F_A = H_1 F_A$, is the distance from the first principal point to the anterior focus; it is also equal to the distance of the second nodal point from the posterior focus, $N_2 F_P$.

The posterior focal distance, $F_P = H_2 F_P$, is the distance from the second principal point to the posterior focus; it is also equal to the distance of the anterior focus from the first nodal point, $F_A N_1$.

Rays in media n_1 parallel to the principal axis meet after refraction in medium n_4 at the point F_P.

Rays diverging from F_A are, after refraction, parallel to the principal axis.

A ray directed toward the first principal point, H_1, appears, after refraction, to proceed from the second, but the direction after refraction is not parallel to its original course.

A ray directed to the second principal point, appears, after refraction, to proceed from the second. The two principal points are images of each other.

A ray directed to the first nodal point, N_1, after refraction appears to come from the second and its direction is parallel to its original

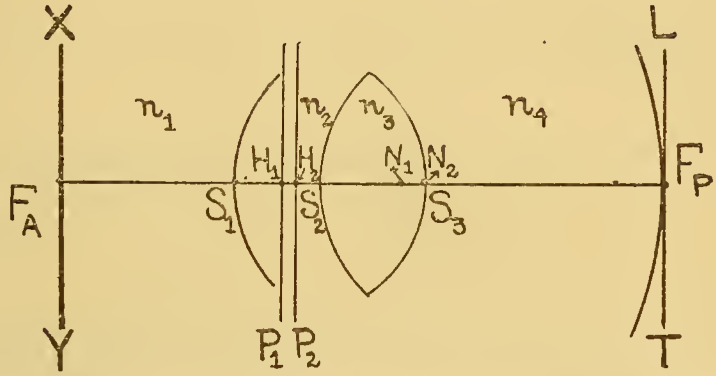


Fig. 5.—The Six Cardinal Points and Dioptric Media of the Eye.

course. A ray directed to the second appears after refraction to come from the first nodal point. In the case of a single refracting surface a ray directed to its nodal point passes through without deviation; but in a compound system, where there are two nodal points, a ray must be directed to the first in order to appear to come from the second. The two nodal points are images of each other.

The distance which separates the two principal points is equal to that which separates the two nodal points, or $H_1 H_2 = N_1 N_2$ (Fig. 5).

Three important relations, therefore, exist in such a compound dioptric system:—

$$\text{(a)}\quad H_1 F_A = N_2 F_P$$
$$\text{(b)}\quad H_2 F_P = N_1 F_A$$
$$\text{(c)}\quad H_1 H_2 = N_1 N_2$$

10. Provided, then, the six cardinal points are known, the most complicated system can be reduced to the simplicity of a single lens. Knowing these six points, the image of a given point or object, *XY*,

Fig. 6, can be found by construction choosing any two of the three following rays:—

(1) The ray XA, parallel to the axis, must cut the second principal plane at D at a distance from the axis equal to AH_1 and it must pass through F_P. It takes the direction DX_1.

(2) The ray XB, passing through the anterior focal point F_A, must after refraction be parallel to the axis YY_1: it will take the direction BX_1.

(3) The ray XN_1, directed toward the first nodal point, takes after refraction the direction N_2X_1 parallel to its original direction.

11. In Fig. 6, let $YF_A = l_1$, $H_1F_A = F_A$, $H_1Y = f_1$, $Y_1F_P = l_2$, $H_2F_P = F_P$ and $H_2Y_1 = f_2$. The triangles XYF_A and BH_1F_A on the

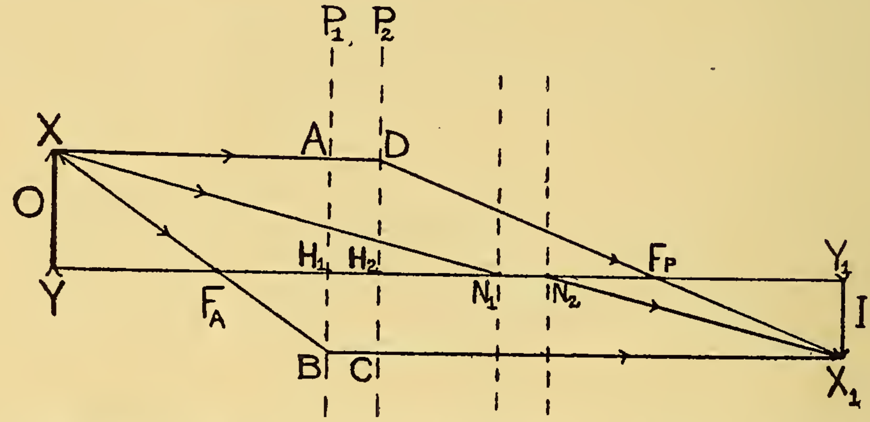


Fig. 6.—Illustrating Image Formation when the Six Cardinal Points are Known, Whereby a Complicated System can be Reduced to a Single Lens Equivalent.

one side and the triangles DH_2F_P and $Y_1X_1F_P$ on the other side being similar give the relation

$$\frac{\text{Image}}{\text{Object}} = \frac{O}{I} = \frac{l_1}{F_A} = \frac{F_P}{l_2}$$

We have then, as before, the relation $l_1 l_2 = F_A F_P$ and we can deduce therefrom the general formula

$$\frac{F_A}{f_1} + \frac{F_P}{f_2} = 1.$$

12. The formulæ which follow are deduced for the purpose of giving the general method and the outline of the mathematical procedure involved in the Gauss equation; the applications to the dioptric system of the eye will then follow. In order to keep the formulæ as symmetrical as possible and to avoid errors in algebraic signs, the

following conventions will be observed: (a) all distances measured to the left of a surface are negative and to the right, positive; (b) all thicknesses are considered negative and when actual numerical values are substituted the minus signs must be employed. Let n_1 be the refractive index of the medium surrounding a lens, n_2 that of the lens, t its axial thickness and r_1 and r_2 the radii of curvature of the first and second surfaces of the lens respectively. Let u be the object distance, v_1 the image distance due to the refraction at the first surface and v the final image distance after refraction at the second surface. The fundamental equation for the refraction at the first curved surface assumes the form

$$\frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{r_1}$$

In order to simplify the formula, $\frac{n_2 - n_1}{r_1}$ is replaced by F_1 and $\frac{n_2}{v_1}$ and $\frac{n_1}{u}$ are replaced by their so-called *reduced* expressions $\frac{1}{v_1}$ and $\frac{1}{u}$. Similarly in the expression for the refraction at the second lenticular surface the expression $\frac{n_1 - n_2}{r_2}$ will be put equal to F_2, and $\frac{n_1}{v}$ and t, the lens thickness, will be given their reduced values, $\frac{1}{v}$ and $\frac{t}{n}$, where n represents the index of the medium in which the quantity is measured. Ultimately, of course, the numerical values obtained by means of these reduced equations must be multiplied by the proper quantities in order to literally "undo" these simplifications. The "reduced" fundamental equation becomes

$$\frac{1}{v_1} - \frac{1}{u} = F_1$$

$$\text{whence} \quad v_1 = \frac{1}{F_1 + \frac{1}{u}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The expression connecting v_1 (the image formed by the first refraction which, in turn, serves as the object for the second refraction) and v, the final image is

$$\frac{n_1}{v} - \frac{n_2}{v_1 + t} = \frac{n_1 - n_2}{r_2}$$

or as a reduced expression $\frac{1}{v} - \frac{1}{v_1 + t} = F_2$, whence

$$v = \cfrac{1}{F_2 + \cfrac{1}{v_1 + t}} \text{.................................} (2)$$

By a substitution of the value of v_1 (equation 1) in equation 2 we obtain

$$v = \cfrac{1}{F_2 + \cfrac{1}{t + \cfrac{1}{F_1 + \cfrac{1}{u}}}} \text{.................................} (3)$$

This continued fraction, when worked out, gives

$$v = \frac{u\,(F_1 t + 1) + t}{u\,(F_1 F_2 t + F_1 + F_2) + F_2 t + 1} \text{..............} (4)$$

This may, for brevity's sake, be written

$$v = \frac{Bu + D}{Au + C} \text{.....................................} (5)$$

where $A = F_1 F_2 t + F_1 + F_2$
$B = F_1 t + 1$
$C = F_2 t + 1$
$D = t$

Equation (5) correlates *u* and *v* when both are finite. When *u* is at infinity, then the focal length measured from the second surface is

$$v = \frac{B}{A} \quad \text{.......... (6)}$$

This value of v in equation (6) is known as the *back focal distance* measured from the pole of the second surface. When v is infinite, then in equation (5) Au + C must equal zero and

$$u = -\frac{C}{A} \quad \text{.......... (7)}$$

This gives the value of the focal length measured from the pole of the first surface.

In order to find the positions of H_1 and H_2, the equivalent points, and N_1 and N_2, the nodal points, it is necessary to derive an expres-

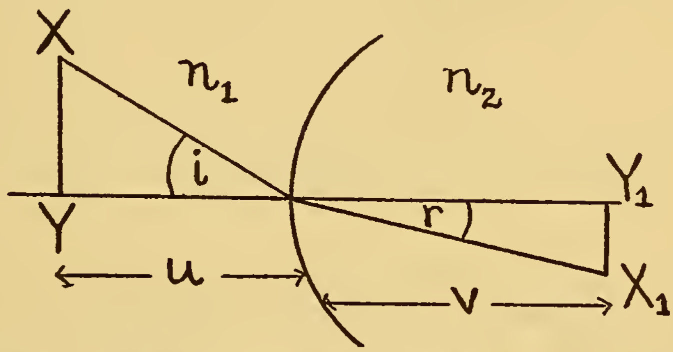


Fig. 7.—Graphically Illustrative of the Magnification Produced by a Refracting Surface.

sion for the total magnification, *M*, produced by the lens system. If m_1 represents the magnification due to the first surface and m_2 that produced by the second similar surface, then $M = m_1 \times m_2$.

13. In Fig. 7 let *XY* be an object in front of the first surface and X_1Y_1 its corresponding image. The angles "*i*" and "*r*" represent incidence and refraction respectively.

Then $m_1 = \frac{X_1Y_1}{XY}$. When the angles are small $\sin i = \frac{XY}{u}$ and

$\sin r = \frac{X_1Y_1}{v_1}$ and $\frac{\sin r}{\sin i} = \frac{n_1}{n_2}$ or $m_1 = \frac{n_1 v_1}{n_2 u}$. When we express this

ratio in reduced quantities we have $m_1 = \frac{v_1}{u}$. In a like manner the magnification, m_2, of the second surface is $m_2 = \frac{v}{v_1 + t}$ and $M = m_1 \times m_2 = \frac{v_1}{u} \times \frac{v}{v_1 + t}$.

From equation (1) $\frac{v_1}{u} = \frac{1}{F_1u + 1}$, and from equations (1) and (2) we have

$$\frac{v}{v_1 + t} = \frac{1}{F_2\left(\frac{u}{F_1u + 1} + t\right) + 1}$$

$$\text{Hence } M = \frac{1}{F_1u + 1} \times \frac{F_1u + 1}{u(F_1F_2t + F_1F_2) + F_2t + 1}$$

$$= \frac{1}{u(F_1F_2t + F_1 + F_2) + F_2t + 1}$$

$$= \frac{1}{Au + C} \qquad \text{.........................(8)}$$

in terms of our previous notation.

14. If now the virtual image and object be equal in size, the magnification will be $+1$. The planes of unit virtual magnification for thick lenses and lens systems lie in the equivalent or principal planes. If such were possible and one could place a small object in one plane, then its virtual image, identical in all points to the object, would be situated in the other. Hence $Au + C = 1$ and

$$u = H_1 = \frac{1 - C}{A} \qquad \text{.................................(9)}$$

measured from the first surface.

Equation (5) reads $v = \frac{Bu + D}{Au + C}$ and by substitution of the value of u from equation (9) in equation (5) there is obtained

$$v = H_2 = \frac{B - B \times C + A \times D}{A}$$

$$= \frac{B - 1}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

since $(A \times D) - (B \times C) = 1$. The distance $v = H_2$ is measured from the second surface.

The values of u and v from equations (9) and (10) must be added to those of u and v in equation (5) in order to find the equivalent focal distances, since equivalent focal distances must always be measured from the equivalent planes to the points representing the back focal distances. Hence

$$v + \frac{B - 1}{A} = \frac{B\,[u + (1 - C)/A] + D}{A\,[u + (1 - C)/A] + B}$$

which reduces to $A = \frac{1}{v} - \frac{1}{u} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$

This expression, the reader will observe, is precisely similar in form to the general thin lens formula involving object and image distances and principal focal lengths.

When u is infinite, $v = \frac{1}{A}$ and when v is infinite, $u = -\frac{1}{A}$. These are, of course, reduced expressions and in obtaining numerical results they must be multiplied by the proper index of refraction in every case.

15. We are now in a position to apply the Gaussian method to the case of the eye having three surfaces, S_1, S_2 and S_3 (see Fig. 5), with thicknesses t_2 and t_4, with the following data:—

r_1 (cornea) = 8 mms.

r_3 (anterior surface of the crystalline) = 10 mms.

r_5 (posterior surface of the crystalline) = 6 mms.

t_2 (thickness of aqueous) = 3.6 mms.

t_4 (thickness of crystalline lens) = 3.6 mms.

$n_1 = \text{air} = 1.$
$n_2 = \text{aqueous} = 1.333.$
$n_3 = \text{lens} = 1.45.$
$n_4 = \text{vitreous} = 1.333.$

Let F_1, F_3 and F_5 represent the reduced focal lengths of the three surfaces S_1, S_2 and S_3. Then

$$F_1 = \frac{n_2 - n_1}{r_1} = \frac{1.333 - 1}{8} = 0.0416 \text{ mm.}$$

$$F_3 = \frac{n_3 - n_2}{r_3} = \frac{1.45 - 1.333}{10} = 0.0117 \text{ mm.}$$

$$F_5 = \frac{n_4 - n_3}{r_5} = \frac{1.333 - 1.45}{-6} = 0.0195 \text{ mm.}$$

The reduced values of t_2 and t_4 are:

$$t_2 = \frac{-3.6}{1.333} = -2.7007 \text{ mm.}$$

$$t_4 = \frac{-3.6}{1.45} = -2.4830 \text{ mm.}$$

Writing an equation for v, similar to equation (3), in the form of a continued fraction and side by side with it the expression with numerical values substituted, we have:—

$$v = \cfrac{1}{F_5 + \cfrac{1}{t_4 + \cfrac{1}{F_3 + \cfrac{1}{t_2 + \cfrac{1}{F_1 + \cfrac{1}{u}}}}}} \qquad v = \cfrac{1}{0.0195 + \cfrac{1}{-2.4828 + \cfrac{1}{0.0117 + \cfrac{1}{-2.7007 + \cfrac{1}{0.0416 + \cfrac{1}{u}}}}}}$$

This equation when worked out gives

$$v = \frac{0.7587u - 5.105}{0.0668u + 0.869} = \frac{Bu + D}{Au + C};$$

whence $A = 0.0668$ $C = 0.869$
$B = 0.7587$ $D = -5.105$

Finally,

$$F_A = \text{anterior focus} = \frac{-n_1}{A} = \frac{-1}{0.0668} = -15 \text{ mms.}$$

$$F_P = \text{posterior focus} = \frac{n_4}{A} = \frac{1.333}{0.0668} = +20 \text{ mms.}$$

$$H_1 = \text{first equivalent point} = \frac{n_1(1 - C)}{A} = \frac{0.1311}{0.0668}$$

$= 1.96$ mm. from r_1 toward the lens.

$$H_2 = \text{second equivalent point} = \frac{n_4(B - 1)}{A} = \frac{-0.3128}{0.0668}$$

$= -4.81$ mms. from r_5, or $t_2 + t_4 - 4.81 = 7.2 - 4.81 = 2.39$ mms. from r_1 toward the lens.

The distance of the nodal points from the equivalent points is always equal to the difference between the anterior and the posterior focal lengths; hence $N_1 = F_P - F_A$ from H_1 and $N_2 = F_P - F_A$ from H_2. The value of $F_P - F_A$, algebraically considered as representing two linear dimensions, is $20 - 15 = 5$ mms. Therefore,

$N_1 =$ first nodal point $= 5 + 1.96 = 6.96$ mms. from r_1.
$N_2 =$ second nodal point $= 5 + 2.39 = 7.39$ mms. from r_1.

In brief tabulation, the six cardinal points of the eye are situated with respect to the cornea as follows:—

$F_A = 13.05$ mms. from cornea	$H_1 = 1.95$ mms.	$N_1 = 6.95$ mms.
$= 15$ mms. from H_1		
$= 20$ mms. from N_1		
$F_P = 22.38$ mms. from cornea	$H_2 = 2.38$ mms.	$N_2 = 7.38$ mms.
$= 20$ mms. from H_2		
$= 15$ mms. from N_2		

16. Gauss's theory supposes that the aperture of the system is very small. This is not true in the case of the eye, and, according to Tscherning, "many errors committed in questions of ocular refraction appear to be due to the fact that we do not sufficiently take into account the large aperture of the system." In optical instruments an aperture of over ten degrees is not considered acceptable. If we assume that the pupil of the eye has a diameter of four millimeters, the aperture of the cornea would be twenty degrees. Ordinarily under normal conditions the pupillary diameter is as great as five or six millimeters. We have shown in a preceding paragraph that the pupil as seen through the cornea has neither its real position nor its true size: it appears moved forward approximately 0.5 mm. and enlarged on account of the refraction through the cornea by nearly the same amount. We view, then, a virtual image of the iris and of the pupil; they are the apparent iris and the apparent pupil and are aerial images. Rays in air which are directed toward a point of the apparent pupil are, after refraction by the cornea, directed toward the corresponding point of the real image. The apparent pupil belongs to the incident rays as does also the first principal point or the first nodal point, and the crystalline image of the pupil belongs to the emergent rays. The luminous cone which enters an eye is therefore limited by the apparent pupil; in its course between the cornea and the crystalline it is limited by the real pupil and in the vitreous by the crystalline image of the pupil. Professor Abbe has proposed the names of pupil of entrance and pupil of exit for the images of the diaphragm. The principal planes are each, in turn, the image of the other and the object and image are of the same size and the general formula $\frac{F_A}{f_1} + \frac{F_P}{f_2} = 1$ holds when all measurements are made from the principal points. Calculations could just as well be made from any other pair of points which are images one of the other; the entrance and exit pupils are such, except that object and image are not of the same size. This then is the essential difference and the point of value in this discussion; if an incident ray meets the first principal plane at a distance from the axis equal to x, the emergent ray also cuts the second principal plane at a distance from the axis equal to x. But if the incident ray meets the pupil of entrance at a distance from the axis equal to x, then the emergent ray cuts the plane of the pupil of exit at a distance from the axis which bears the same relation to x as the diameter of the pupil of exit does to that

of the pupil of entrance. If we assume an actual pupillary diameter, or exit pupil, of 4 mms., then the entrance pupil will be about 4.5 mms. and the ratio would be 40/45.

III. THE FUNDAMENTALS OF THICK LENS OPTICS AND THEIR APPLICATIONS TO OCULAR CALCULATIONS

17. The student of physiologic optics, desirous of making detailed ocular calculations as to the various optical constants of the different media of the eye and of computing the combined dioptric system of the eye and an auxiliary lens, will find the following brief presentation of the essentials of thick lens optics of service. We shall proceed to find or to give expressions for the effectivity, the equivalent focal

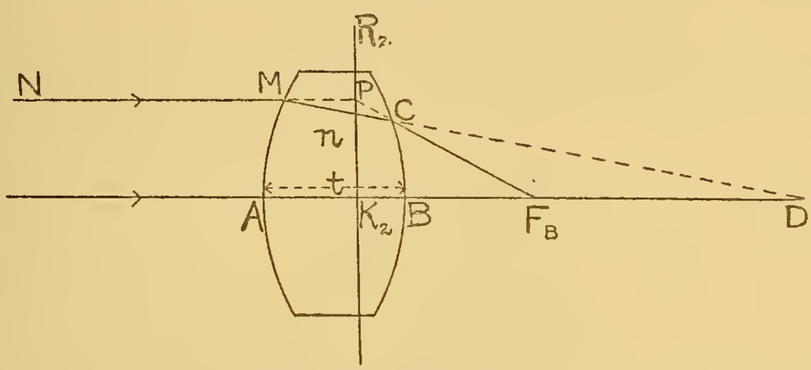


Fig. 8.—Back Focal Length and Equivalent Points of a Thick Lens.

length and the positions of the equivalent points in terms of the radii, thickness and index of the lens. Let

F_E be the equivalent focal length;
K_1 and K_2 be the first and second equivalent points;
T be the distance between K_1 and K_2, or the optical interval;
r_1 and r_2 be the radii of curvature of the surfaces of the lens;
n be the index of the medium;
A and B be the first and second surface points, respectively, on the principal axis.

In Fig. 8, let NM be a parallel ray incident at M; this will be deviated towards the axis by the first surface and would, if not intercepted by the second surface CB, intersect the principal axis AD at D. It is brought, however, by the refraction at the second surface to F_B. Hence, by definition, D is the posterior focus of the first surface, F_B the principal focal point of the lens as a whole and BF_B the back focal length. Project MN and CF_B until they intersect at P; a plane drawn perpendicular to the principal axis through the point P will locate the second equivalent point K_2. All the refraction of light therefore appears to take place at R_2K_2. The distance K_2F_B is the equivalent focal length, since it is the focal length of the thin lens which, if placed at K_2, would have the same effect as the original thick lens. For parallel light, the image by refraction at the first surface A will be at a distance $AD = F_A$ from this surface, and

$$\frac{n-1}{r_1} = \frac{n}{F_A} \qquad (1)$$

Therefore, the distance of the first image from the second surface will be $AD - AB = F_A - t$. If F_B is the distance of the second, or final, image from the second surface, then

$$\frac{n}{F_A - t} - \frac{1}{F_B} = \frac{n-1}{r_2} = \frac{n}{F_{A_1}} \qquad (2)$$

Hence

$$\frac{1}{F_B} = \frac{n}{F_A - t} + \frac{n}{F_{A_1}}$$

$$= n\left(\frac{t + F_A + F_{A_1}}{F_{A_1}(F_A - t)}\right)$$

or $$F_B = \frac{F_{A_1}(F_A - t)}{n(F_A + F_{A_1} - t)} = BF_B \qquad (3)$$

Substituting values of F_A and F_{A_1} in equation (3) we obtain

$$F_B = \frac{\dfrac{nr_2}{n-1}\left(\dfrac{nr_1}{n-1} - t\right)}{n\left(\dfrac{nr_1}{n-1} + \dfrac{nr_2}{n-1} - t\right)} \qquad (4).$$

This equation, when algebraically simplified, becomes

$$F_B = \frac{nr_1r_2 - tr_2(n-1)}{n(n-1)\left(r_1 + r_2 - \dfrac{t(n-1)}{n}\right)} \qquad (5)$$

$= \text{back focal length.}$

In Fig. 8, the triangles PK_2F_B and CBF_B, considering CB as a straight line, are similar; also the triangles MAD and CBD. Hence

$$\frac{K_2F_B}{BF_B} = \frac{PK_2}{CB} = \frac{MA}{CB} = \frac{AD}{BD} \qquad (6)$$

K_2F_B is the equivalent focal length which we desire to find; AD is the posterior focal length of the first surface; BD is equal to $AD - AB$ or $AD - t$, and $BF_B = F_B$ is the back focal length. Equation (6) may be written

$$K_2F_B = F_E = BF_B \times \frac{AD}{BD} \qquad (7)$$

Substituting in this equation the value of BF_B and carrying out the processes involved gives

$$F_E = \frac{r_1 r_2}{(n-1)\left(r_1 + r_2 - \frac{t(n-1)}{n}\right)} \qquad (8)$$

= the equivalent focal length.

The distance of K_2 from the pole B of the second surface is K_2B and can be found from $K_2B = K_2F_B - BF_B = F_E - F_B$, or

$$K_2 = \frac{r_2 t}{n\left(r_1 + r_2 - \frac{t(n-1)}{n}\right)} \qquad (9)$$

and similarly

$$K_1 = \frac{r_1 t}{n\left(r_1 + r_2 - \frac{t(n-1)}{n}\right)} \qquad (10)$$

The optical thickness, T, is the distance between the equivalent points, and is

$$T = t_1 - (K_1 + K_2) \qquad (11)$$

For purposes of convenience in applying these equations, let

$$Q = r_1 + r_2 - \frac{t(n-1)}{n} \quad \ldots\ldots (12)$$

Then

$$F_E = \frac{r_1 r_2}{(n-1)Q} \quad \ldots\ldots (13)$$

$$K_1 = \frac{r_1 t}{n \cdot Q} \quad \ldots\ldots (14)$$

$$K_2 = \frac{r_2 t}{n \cdot Q} \quad \ldots\ldots (15)$$

The application of these formulæ to ocular calculations follows in the succeeding paragraphs.

18. *Schematic eye No. A*

This is calculated for an axial length of 22.2 mms. The data used is as follows (see Fig. 5) :—

r_1 = radius of curvature of cornea, $S_1 = 8$ mms.
r_2 = radius of anterior surface of crystalline, $S_2 = 10$ mms.
r_3 = radius of posterior surface of crystalline, $S_3 = 6$ mms.
Distance S_1S_2 (from cornea to crystalline) = 3.6 mms.
$S_2S_3 = t$, the thickness of crystalline = 3.6 mms.
$n_1 = 1$ $n_2 = 1.333$ $n_3 = 1.45$ $n_4 = 1.333$.

19. *Optics of the cornea*

$$F_A = \text{anterior focal length} = \frac{n_1 r_1}{n_2 - n_1} = 24 \text{ mms.}$$

$$F_B = \text{posterior focal length} = \frac{n_2 r_1}{n_2 - n_1} = 32 \text{ mms.}$$

$$\frac{F_A}{F_B} = \frac{24}{32} = \frac{1}{1.333} = \frac{3}{4} = \frac{n_1}{n_2}$$

20. *Optics of the crystalline lens*

The index is presumed to be 1.45 and the lens is situated with media on both sides of index practically equal to 1.333. The relative index, n_r, of the crystalline is

$$n_r = \frac{n_3}{n_2} = \frac{1.45}{1.333} = 1.087$$

Equation (12) reads: $Q = r_1 + r_2 - \frac{t(n-1)}{n}$, or $Q = 10 + 6 - 3.6$

$$\times \frac{1.087 - 1}{1.087} = 15.72 \text{ mms.}$$

$$K_1 = \frac{r_1 t}{n_r Q} = \frac{10 \times 3.6}{1.087 \times 15.72} = 2.1 \text{ mms.}$$

$$K_2 = \frac{r_2 t}{n_r Q} = \frac{6 \times 3.6}{1.087 \times 15.72} = 1.26 \text{ mms.}$$

The anterior and posterior focal lengths are equal, since the aqueous and vitreous have the same indices, and will be represented by F_C.

$$F_E = F_C = \frac{r_1 r_2}{(n_r - 1)Q} = \frac{10 \times 6}{0.087 \times 15.72} = 43.86 \text{ mms.}$$

The distance $K_1K_2 = T_C$ of the lens. Hence

$$T_C = 3.6 - (2.1 + 1.26) = 0.24 \text{ mm.}$$

21. *Optics of the combined cornea and crystalline lens*

Having thus considered the optics of the crystalline lens and having deduced mathematical expressions representing the focal length, F_E, of the reduced or thin lens equivalent, the two optical systems *A* and *L* (Fig. 9) representing the cornea and crystalline may be combined and the anterior, F_1, and posterior, F_2, focal lengths and other optical constants derived from the formulæ which hold for thin lenses separated by an interval. If we have two thin lenses of focal lengths F_1 and F_2 separated by a distance, *d*, from center to center, the following expressions are valid:

F_E = equivalent focal length of combined lenses.

$$= \frac{F_1F_2}{F_1 + F_2 - d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

K_2 = distance of the second equivalent point from the second lens.

$$= \frac{F_2d}{F_1 + F_2 - d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

K_1 = distance of the first equivalent point from the first lens.

$$= \frac{F_1d}{F_1 + F_2 - d}$$

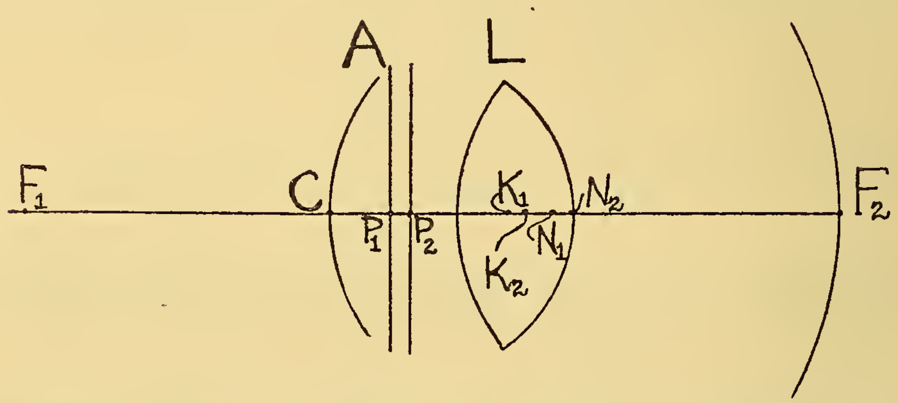


Fig. 9.—The Combined System of the Cornea and the Lens.

and t = equivalent thickness or optic interval.

$$= d - (K_1 + K_2) = \frac{d_2}{F_1 + F_2 - d} \quad \ldots\ldots\ldots\ldots (18)$$

The distance K_1 is measured backwards from the first lens and K_2 forward from the second lens; that is, in each case toward the other lens. This means that the algebraic signs attached to these various quantities will be positive if they are to be measured in the directions specified above.

22. Combining the two systems *A* and *L* as diagrammed in Fig. 9, the distance between the adjacent principal points of *A* and *L* is $CK_1 = d$, or

$$d = CK_1 = 3.6 + 2.1 = 5.7 \text{ mms.}$$

We have previously represented the posterior focal length of the cornea by F_B and the anterior and posterior focal lengths of the crystalline lens, since they are equal, by F_C.

Then $F_B + F_C - d \backsimeq F_1 + F_2 - d$ (equation 16). For the sake of convenience let

$$F_B + F_C - d = Q.$$

Then $Q = 32 + 43.86 - 5.7 = 70.16$ mms. The distance of the first principal point, P_1, behind the cornea is

$$P_1 = \frac{F_A \cdot d}{Q} = \frac{24 \times 5.7}{70.16} = 1.95 \text{ mm.}$$

The distance of the second principal point, P_2, in front of K_2, the second equivalent point of the crystalline, is

$$P_2 = \frac{F_C \cdot d}{Q} = \frac{43.86 \times 5.7}{70.16} = 3.56 \text{ mms.}$$

Hence P_2 lies behind the cornea at a distance

$$CP_2 = 3.6 + 3.6 - (1.26 + 3.56) = 2.38 \text{ mms.}$$

The anterior focal length, F_1, of the whole ocular system is

$$F_1 = \frac{F_A \cdot F_C}{Q} = \frac{24 \times 43.86}{70.16} = 15 \text{ mms.}$$

The posterior focal length, F_2, is

$$F_2 = \frac{F_B \cdot F_C}{Q} = \frac{32 \times 43.86}{70.16} = 20 \text{ mms.}$$

The distance, T, between the principal points is

$$T = P_2 - P_1 = 2.38 - 1.95 = 0.43 \text{ mm.}$$

The nodal points, N_1 and N_2, are found as follows:—the distance N_1P_1 or N_2P_2 is equal to $F_2 - F_1 = 5$ mms. Hence

$$CN_1 = 1.95 + 5 = 6.95 \text{ mms.}$$
$$CN_2 = 2.38 + 5 = 7.38 \text{ mms.}$$

23. *Schematic eye No. (B)*

This will be calculated for an axial length of 24 mms. All the data correspond with schematic eye No. (A) except that n_c, the index of the crystalline, is 1.418. The cornea is as previously calculated.

In the case of the crystalline $n_r = \frac{1.418}{1.333} = 1.0635$. For the crystalline we obtain $K_1 = 2.15$ mms., $K_2 = 1.28$ mms., $F_C = 60$ mms. and $T_C = 0.17$ mm. Combining the two systems, the corneal and lenticular, the approximate values for this eye are

$F_1 = 16.7$ mms. $P_1 = 1.6$ mm. $N_1 = 7.16$ mms.
$F_2 = 22.26$ mms. $P_2 = 1.92$ mm. $N_2 = 7.48$ mms.
$T = 0.32$ mm. $F_2 - F_1 = 5.56$ mms.

24. *The reduced eye*

For convenience in calculations, the eye may be reduced to a single refracting surface so that there is only one refracting surface and medium, the cornea, which is at the ideal refracting plane or principal plane of the schematic eye. This special case has been discussed, with examples, under the caption "Refraction at Curved Surfaces with Applications to Ocular Calculations."

In closing this discussion of the optical system of the eye it is pertinent to add for reference the following table. Other tables, based upon slightly different data, may be found in the works of von Helmholtz, Donders and Tscherning.

OPTICAL CONSTANTS OF THE EYE

	Calculated Eye No. (A)	Calculated Eye No. (B)
Cornea		
Thickness	1 mm.	......
Index	1.333	......
Radius	8 mm.	......
Principal point at cornea.......	......	......
Nodal point behind it...........	8 mm.	......
Anterior focal length...........	24 mm.	......
Posterior focal length..........	32 mm.	......
Anterior focal power...........	42 D.	......
Posterior focal opwer...........	31 D.	......
Index of aqueous and vitreous.....	1.333	......

	Calculated Eye No. (A)	Calculated Eye No. (B)
Crystalline		
Thickness	3.6 mm.	......
Computed index	1.45	1.418
Relative index	1.089	1.0635
Anterior radius	10 mm.	......
Posterior radius	6 mm.	......
First equivalent point	2.1 mm.	2.15 mm.
Second equivalent point	1.26 mm.	1.28 mm.
Focal length	43.86 mm.	60 mm.
Focal power	23 D.	16 D.
The Schematic Eye		
Length of optic axis	22.2 mm.	24 mm.
Principal point	2.2 mm.	1.75 mm.
Nodal point	7.2 mm.	7.5 mm.
Principal point to nodal point	5 mm.	5.75 mm.
Anterior focal length	15 mm.	16.5 mm.
Posterior focal length	20 mm.	22.25 mm.
Anterior focal power	66.66 D.	60 D.
Posterior focal power	50 D.	45 D.
Cornea to front of crystalline	3.6 mm.	......
Cornea to back of crystalline	7.2 mm.	......
Cornea to center of rotation	13.2 mm.	14 mm.
Nodal point to center of rotation	6 mm.	6.5 mm.
Retina to center of rotation	9 mm.	10 mm.

IV. EXPERIMENTAL DETERMINATIONS OF THE OPTIC CONSTANTS OF THE EYE

25. Tscherning in his *Physiologic Optics,* gives the following brief table showing the constants of an eye measured as accurately as possible; some of the methods which he employed will be discussed elsewhere in this article.

Optic Constants of the Eye

Position of	anterior surface of the cornea	= 0.
	posterior surface of the cornea	= 1.15 mm.
	anterior surface of the crystalline	= 3.54 mms.
	posterior surface of the crystalline	= 7.60 mms.
Radius of	anterior surface of the cornea	= 7.98 mms.
	posterior surface of the cornea	= 6.22 mms.
	anterior surface of the crystalline	= 10.20 mms.
	posterior surface of the crystalline	= 6.17 mms.

The accepted values of the indices are:—index of air = 1, cornea = 1.377, aqueous = 1.3365, total index of the crystalline lens = 1.42, vitreous = 1.3365. Tscherning's data give for the *thickness of the crystalline lens* the value 7.60 — 3.54 mms. = 4.06 mms., while all the computations which we have made have been on the basis of von Helmholtz's determination of 3.60 mms. Tscherning's opinion is that the Helmholtz value is too small a number to be considered an average. According to Merkel it is 3.7 mms. The lack of agreement between these figures is doubtless due, in part at least, to the fact that, on account of the difficulty of observation, the number of individuals examined by any single investigator has not been large enough to establish a correct average. A thickness of 4 mms., as given by Listing, may be accepted as the normal standard.

26. A reference to the table of optical constants will disclose the fact that the *anterior surface of the cornea* is the most effective of the refracting media of the eye. The form of this surface has been most carefully studied by numerous investigators since the introduction of ophthalmometry by Helmholtz. A measurement of the cornea is laborious with Helmholtz's device, but the invention of the Javal-Schiötz instrument has made clinically possible that which was but previously essentially a laboratory experiment. Prior to the invention of the ophthalmometer it was customary to regard the normal cornea as the small end of an ellipsoid of revolution turning about the long axis. The early measurements showed that the cornea had a greater curvature at the center than at the periphery. But by use of the Javal-Schiötz ophthalmometer it was found that the cornea did not, as a whole, conform to any symmetrical surface. The conclusions reached by Sulzer are briefly as follows:—(1) The central region of the normal cornea differs but little from the segment of a sphere. (2) At a distance of about 2 mms. from the point of intersection of the visual line with the cornea the curvature begins to abruptly diminish. From this point on to the periphery the corneal surface shows a progressively decreasing curvature. (3) Whether we regard the point of intersection of the visual line with the corneal surface or the point of maximum curvature as representing the center of the cornea, the curvature does not diminish proportionately to the distance from this center. The average radius of curvature of the central portion of the anterior surface of the cornea is 7.829 mms., according to Helmholtz. Other averages do not differ much from this and a radius of 7.8 mms. is commonly accepted as the standard for the normal eye, representing a dioptric value of 43 D. approximately. From the researches of Schiötz on some 969 eyes we learn that the

radii may vary from 7.2 mms. to 8.6 mms. Examinations made by Tscherning on emmetropes showed that 35 per cent. of all such persons examined had a corneal radius of curvature of approximately 7.66 mms.

27. The *curvature of the posterior surface of the cornea* follows, in general, that of the anterior surface but approximates more closely to the spherical form. The average radius of curvature of the posterior surface of the cornea, which was first determined by Tscherning optically in a living eye by the use of the ophthalmophakometer, is given by him as 6.22 mms. These methods will be described and discussed under a separate caption farther along herein.

28. The refracting power of the cornea is about two and a half times greater than that of the crystalline lens. The sum of their refracting powers is not far from the total refracting power of the eye because the nodal points of the cornea, situated practically at its radius of curvature, some 8 mms. back of the anterior surface of the cornea, are very close to the nodal points of the lens. The following table shows the refracting powers of each of the surfaces:—

Anterior surface of the cornea	= + 47.24 D
Posterior surface of the cornea	= — 4.73 D
Anterior surface of the lens	= + 6.13 D
Posterior surface of the lens	= + 9.53 D
Total	= + 58.17 D

We see that the value of the posterior surface of the cornea is negative and almost equal in power to the anterior surface of the crystalline. But little error is made, however, by supposing that the substance of the cornea does not exist. By elimination of the negative effect of the posterior corneal surface the total refraction of the cornea would increase, but the power of the anterior surface diminishes nearly as much since we replace the index of the cornea by the weaker index of the aqueous. This simplification, neglecting the posterior corneal surface, gives the refracting power as 40.98 D instead of 42.16 D, which is an error of 1.2 D or about $1/50$ of the total power of the eye.

The following data are taken from Donders's *Accommodation and Refraction of the Eye,* as showing the influence of age and sex upon the radius of the anterior surface of the cornea:

(1) Radius in the line of vision—

	Maximum	Minimum	Average
In men	8.396	7.28	7.858 mms.
In women	8.487	7.115	7.799 mms.

(2) As to the influence of time of life—

In 79 men, average radius	= 7.858 mms.
In 20 men, under 20 years, average	= 7.932 mms.
In 51 men, under 40 years, average	= 7.882 mms.
In 28 men, above 40 years, average	= 7.819 mms.
In 28 men, above 60 years, average	= 7.809 mms.
In 38 women, average radius	= 7.799 mms.
In 6 women, under 20 years, average	= 7.720 mms.
In 22 women, under 40 years, average	= 7.799 mms.
In 16 women, above 40 years, average	= 7.799 mms.

29. The *curvatures of the crystalline lens* have been measured by the ophthalmophakometer. Much greater difficulty has been experienced in determining the exact forms of the surfaces of the crystalline lens than in the case of the cornea; but measurements made by Donders, Helmholtz, Knapp, Tscherning and others, show that when the ciliary muscle is relaxed the central portion of the anterior surface does not differ appreciably from the segment of a sphere having a radius of 10 millimeters and that the corresponding portion of the posterior surface equally resembles a segment of a sphere having a radius of curvature of 6 mms. Measurements made on 9 eyes by Stadfeldt and on 86 eyes by Awerbach show

	Stadfeldt	Awerbach
Anterior radius	10.95 mms.	10.43 mms.
Posterior radius	6.0 mms.	6.1 mms.
Depth of anterior chamber......	3.85 mms.	3.59 mms.
Thickness of the lens...........	3.65 mms.	3.90 mms.

The curves in Fig. 10 are taken as a resumé of the work of Awerbach. The number of eyes examined was not sufficiently large to be productive of regular or "smooth" curves but they do indicate admirably the variations in the different dimensions of the optical apparatus of the eye. The abscissæ indicate the dimensions in millimeters; the ordinates show the number of eyes examined which gave that dimension. In these diagrams R_1 represents the radius of the cornea, R_3 the radius of the anterior surface and R_4 the radius of the posterior surface of the lens, P the depth of the anterior chamber and E the thickness of the lens. It is probable, as Tscherning remarks with reference to these curves, that the differences of the dimensions of the optical portions of the eye correspond to analogous differences in other portions of the eyeball. One would expect to find larger radii of curvature of cornea and crystalline in large eyes. For a radius of curvature of the anterior surface of the lens lying between 9 and 10 millimeters, there should correspondingly be an average corneal radius of 7.9 millimeters. In like manner one finds that large radii of curvature are correlated in turn with larger dimensions of the anterior chamber and of the crystalline. This is the significance of the double set of abscissæ attached to several of the diagrams given in Fig. 10.

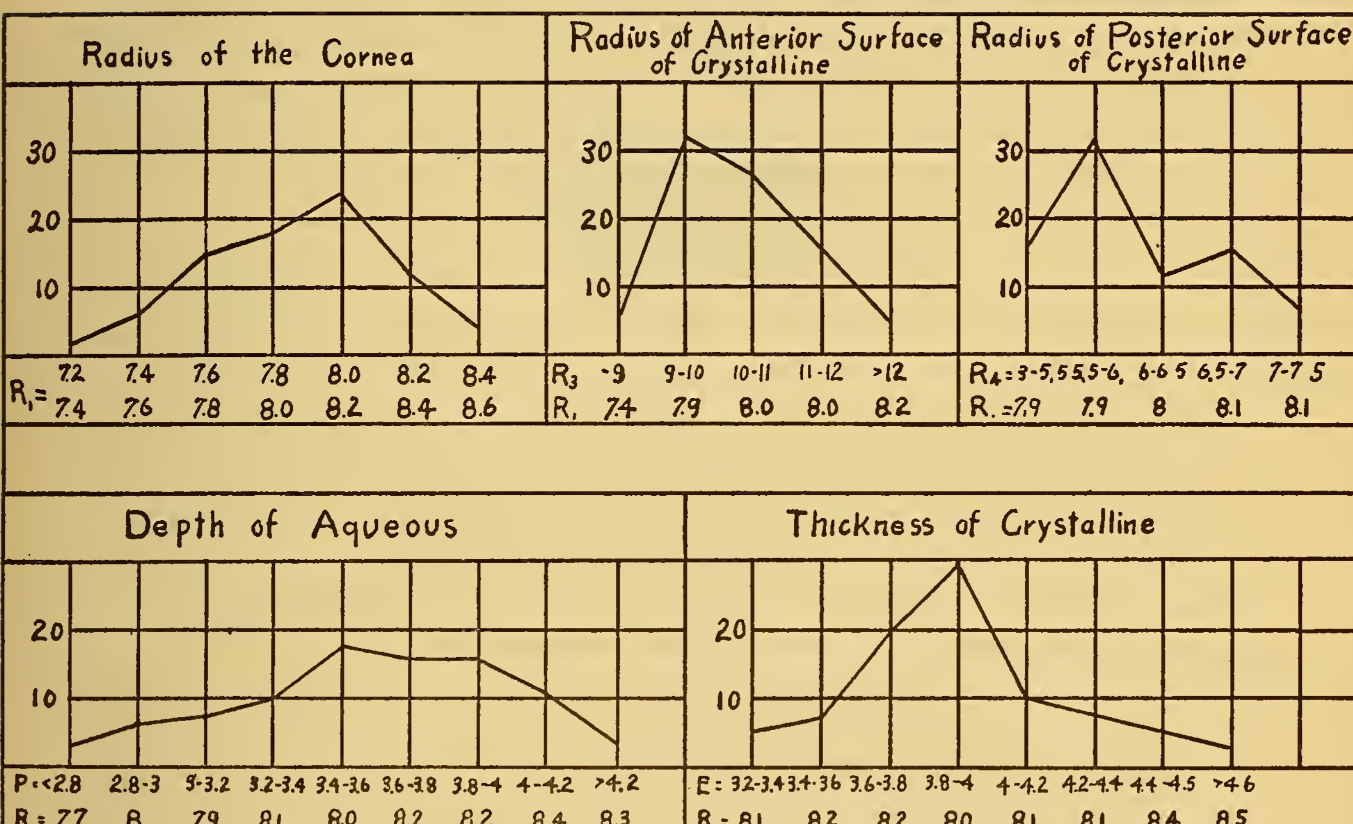


Fig. 10.—Curves Showing the Variations of the Dimensions of the Optical System of the Eye. (After Awerbach.)

Age and sex cause considerable variation in some of the radii of curvature of the various surfaces. The eye of the new-born child is much smaller than that of the adult, being about 17 millimeters axial length instead of 24 millimeters, and hence we might expect to find the curvatures of all surfaces increased in the same proportion. This is not so: Holth, by the clever device of injecting a solution of gelatine under the cornea as the aqueous was drained, measuring the anterior curvature and then carefully removing the cornea and thus, by contact of the gelatine with the inner side of the cornea, having a curve exactly mating it, obtained

	Man 44 yrs.	Woman 76 yrs.	Child 2 months
R_1	7.75 mms.	7.86 mms.	7.41 mms.
R_2	6.58 mms.	6.25 mms.	6.98 mms.

Axenfeldt's results are concordant with those of Holth. The dioptric power of the cornea varies from 40 to 47 dioptries and the values which are found in very young children are near to this upper limit. Compensation for the diminution in the length of an infant's eye is made by the crystalline. From the researches of Stadfeldt we learn that the thickness of the crystalline lens of a new-born child is the same as that of an adult, but the diameter is about 6 mms. instead of 8 or 9 mms., hence it follows that the curvatures of the crystalline surfaces are greater than for an adult. Stadfeldt's results are:—

	Anterior Radius = R_3	Posterior Radius = R_4	Thick-ness	Diameter
Adult	11 mms.	6 mms.	3.6 mms.	8-9 mms.
New-born ...	4.5 mms.	4 mms.	3.9 mms.	6 mms.

If the index of the child's crystalline is the same as that of the adult, its power would be nearly twice that of the adult or 32 D. power instead of approximately 16 D., and the crystalline refracting power would approach that of the cornea. We may reasonably doubt whether the index of the new-born crystalline is that of the adult eye since the hard nucleus does not exist in early childhood.

30. The *refractive index of the cornea,* which cannot be measured directly on the living eye, has a value of 1.377 as determined by Aubert, Matthiessen and Tscherning. The *indices* of the normal *aqueous* and *vitreous* are very exactly known and can be readily determined by various refractometer methods. Each has a value of 1.3365. This same value was also assumed by Helmholtz in his schematic eye as the index of the cornea. Fleischer gives 1.3373.

31. Less is known about the *index* of the *crystalline lens* than of any of the optical constants of the eye. The crystalline lens is not a homogeneous substance. Its refractive index gradually increases from the cortex to the nucleus. The curvature of the layers also diminishes as we pass from the center out to the peripheral layers. Each layer takes the form of a meniscus, the concavity of which is greater than the convexity. This conclusion follows both from anatomical and optical observations.

After death there is frequently produced a differentiation between the cortical and nuclear masses probably caused by the soaking up of water by the superficial parts. In consequence of this there are five, instead of three, images of Purkinjé. The three reflections of, for example, a candle come from the convex cornea, the convex anterior surface of the crystalline and the concave posterior surface of the crystalline, all being regarded as reflecting or mirror surfaces. The two extra images come from the convex anterior surface of the nucleus and the concave posterior surface of the nucleus. Fig. 11 (A) gives the Purkinjé images of an ox eye (dead). The positions of these images indicate that the curvature of the surfaces of the nucleus is considerably greater than that of the true crystalline surfaces. In Fig. 11 (A), (a) is the image of the cornea, (b) image of anterior

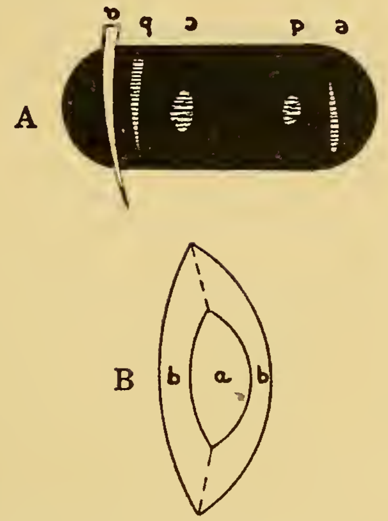


Fig. 11.—(A) Images of Purkinjé of the Eye of an Ox (dead)—Flames of a Candle. (After Tscherning.)

a. Image of cornea; *b.* image of the anterior surface of the crystalline lens; *c.* image of the anterior surface of the nucleus; *d.* image of the posterior surface of the nucleus; *e.* image of the posterior surface of the crystalline lens.

— (B) Illustrating the Curvature of the Nucleus and Cortical Parts of the Eye.

surface of the crystalline, (c) image of anterior surface of the nucleus, (d) image of posterior surface of the nucleus, (e) image of posterior surface of the crystalline lens. Fig. 11 (B) roughly diagrams the relative curvatures of nuclear and superficial layers. Demicheri has described alterations of the crystalline lens giving four images (since the images from the posterior nucleus and posterior crystalline surface coincide) which he has called *faux lenticoné* and giving the same phenomena which one observes in keratoconus.

The crystalline lens is, therefore, not homogeneous but is made up of a great number of superimposed layers whose curvatures increase in passing from the surface toward the center so that the nucleus is nearly spherical. The index of refraction also increases from without inward, so that the crystalline lens may be considered as composed of so many divergent menisci increasing in power proportionately to their proximity to the nucleus, which in turn is a convex lens of a very short radius and high index of refraction. The effect of such a structure of the lens is to diminish spherical aberration, i. e., to permit of the formation of distinct images even by rays which enter the eye at a considerable angle with the optic axis. A homogeneous lens brings to a single focus only such rays as pass through near its center. The lamellar structure of the crystalline lens peculiarly adapts it for indirect vision in which retinal images are produced by the rays which form a considerable angle with the axis of the eye.

32. The peculiar structure of the crystalline lens has still another effect, for it renders the dioptric power of this lens greater than if it were homogeneous. Indeed, its *total index*, which is to say the index of an imaginary lens having the same form and same focal length as the crystalline lens, is greater than not only the mean value of the crystalline layers but even that of the nucleus. This seems paradoxical, but it is easy to convince one's self of its truth. Suppose the crystalline lens divided into two parts, the cortical and nuclear, as depicted in Fig. 11 (B), and suppose the index uniform in each part but greater in the nucleus. The cortical menisci, being divergent in form and power, neutralize a part of the positive refractive power of the nucleus. This neutralization is greater in proportion as the index of refraction of the cortical layers increases, because the refractive powers of the menisci increase in like proportion. Hence the refractive power of the crystalline lens would be less if the index of refraction of the cortical laminæ was equal to that of the nucleus.

The question of the crystalline refraction and its total index of refraction is one of the most complicated problems in ocular dioptrics. Since the indices increase in value from without toward the center, the problem becomes analogous to that of the refraction of the atmosphere in which the indices change from layer to layer.

33. Thomas Young was the first to establish the total index of the crystalline. He measured the index of the center of the nucleus and placed it at 1.0588 with respect to the aqueous or 1.412 with reference to air. In order to calculate the index he assumed a crystalline of spherical shape of radius R with a nuclear part having a radius r_a and index equal to n_o, which index decreased in value toward the periphery in the inverse ratio to some power, k, of its distance from the center. The index, n_1, at any point is

$$n_1 = n_o \left(\frac{r_a}{r} \right)^k$$

The total index, N, was calculated from the formula

$$N = n_0 \frac{1 - K}{\frac{n_0}{n_1} - K\left(\frac{n_0}{n_1}\right)^{\frac{1}{K}}}$$

Since the values of the central and peripheral indices, n_1 and n_0 respectively, are known and their radii R and r_a then K can be found as

$$\left(\frac{n_0}{n_1}\right)^{\frac{1}{K}} = \frac{R_a}{R} = 4$$

The ratio $\frac{n_0}{n_1} = 1.0164$ and therefore

$$\sqrt[K]{1.0164} = 4 \text{ and } K = \frac{\log. 1.0164}{\log. 4}$$

This value, when substituted in the equation for N, gives the value of $N = 1.075$ relative to the aqueous or 1.436 as the total index with reference to air. This optical procedure is an interesting application of the mathematical methods involved in the Newtonian theory of attraction. Matthiessen, in his work, deduced a theorem known as the law of Matthiessen, which states that the total index may be found by taking the difference between twice the central index and the cortical or, in terms of the notation as used in this paragraph,

$$N = 2\,n_0 - n_1$$

This gives a total index of 1.437. Stadfeldt and Tscherning determined the total index of the crystalline by entirely different methods and arrived at a value of 1.42. This value is considered rather low by most authorities. Stadfeldt's results on 11 eyes, enucleated within thirty-six hours after death, show the following data on the crystalline lens.

Sex	Age	Thickness (Lens)	Anterior Radius = R_1	Posterior Radius = R_2	Focal Length (mms.)	Total Index = N
M	45	3.67	10.89	6.49	51.933	1.4390
M	45	3.78	11.25	6.43	50.353	1.4430
M	50	4.06	10.55	6.404	54.482	1.4322
M	50	3.99	10.714	6.193	52.618	1.4341
M	40	3.85	9.782	6.428	50.138	1.4371
M	40	3.78	11.175	6.424	54.61	1.4339
F	31	4.24	10.887	5.720	48.832	1.4370
F	25	3.99	8.437	5.819	41.994	1.4434
F	25	3.64	8.654	5.72	43.014	1.4410
M	32	3.64	12.981	6.25	59.44	1.4297
M	32	3.50	12.981	6.08	55.839	1.4339
	Average	3.83	10.75	6.18	51.21	1.4368

W. Krause found as the index of the external layer 1.4053, for the intermediate 1.4294 and for the nucleus 1.4541. Woinow gives 1.3932, 1.4199 and 1.4315 respectively.

V. OCULAR CATOPTRICS AND THE MATHEMATICAL PRINCIPLES OF OPHTHALMOMETRY AND OPHTHALMOPHAKOMETRY AND THEIR APPLICATIONS

34. Ophthalmometry, or keratometry, is the measurement of corneal curvature and particularly, in practice, of the determination of the differences in the curves in order to determine the amount of corneal astigmatism.

The cornea, having a clear reflecting surface, acts as a convex mirror and the laws governing the conjugacy of foci of mirrors may be applied here.

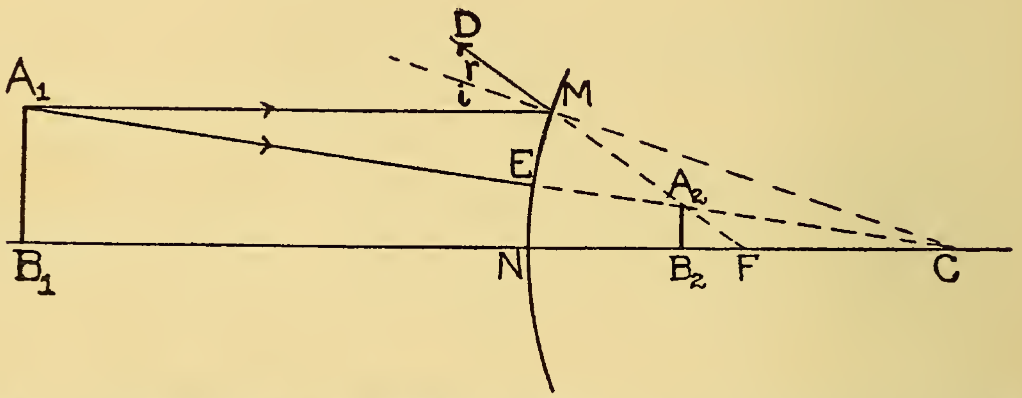


Fig. 12.—Illustrative of the Laws of Reflection and Formation of Images in Convex Mirrors.

35. Let MN be the reflecting surface of a convex mirror. Let A_1B_1 be the object. In order to find the image position we proceed as in Fig. 12. CN represents the radius of curvature; the point N is the pole.

We proceed as follows:—(1) A ray of light, A_1M, parallel to the principal axis, B_1C, will strike the convex mirror at the point M. According to the laws of reflection at plane surfaces, considering an infinitesimal unit plane at M, the angles of incidence (i) and reflection (r) are equal and the ray after reflection travels in the direction DM. This ray when projected backwards will cut the principal axis at a point F, half way between the pole N and the center C, known as the principal focus.

(2) A ray A_1E, directed toward the center of curvature, will proceed normally to the spherical surface and will, therefore, after reflection travel back upon itself. The return ray, EA_1, when projected backward, will necessarily cross the principal axis at C.

(3) The projections of the two rays DM and EA_1, which are the paths of the incident rays A_1M and A_1E proceeding from A_1, intersect at A_2 behind the mirror. This locates the image of the object point A_1 at A_2; the image is virtual, erect and smaller than the object. The rays, MD and EA_1, actually reflected, are divergent in the initial medium.

$$\text{Let } FC = F = \frac{R}{2} = \text{principal focal length}$$

$$NB_2 = f_2 = \text{image distance}$$
$$NB_1 = f_1 = \text{object distance}$$
$$B_2C = NC - NB_2 = R - f_2$$

The triangles A_1CB_1 and A_2CB_2 are similar, hence

$$\frac{O}{I} = \frac{A_1B_1}{A_2B_2} = \frac{B_1C}{B_2C} = \frac{f_1 + R}{R - f_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

From triangles MNF (considering MN as a straight line, which is permissible when the aperture of MN is small) and A_2B_2F,

$$\frac{O}{I} = \frac{MN}{A_2B_2} = \frac{A_1B_1}{A_2B_2} = \frac{NF}{B_2F} = \frac{F}{F - f_2} \quad \ldots\ldots\ldots\ldots (2)$$

Hence $\dfrac{f_1 + R}{R - f_2} = \dfrac{F}{F - f_2}$ or, by clearing,

$$f_1F - f_1f_2 - Rf_2 = - f_2F.$$

But $Rf_2 = 2\,Ff_2$, hence

$$f_1F - f_1f_2 = Ff_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Dividing all terms of equation (3) by f_1f_2F, we obtain

$$\frac{1}{f_2} - \frac{1}{F} = \frac{1}{f_1} \text{ or}$$

$$\frac{1}{f_1} - \frac{1}{f_2} = \frac{1}{F} = \frac{2}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

This is the fundamental law connecting the conjugate foci f_1 and f_2 in the case of reflection. The general form of this equation is

$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2}{R}$; in the case of a convex mirror f_2 and R are negative quantities, as to direction only, since they are measured from the pole in a direction opposite to that in which the object distance f_1 is measured.

Likewise it can be demonstrated that

$$\frac{A_1B_1}{A_2B_2} = \frac{O}{I} = \frac{f_1}{f_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

If *MN* be considered the cornea, which has a large curvature, and B_1N, the object distance, be great in comparison with B_2N, the image distance, we may then assume without appreciable error that the image A_2B_2 will be formed at the principal focus, F. Hence we may write

$$f_2 = F = \frac{R}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Therefore, the equation (4), $\frac{1}{f_1} - \frac{1}{f_2} = \frac{-2}{R}$, becomes, upon substitution of the value of f_2 from equation (6), $\frac{1}{f_1} - \frac{2}{R} = \frac{-2}{R}$ or f_1 is infinity. This point is inserted at this place to call the attention of the reader to the fact that infinity, optically considered, is after all nothing but a relative quantity, that is to say—a ratio.

A substitution of f_2 (equation 6) in equation (5) gives

$$\frac{R}{2f_1} = \frac{I}{O} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

whence $R = \frac{2f_1I}{O} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$

The exact formula, which can be derived from the preceding expressions, is

$$R = \frac{2f_1I - R}{O}$$

36. Equation (8) is the *fundamental equation of ophthalmometry;* one must of necessity then measure the distance of the object and its size before the curvature of the cornea can be determined. The size of the image is the difficult measurement to obtain. Two luminous objects, called *mires,* are used in an ophthalmometer and the distance between them is a measurable quantity. The image, then, is the distance separating the images of the mires. Physicists use a micrometer placed at the focus of the objective of a telescope with which the image is observed. The objective forms an image on the micrometer, the graduations of which permit the size of the image to be read directly by observing it through the eyepiece. This method cannot be employed with the human eye because the observed eye cannot be kept absolutely stationary. To obviate this defect, Helmholtz intro-

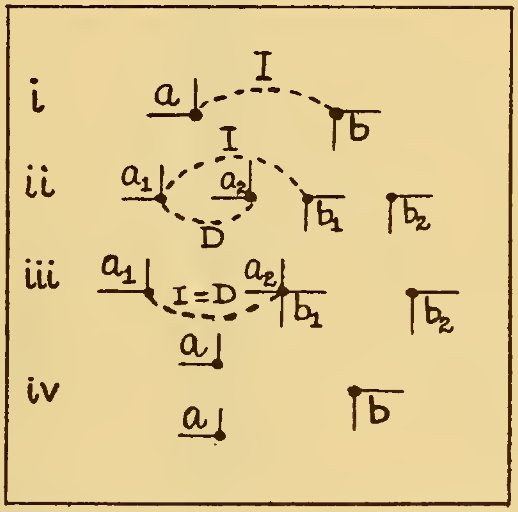


Fig. 13.—Illustrating the Principle, of Doubling (dédoublement) as Applicable to Ophthalmometry.

duced the principle of doubling (dédoublement); Thomas Young (1801) had, however, already made use of this same method, borrowed from astromony, for the same purpose. Suppose, therefore, that we wish to measure a distance I separating the two points a and b and that we have some process (such, for example, as plane parallel plates of glass placed obliquely behind the objective but in a symmetrical manner in relation to the axis of the telescope), which permits us to see everything doubled at a certain distance D. In Fig. 13 (i) let a and b be two such points originally. By the doubling process we shall see four points, Fig. 13 (ii) a_1 and a_2, b_1 and b_2, and the distance a_1a_2 would be equal to b_1b_2 and to D, while the distance $a_1b_1 = a_2b_2 = I$. Suppose we now vary the doubling; for example, by changing the inclination of the plates obliquely placed behind the objective of a telescope. By increasing the doubling it will be possible to cause a_2

and b_1 to coincide, Fig. 13 (iii), and at that instant $I = D$. If we know the amount of doubling we shall then have measured $I = ab$. Instead of causing the doubling to vary it is possible to make I vary; this is accomplished by varying the distance between mires, the doubling device remaining constant in its value.

37. It is generally desirable to employ a certain magnification in order to make measurements easily and with accuracy. This is accomplished by a telescope. Essentially, then, an ophthalmometer consists of a telescope, the luminous mires and the doubling apparatus. The cornea under observation is approximately at the one symmetrical plane of the objective of the telescope and its real image at the conju-

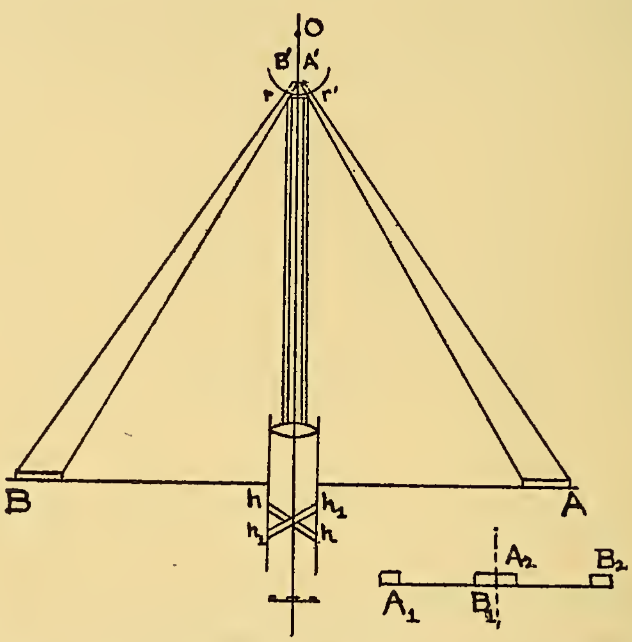


Fig. 14.—Optical Principles of the Ophthalmometer.

gate point for the objective; this first image is at the focal plane of the eye-piece which furnishes the observer with a magnified inverted image of the corneal reflections. The doubling apparatus is placed either between the components of the objective, as in the Javal instrument, or in the convergent light from the latter. In the modern form of instrument of Javal and Schiötz two luminous mires are made to slide along a metallic arc by means of a revolving drum device and it is the distance between them which serves as the object. By moving the mires on the arc the size of the object is made to vary until it corresponds to the constant doubling value of the prism device. The doubling device is a Wollaston prism composed of two rectangular quartz prisms which are cemented together so as to form a single, thick, plane parallel plate; the two prisms are cut from quartz so that the apex of one is parallel to the optic axis of the crystal and the other perpendicular to it. The Wollaston prism is so placed as to double in a direction exactly parallel to the plane of the arc. When the second and third images of the four produced by the doubling prism are in contact, the radius and the power of the cornea can be read off on the scale attached to the disc of the instrument. A diagrammatic scheme of the essential principles of ophthalmometry is shown in Fig. 14. Fig. 15 gives the Javal-Schiötz doubling device.

38. We have stated that the disc or the arc is graduated in diopters

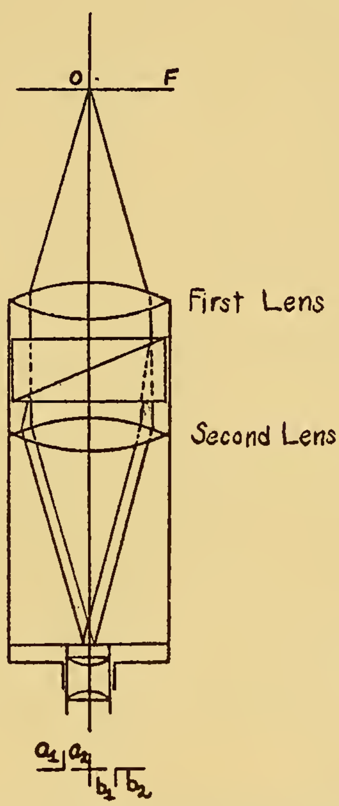


Fig. 15.—Optical Apparatus of Javal's Ophthalmometer (with contact of doubled images in lower diagram.)

in such a manner that one degree corresponds to one diopter. This can be explained and the method of computing the amount of doubling produced by the prism calculated. Javal and Schiötz took as the index of the cornea and aqueous the value 1.3375. The focal length of the cornea is

$$F_C = \frac{R}{n - 1} = \frac{R}{1.3375 - 1}$$

The refracting power, D_C, is $D_C = \frac{1}{F_C} = \frac{n-1}{R}$ or, expressing

R_C in millimeters,

$$D_C = \frac{337.5}{R_C} \text{ or } R_C = \frac{337.5}{D_C}$$

By means of this formula one can calculate the relations between refracting powers of the cornea and the corresponding radii expressed in millimeters. Some of the values thus obtained are:—

Refraction (Diopters)	Radius (Millimeters)
50	6.75
48	7.03
46	7.34
45	7.50
43	7.85
41	8.23
39	8.65
38	8.89

The general ophthalmometric formula is

$$\frac{O}{I} = \frac{2f_1}{R_C} \quad \ldots\ldots (1)$$

and since $R_C = \dfrac{337.5}{D_C}$ .... (2),

therefore, $O = \dfrac{2f_1 D_C I}{337.5}$ .... (3),

in which I designates the image which, at the moment of contact, is equal to the doubling. This is the condition diagrammed in Fig. 13 (iii). Let a denote the linear length of a degree on the scale of the arc: this can be readily calculated from the relation that the radius multiplied by the angle at the center, expressed in radians, is equal to the arc. If this length must correspond to one diopter, the object which corresponds to the image I must have the linear size $D_C \cdot a$. Hence

$$D_C \cdot a = \frac{2f_1 \cdot D_C \cdot I}{337.5}$$

$$\text{or} \quad a = \frac{2f_1 \cdot I}{337.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

But $\quad a = 1°$ in length, hence

$$\frac{1°}{360°} = \frac{a}{2\pi f_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where $2\pi f_1$ represents the circumference of the circle of which the ophthalmometric arc is a portion and a represents an arc corresponding to the linear length of a degree. Therefore, from equations (4) and (5)

$$a = \frac{2\pi f_1}{360} = \frac{2f_1 \cdot I}{337.5}$$

$$\text{or } I = \pi \frac{337.5}{360} = 2.94 \text{ mms.}$$

In order, therefore, that a degree of the arc may correspond with one dioptry the doubling of the prism must be 2.94 mms. The radius of the arc can be so selected as to give the linear length of a degree any value desired. Suppose one desires this length to be 5 millimeters. Then, since the value of the radius of the arc, f_1, in millimeters, multiplied by the angle in radians is equal to the arc in millimeters and one radian is approximately 57.3°, we have

$$f_1 \times \frac{1}{57.3} = 5 \text{ millimeters}$$

$$\text{or } f_1 = 286.5 \text{ millimeters.}$$

Now suppose a living cornea to be examined. Having focused the telescope, four images are seen, the two central ones being closer together: we regard these two images only, wholly neglecting the outer two. The central images can be made to touch by means of the rack controlling the mires. If the cornea is spherical these images will remain in contact as the telescope and arc are rotated through any meridian. If there is corneal astigmatism, the distance between the two images will vary, as the arc is rotated, on account of the variation in curvature; likewise the images will have an eccentric sliding motion with respect to each other and the central lines of the mires will be broken. Rectangular objects, for instance, will give rectangular images when the plane of deviation of the prism corresponds with either one of the principal meridians. In any oblique meridian, the rectangles will be distorted so as to appear as oblique parallelograms. This phenomenon is due to toric reflection from surfaces having compound curvatures; one of the images will be higher than the other, which accounts for the discontinuity of the middle lines of the mires.

39. *Artificial astigmatism.* A standard reflecting surface comes with many ophthalmometers, used with cylindrical lenses from trial cases to produce artificial astigmatism. A truly astigmatic artificial cornea does not permit of the obtainance of any variation in the amount of astigmatism however. If a convex cylinder is placed with its curved surface in contact with the sphere there is a reduction of the negative reflecting power across the axis, while a concave cylinder similarly placed increases it; in both cases the power parallel to the axis is unchanged. Let R be the reflecting power of the sphere, D the refracting power (on an index of 1.3375) and P the power of the cylinder. Suppose the cylinder to be convex. Then the combined reflecting power of the sphere and cylinder is $R - 2P$ since the focal length of a plano-convex cylinder is zero in one meridian and has a value in a meridian at right angles thereto equal to $2r$, where r is the radius of the curved surface, when the index equals 1.5, hence a reflecting power of $2P$. The refracting power across the axis as compared with the meridian parallel to the axis is $\frac{D(R - 2P)}{R}$, so that the artificial astigmatism produced is:

$$\text{Astigmatism} = \frac{D(R - 2P)}{R} - D = -\frac{2P \cdot D}{R}$$

The normal cornea can be assumed as having a radius of 7.5 mms. The focal length of a mirror of radius 7.5 mms. is one-half the radius or 3.75 mms. Hence the reflecting power of a standard sphere representing the cornea is 266 D. Its refracting power is 45 D. Therefore the ratio of the reflecting to the refracting power is nearly 6 to 1. Hence D = 45 diopters and R = 266 diopters and as a consequence

$$\text{Astigmatism} = \frac{2 \times 45 \times P}{266} = \frac{P}{3}$$

We thus find that the artificial astigmatism produced is approximately one-third the power of the cylinder producing it. Thus, if the cylinder used be + 3 D., the astigmatism produced is + 1 diopter; if P = — 6 D., then the astigmatism created is — 2 diopters; the signs refer only to the character of the astigmatism produced; the addition of plus cylinders produces an artificial hyperopic astigmatism while minus (concave) cylinders produce an artificial myopic astigmia. Furthermore, it will be found that the artificial astigmatism produced by the addition of a cylindrical lens to the standard cornea (of glass) is approximately *one-half* of the power of the cylinder producing it if we make use of the artificial corneas furnished by American manufacfacturers of ophthalmometers. In a preceding paragraph we gave the ratio as one-third instead of one-half. The following abbreviated calculation shows that, while the power of the artificial cornea may be the same, i. e., 45 diopters, the index of glass is approximately 1.5, whereas that of the true cornea is 1.3375. Hence, since D = 45 diopters, n = 1.5 and D = (n — 1) 1000/r (mms.), then r = 500/45 = 11 mms. (approximately). Therefore the reflecting power of this standard cornea is equal to 1000/5.5, or, 180, practically. Hence

$$\text{Astigmatism} = \frac{2 \times 45 \times P}{180} = \frac{P}{2}$$

Three points in practice need mention:—(1) the distance from the mires to the standard reflecting sphere may be neglected as not affecting the calculations, (2) the curved surface of the cylinder must be in contact with the cornea and (3) the ophthalmometer must not be refocused between the first and second positions even though the thickness of the glass along the axis causes a slight blurriness in the image.

40. *Results of the measurements on the human cornea.* Only a very small part of the cornea is used for measurement; likewise the portion upon which measurements are made is at or near the visual line, since the patient is ordinarily directed to look into the telescopic opening unless the peripheral parts of the cornea are desired. In Fig. 16, *AB* represents the distance apart of the mires considered as points; *CN* represents the distance of the mires from the cornea. *F* represents the focal length of the cornea considered as a convex mirror; we shall assume at this point so small an image of *AB* at *F* as to practically amount to a single image point only. The triangles ABF and DEF are similar; hence

$$\frac{AB}{DE} = \frac{CF}{NF}$$

CN in the ordinary ophthalmometer is about 300 millimeters; *NF* for the assumed normal cornea is 3.75 mms.; our previous calculations of 5 mms. as representing the linear value of 1 diopter refracting power and the cornea having a power equal to 45 D., gives *AB* the value of 225 mms. Then $\frac{225}{DE} = \frac{300}{3.75}$ or DE = 2.8 mms. This shows that the images of the mires are formed by reflection from two small parts of the cornea situated about 1.4 mms. from the visual line CNF (Fig. 16).

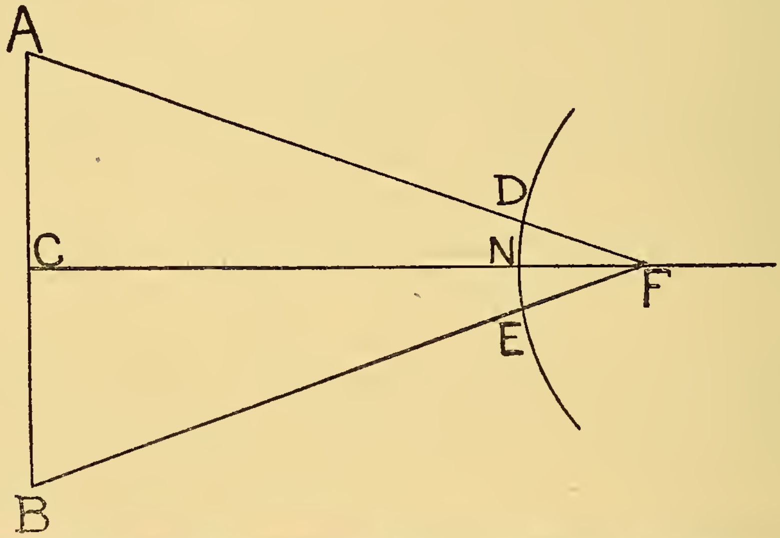


Fig. 16.—Simple Diagram Representing Relative Positions of Mires (A and B), and Cornea (D E) in Ordinary Ophthalmometer.

41. The radius of the cornea at the summit varies between 7 and 8.5 mms. Cases of keratoconus exceed this limit. Tscherning and Bourgeois examined a considerable number of emmetropes and found an average radius of curvature of 7.8 mms. Fig. 17 gives the curve obtained from the researches of these men; the abscissæ indicate the radii of curvature of the cornea in millimeters and the ordinates the number per hundred of emmetropes in whom there was found the radius of curvature specified. The value of R = 7.8 mms. is probably high, for Tscherning states that the persons examined were tall in stature and of large cranial circumference. Likewise Steiger has since found a more manifest relation between the radii of curvature of corneæ and the distance between the eyes.

Discussing corneal curvatures, Edward Jackson *(Ophthalmology,* Vol. XIII) gives the following tabulation:

Corneal Curvatures Among 2,000 Eyes

Radius of Curvature	Number of Eyes	Percentage
Under 6.5 mm..................	6	0.3
6.5 to 7 mm....................	10	0.5
7 to 7.5 mm....................	305	15.25
7.5 to 8 mm....................	1,263	63.15
8 to 8.5 mm....................	409	20.45
8.5 to 9 mm....................	2	0.1
Over 9 mms....................	5	0.25

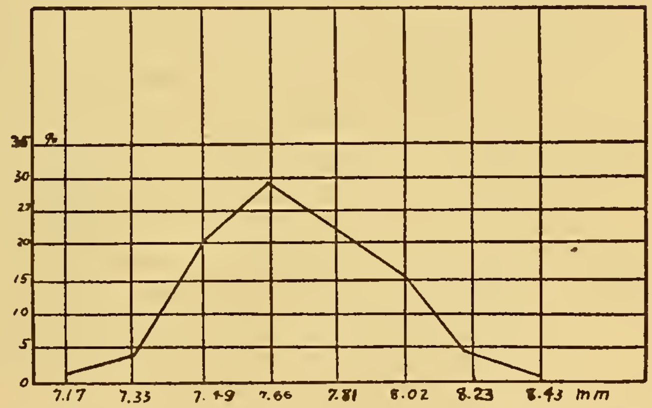


Fig. 17.—Curve Showing Relation between Radii of Curvature of Cornea (in millimeters) and Percentage of Emmetropes in whom the Radius of Curvature in Question Occurs. (After Tscherning.)

It would be an error to presume that one radius rather than another corresponds to emmetropic conditions. It may be safe to say with Tscherning "that in emmetropic eyes there exists a constant relation between the radius of curvature of the cornea and the length of the ocular axis, so that the ocular shell of different emmetropic eyes would always be a reproduction of the same type, a little enlarged or a little diminished." It is still an open question as to whether there exists such a thing as myopic and hyperopic corneal curvatures, although Sulzer presented some evidence in support thereof at the 1896 Congress of the French Society of Ophthalmologists; it doubtless does exist in cases of very high hyperopia which approach microphthalmia. Even in cases of anisometropia, excepting, however, cases of astigmatism, there is generally under a half diopter's difference between the corneal refraction of the two eyes.

42. *The form of the anterior surface of the cornea.* Previous to the invention of the Javal-Schiötz instrument, ophthalmometry was a difficult procedure. By their invention they made the ophthalmometer a clinical instrument. Up to their time but little was known as to the form of the cornea. The ophthalmometer of Helmholtz was a complicated device; one was forced to measure three points on a meridian, one corresponding to a point on the visual axis and the other two situated at some distance from it, there being one on each side of this line. The peripheral rays were found to have a greater radius than those which came from the central portion; hence it was obvious that the cornea could not be assumed to be a portion of a spherical surface.

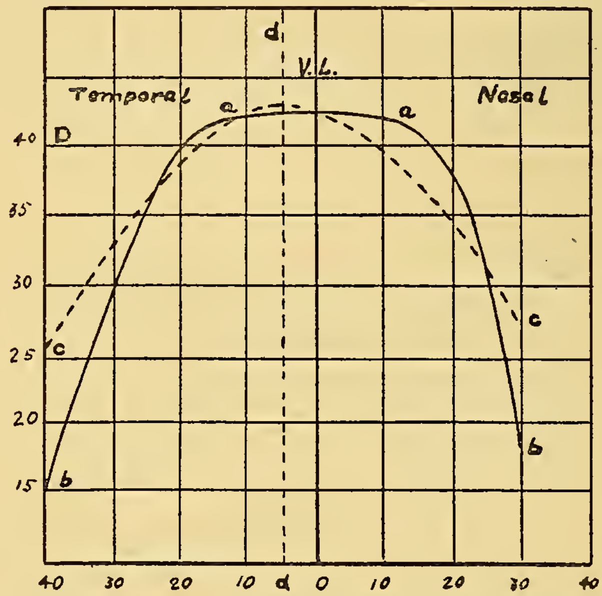


Fig. 18.—Diagram of Corneal Refraction. (After Ericksen.)
The abscissas indicate the distance of the visual line in degrees and the ordinates the corneal refraction in diopters.

As a consequence the curvature of the second degree which approached most closely the observed data was constructed and calculated. Hence there arose the belief that the form of a non-astigmatic cornea was an ellipsoid of revolution around the long axis, which axis departed from the visual line outwardly, forming an angle, known as the angle *alpha* (α), of about 5 degrees with this line. (The angles *alpha, gamma* and *kappa,* so-called, will be discussed elsewhere in this article.) The dotted curve *cc* in Fig. 18, corresponds to an ellipsoid calculated from the three measurements taken at 0° and 25° on the right and left sides of the visual axis: the line *dd* is the axis of this ellipsoid and passes to the temporal side of the visual line, *V.L.*, or the zero degree line by an angular amount of about 5°, representing the angle alpha. The full line curve, *bb,* shows the refraction of the horizontal meridian of a cornea measured in graduations of five degrees. We see that the true form of the cornea differs considerably from the ellipsoid. The method as employed by Javal and Schiötz, which, parenthetically it may be said, can be readily repeated by anyone so desiring, was to divide the keratoscopic disc into 5 degree graduations by narrow concentric rings. After having made the measurements along the visual line when the patient looks at the center of the objective, the measurements are repeated with the observed party fixing upon the 5°, 10° and so forth points, both to the right and left of the central fixation point, the head remaining immovable. Every meridian may be tested in the same manner. Some measurements made in this manner by Sulzer and Eriksen are diagrammed in Fig. 18; these results confirm the statements of Aubert and Matthiessen who made use of the Helmholtz ophthalmometer and affirmed that the cornea could be divided optically into two parts:—(a) a central one (*aa* Fig. 18) approximately spherical and normally chiefly operative in vision and (b) a peripheral portion (*ab* or *ac* Fig. 18) which is much flattened. Eriksen, from the averages of measurements made on 24 eyes, gives the following as the limits of the optic part of the cornea in comparison with its entire value in degrees.

	Optic Portion	Total Cornea
Outwards	16.5°	44.7°
Inwards	14°	40.1°
Above	12.5°	38.5°
Below	13.5°	42.2°

The total of the optic part of the cornea is about 30° horizontally and 25° vertically; the total width of the cornea is about 85° horizontally and 80° vertically.

No axis of symmetry, properly speaking, has been found by any observer. Most of the results, however, show a tendency to symmetry about an axis directed about 5° outward and a little below the visual line.

43. Eriksen tried to obtain information as to the variation of the peripheral radii by examining the form which the image of a white square assumes in the horizontal meridian at different distances from the visual line. These results are shown in Fig. 19, in which the upper numerals indicate angular distances from the visual line; the lower figures show the dioptral refraction. At about 30° from the visual line, the horizontal meridian being under test, the image is horizontally some two and a half times greater than at the center.

At the extreme periphery the image becomes narrower and at 33° is an upright rectangle; at this point double images may be obtained, one of which may be inverted in the horizontal direction indicating that the curvature increases very considerably toward the border and that there may be, and doubtless is in many eyes, a concavity at the sclero-corneal border.

The obliquity of the cornea plays but a small part as far as the optics of the eye is concerned since the optic part of the cornea is nearly spherical; this sphericity holds over a linear cross-sectional diameter of about 4 mms. When the pupil is very large the *basilar* or peripheral parts may produce certain changes in the refractivity of the eye in the peripheral regions thus producing marked spherical aberration effects. But the position of the pupil varies considerably in different eyes; Sulzer found that on an average the center of the pupil is temporalward from the visual line about 5° but may be dis-

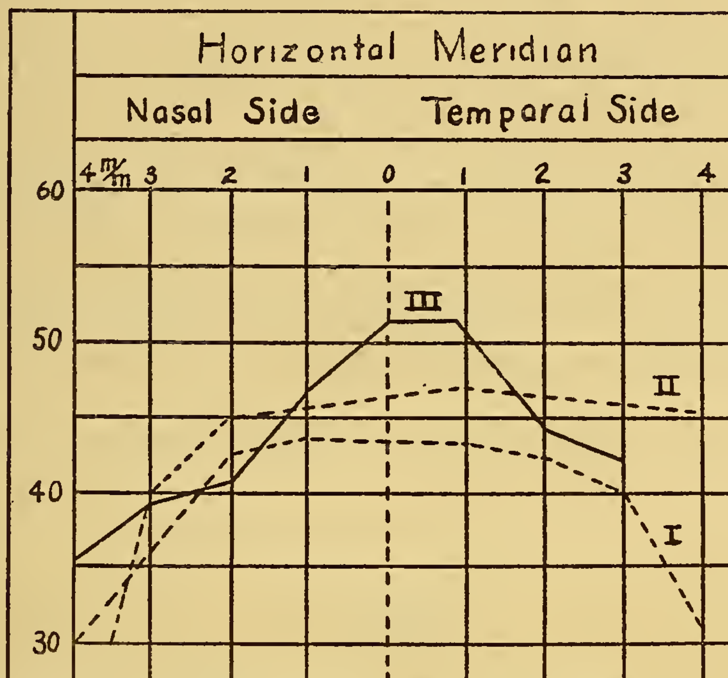
Fig. 19.—Forms of the Image of a White Square at Different Parts of the Cornea. Horizontal meridian, internal half. (After Ericksen.)

The figures at the top of the squares indicate the distance in degrees from the visual line. Those at the bottom the refraction (in the horizontal meridian) in dioptries.

placed either upwards or downwards. This decentering of the pupil may compensate for the obliquity of the cornea, therefore the most marked effect of the peripheral flattening would be temporalward.

44. While the curvature of the normal cornea varies between 40 and 48 diopters, higher values are found in cases of keratoconus. It is not uncommon to find cases of 60 to 80 diopters; indeed Cordiale found a case in which the curvature attained a value of 100 diopters which corresponds to a radius of curvature of 3.4 millimeters. Another case, more pronounced but irregular, had an apex radius of curvature of 2.2 mms. The summit of the curve showing corneal curvatures is usually situated about 1 millimeter from the visual line. Fig. 20 (A) gives the results on the radii of corneal surfaces in the horizontal meridian; Curve I is for a cornea of low curvature, Curve II of high curvature and Curve III keratoconus. Fig. 20 (B) gives similar results for the vertical meridian, the numbers of the curves having the same significance as in Fig. 20 (A). In both figures the abscissæ

indicate distances from the visual line in millimeters while the ordinates represent the refractive power in diopters.



Corneal Radii of Curvature. (After Cordiale.) Curve I. Cornea of low curvature; Curve II, of high curvature, and Curve III, for Keratoconus.

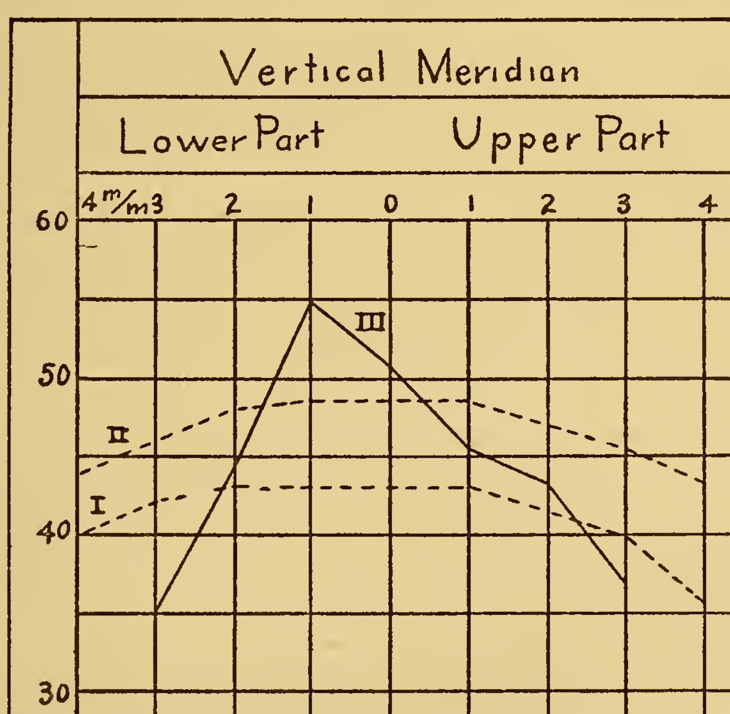


Fig. 20.—The abscissas indicate the distances from the visual lines in millimeters; the ordinates the refraction in diopters.

45. The methods of Sulzer and Eriksen afford very excellent notions as to the form of the cornea but they do not give any direct

conclusions as to the refraction at its periphery and are not directly applicable to ocular dioptrics. The reason is obvious when we consider the fact that, while the cornea is not a true ellipsoidal surface, it crudely approaches it and is, roughly speaking, a surface of revolution of the second degree. In Fig. 21 let *ES* be a parabolic surface, *R* a luminous point, F_2 its image, *EC* the normal and *EH* the radius of curvature. The refraction of the light from the luminous point *R* at the surface point *E* takes place in the same manner as if the surface were replaced at this point by a sphere drawn around the point *C* where the normal *EC* meets the axis. If *N* designates the value of *EC*, the normal, then, as previously proven, the dioptric power is $D_C = \frac{n-1}{N}$. It is experimentally necessary, therefore, to calculate the

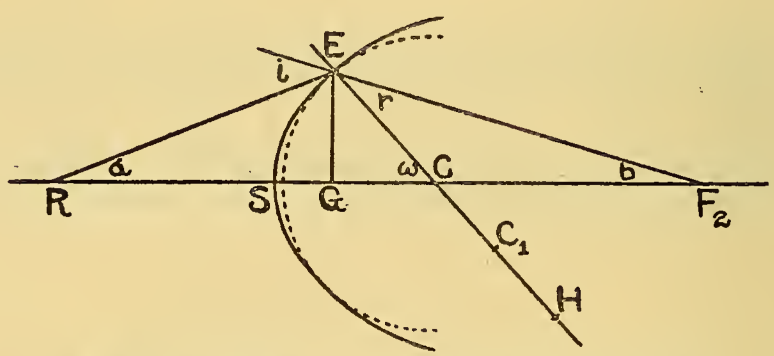


Fig. 21.—Refraction by a Parabolic Surface.

aberration produced by a peripheral flattening of the cornea and in order to do so the values of the normals must be known. To this end Brudzewski replaced the arc of the Javal ophthalmometer by one reaching to 170 degrees. One of the mires was fixed at the middle of the arc so that its border when prolonged would pass through the axis of the telescope, while the other mire was moved on the arc until it made "contact" of images. The observed party fixed the middle of the objective throughout the tests. Brudzewski used prisms of different doubling powers. Having obtained contact he used, for example, a prism doubling 1 millimeter. The arc being placed horizontally he determined the position on the nasal side which the movable mire must have so that contact might be obtained. This process was repeated on the temporal side, putting the arc vertical. These measurements gave the lengths of the *normals* at four points situated one millimeter from the visual line. These measurements were then repeated with prisms of 2, 3 and 4 millimeters doubling power. Knowing the normals, the aberration produced by the corresponding part of the cornea can be found. Fig. 22 will aid in elucidating this method. By the method of Brudzewski one measures the normal at a given point of the cornea; that is to say, the portion of the normal comprised between the surface and the visual line. Let *BL,* Fig. 22, represent the cornea; let *BO* and *EO* be two normals to the cornea and *KC* the visual line. In order to measure the cornea at *E* according to the method of Eriksen the ophthalmometer is directed along the prolongation of *CE* and hence measures the length of *OE* or the radius of curvature at *E.* According to the method of Brudzewski, on the contrary, one directs the ophthalmometer along the visual line *KC.* Considering, then, the instrument placed at a great distance, relatively speaking, from the

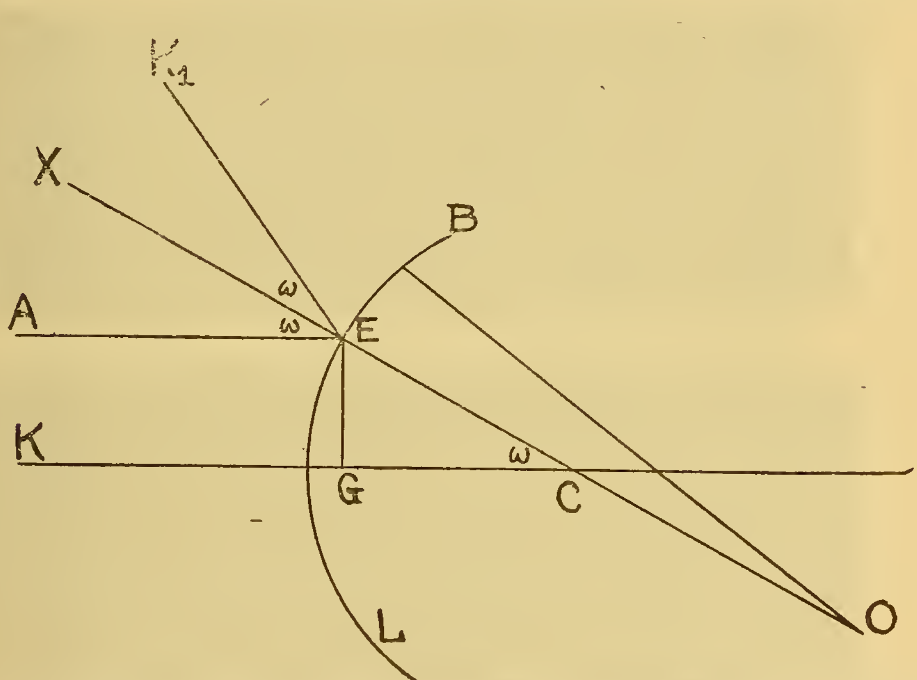


Fig. 22.—Sketch Illustrating Principle Used by Brudzewski for Obtaining the Form of the Cornea.

eye the observer will see the image of the object K_1 in the direction *AE* such that the angle of incidence K_1EX, equals the angle of reflection *XEA,* equals the angle *ECK* which the normal *XC* makes with the visual axis *KC.* Let the symbol ω represent the angle *ECK.* The image of *K* will be seen in the direction *KC* since there is normal incidence. When the observer obtains contact of images it can be said that the distance $EG = y$ is equal to the doubling employed. Knowing y and the angle ω, one can calculate the value of the normal from the expression

$$EC = N = \frac{y}{\sin \omega}$$

In conjunction with Fig. 21 there was deduced the formula that the power at any point, D_C, is given by

$$D_C = \frac{n-1}{N}$$

and therefore we see that

$$D_C = \frac{(n-1)\sin\omega}{y}$$

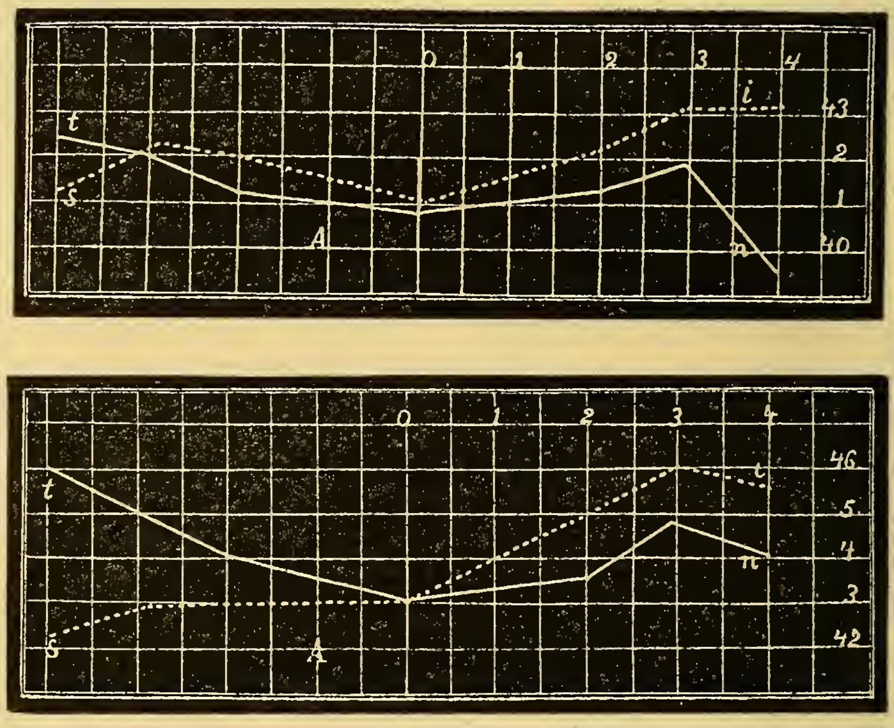


Fig. 23.—Curves Showing the Spherical Aberration of the Cornea. (After Brudzewski.)

The abscissæ indicate the distance from the visual axis in millimeters; the ordinates show the refraction in diopters.

where the angle ω and the distance y must be variable and experimentally obtainable.

46. It is known that a spherical surface has positive aberration. A spherical cornea of 40 diopters has at 4 millimeters from the axis an aberration of about 3 diopters. But such a spherical surface may be made *aplanatic,* i. e., free from aberration and therefore focussing all incident parallel light at a point, if the curvature is sufficiently and properly "flattened out" at the borders. An ellipsoid of revolution which has an eccentricity equal to the inverse of the index of

refraction is such an aplanatic surface. The question which then naturally arises is whether or not there is sufficient flattening of the cornea in the peripheral portions as to correct its aberration. The experimentation of Brudzewski has proven that it is *not;* the aberration is nearly always a positive quantity up to about three millimeters from the axis. Fig. 23 gives some experimental results as to the spherical aberration of the cornea; the abscissæ indicate the distances in millimeters from the visual axis and the ordinates the refraction in diopters. The letter *s* signifies superior, *i* inferior, *t* temporal and *n* nasal regions. The curves of Fig. 23 show a cornea affected but

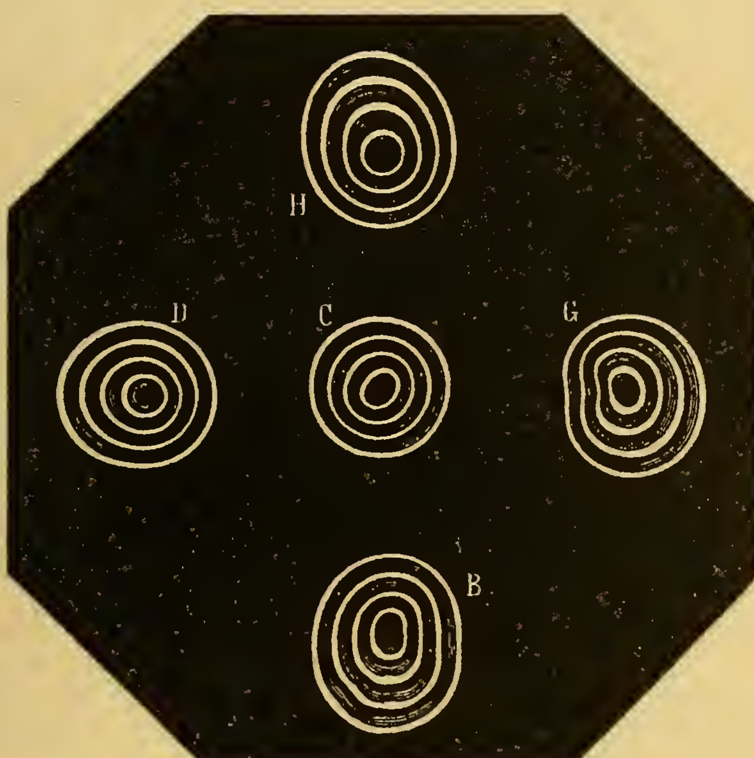


Fig. 24.—Keratoscopic Figures of a Cornea Presenting a Considerable Astigmatism at the Central Part. (After Javal.)

slightly by spherical aberration. The positive aberration is most noticeable temporally and downward. Within a zone lying between the 3 mm. mark and the 4 mm. mark there will always be found a *positive* aberration *temporally* and either "infra" ly or "supra" ly. *Nasally* within this zone the aberration generally becomes *negative.* The maximum amounts of corneal spherical aberration recorded lie between + 4.5 D and — 2.2 D.

47. *Examination of the cornea with the keratoscopic disc.* The Placido's disc is a common form of keratoscopic disc; it consists essentially of a circular plate on which is painted alternate rings of black and white. At the center there is a circular aperture or a convex lens

which acts as a simple microscope. When the patient looks toward the center of the disc the images of the circles seen apparently just back of the cornea are, in the normal eye, circles; in an astigmatic cornea they are elongated along the meridian of least refraction. By having the patient look toward the border of the disc it is easy to see and establish the peripheral flattening of the cornea. Fig. 24 presents some keratoscopic figures copied from Javal. *C* represents direct fixation; *H,* upward; *B,* downward; *D,* to the right and *G,* to the left. These figures show that the central part of the cornea (Fig. 24, C) was affected with a pronounced astigmia while the middle zones are

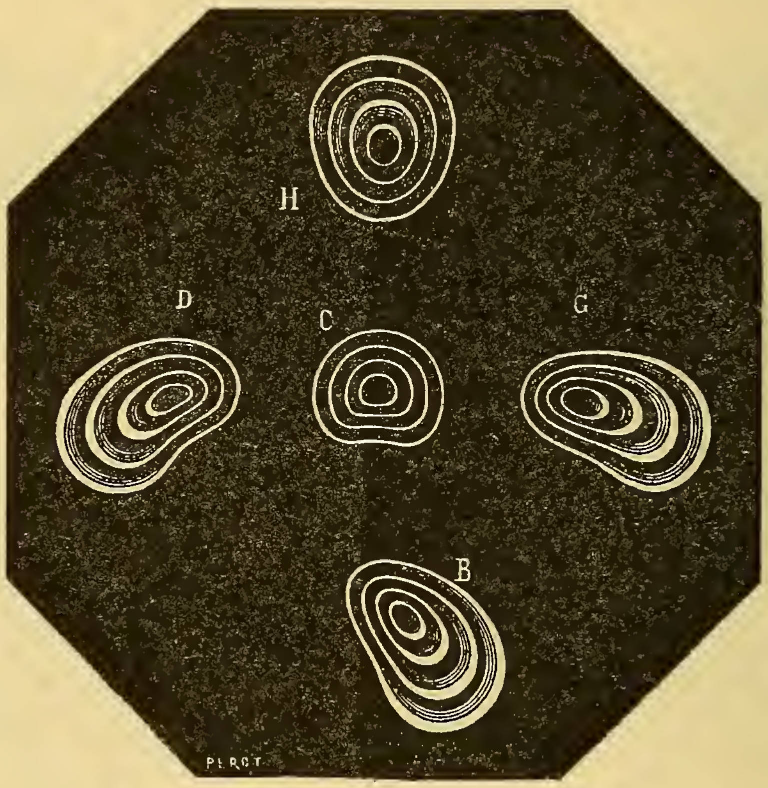


Fig. 25.—Keratoscopic Figures of a Case of Keratoconus. (After Javal.)

scarcely affected at all; in fact the central ring of figure C, which corresponds to the middle of the cornea, is much lengthened while the more peripheral rings are almost circular. The aberration effects at the periphery are noticed in H, G, B and D. In cases of irregular astigmatism the circles assume irregular forms and one may obtain important information from them. Fig. 25 is a reproduction of the keratoscopic figures in a case of keratoconus after Javal. In such cases the image of the disc is small at the summit but a slight deviation of the look causes a change of form by lengthening the image radially.

In Fig. 26 are reproduced the forms of the horizontal section of the cornea according to Cordiale. Curve I is for a normal cornea; Curves

II, III and IV for a case of keratoconus. Curve I, for example, is constructed as follows:—the arc *ad* is struck, with its center on the visual line, having a radius corresponding to that which the cornea possesses along the visual axis. A distance is then taken graphically which corresponds to the *normal* at one millimeter on the temporal (T) side; one of the compass points is placed upon *a*, the other is made to fall upon the visual line and the arc *ab* then drawn; the same procedure is followed in obtaining the remaining portions of the curve.

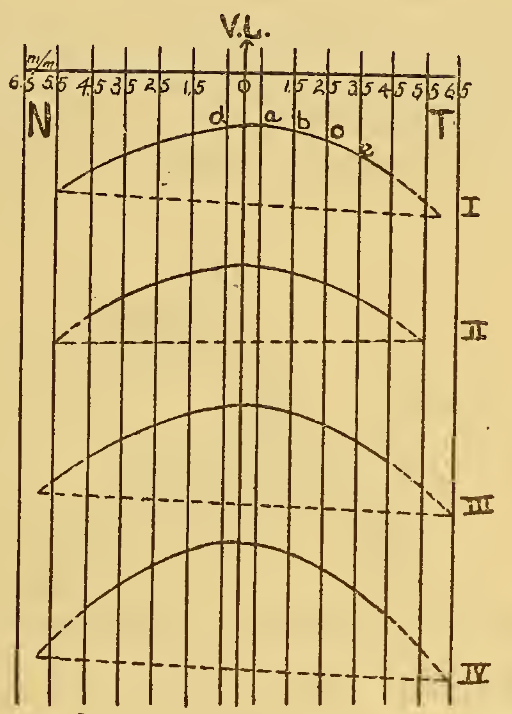


Fig. 26.—Form of the Horizontal Section of the Cornea. (After Cordiale.) Curve I, Normal cornea. Curves II, III and IV, Keratoconus.

48. *The images of Purkinje and ophthalmophakometry.* When a ray of light encounters a polished surface separating two transparent media it will be separated into a reflected ray portion, which will travel back into the first medium, and a refracted ray portion passing on into the second medium. If we represent the incident intensity by unity, then, according to the formula of Fresnel, the intensity (I_{refl}) of the reflected portion will be

$$I_{reflected} = \frac{1}{2}\left\{\frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{\tan^2(i-r)}{\tan^2(i+r)}\right\}$$

where *i* and *r* are the angles of incidence and reflection respectively. When the angles are small their values approach those of the sines and tangents; likewise, it is permissible to write the law of refraction as $i = nr$. Hence, making these assumptions and substitutions in the above equation we have

$$I_{\text{reflected}} = \left(\frac{i - r}{i + r}\right)^2 = \left(\frac{n - 1}{n + 1}\right)^2$$

49. If a flame is placed at some distance from a lens in a dark room two images, one by reflection from each side respectively, will be observed on the same side as the light is situated. If the observations are made on the reverse side, the eye being properly situated, there will be found the dioptric image which is real and inverted, and in addition a small, feeble image due to the double reflection at the internal surfaces of the lens. In Fig. 27, *AB* represents the incident

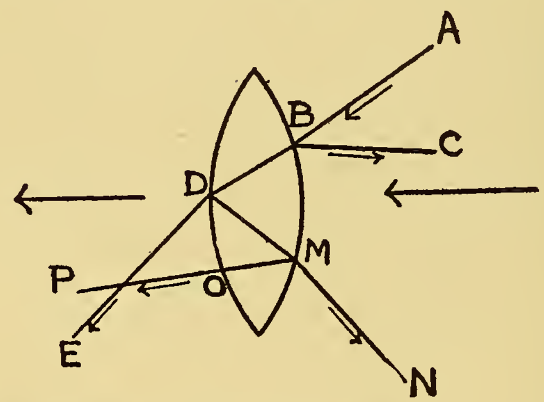


Fig. 27.—Reflections and Refractions by a Lens.

light, *BC* the direction of the reflected ray at the surface *BM, DE* the emergent refracted ray, *MN* the emergent ray after one internal reflection at *D* and *OP* the emergent ray after two internal reflections at *D* and *O*. The incident light is thus divided into three parts *visually* considered: (1) the *useful* ray, such as *DE,* which contributes to the formation of the dioptric image, (2) the *lost* rays, such as *BC,* which dissipate energy by reflection and (3) the *harmful* rays, such as *OP,* which cause indistinctness of image. Such harmful rays may indeed enter an eye which is observing the useful image and be a source of disturbance. This term "harmful rays" need not be limited to doubly internally reflected rays for they exist, for instance, in connection with toric or meniscus lenses whose inner surfaces serve as reflectors of light incident from the rear of the eye.

To return to the formula of Fresnel, we find that for a simple lens of index 1.50 there is a loss by reflection of $I_{\text{reflected}} = \left(\frac{n - 1}{n + 1}\right)^2 =$

$\left(\frac{0.5}{2.5}\right)^2 = \frac{1}{25}$ or approximately 4 per cent. The refracted ray (*DB*) intensity must then be 96 per cent., which is calculable from the relation $I_{refracted} = \frac{4n}{(n+1)^2} = \frac{6.0}{6.25} = 96$ per cent. These figures hold for normal incidence only; in general the loss by reflection is 8 per cent.

For the doubly internally reflected case we find that the percentage of the refracted ray, *DB*, which passes out at *PO* as a harmful ray, is given approximately by

$$96\% \times \left(\frac{0.5}{2.5}\right)^2 \times \left(\frac{0.5}{2.5}\right)^2 = 0.16\% = 0.2\% \text{ (app.)}$$

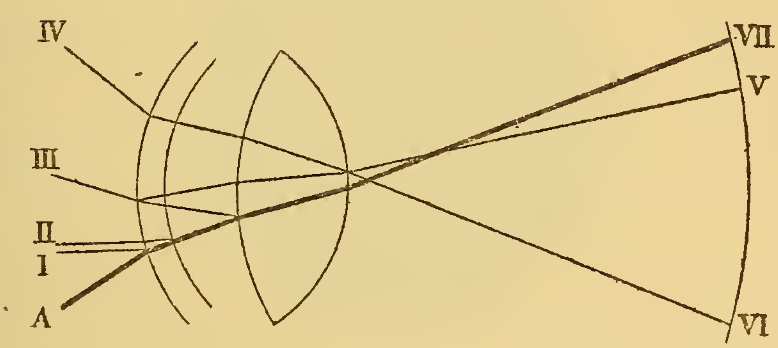


Fig. 28 A.—Manner of Division of Luminous Ray in the Eye. Ray VII is the useful ray.

When these mathematical processes are applied to the eye we can calculate the percentage losses by reflection and refraction at various surfaces. The index of the cornea is 1.377, that of the aqueous is 1.3365; the relative index is 1.03. The relative index of the crystalline with respect to the aqueous is 1.07. There results, then, a loss by reflection of about 2.5 per cent. at the anterior surface of the cornea, as calculated by the Fresnel formula, and a loss at the anterior surface of the cornea of 0.02 per cent. The reflected losses at the crystalline amount to about 0.1 per cent. It is of extreme importance that the losses by reflection at the internal surfaces of the eye are so small. This causes the amount of harmful rays reaching the retina to be reduced to a minimum; but feeble as is this amount it is, nevertheless, sufficient to be visible.

50. The dioptric image formed upon the retina is the useful image for visual purposes: the lost light forms four false images of the first

order, known as the *images of Purkinje,* one for each surface. They correspond to the rays I, II, III and IV of Fig. 28 (A). The harmful rays form a series of false images of the second, or less intense, order, of which only one is visible as V and VI in the figure. Ray VII is the useful ray falling upon the retina.

The positions of the seven images in the eye when the object is situated some 20 degrees below the visual line are shown in Fig. 28 (B).

51. *The images of Purkinje.* The first of these images, that due to the anterior surface of the cornea, is produced by a single reflection. The others are formed by rays which, after having been refracted once or twice, are then first of all reflected and then suffer other refractions before emerging from the eye. Three of these Purkinje-Sansom images,

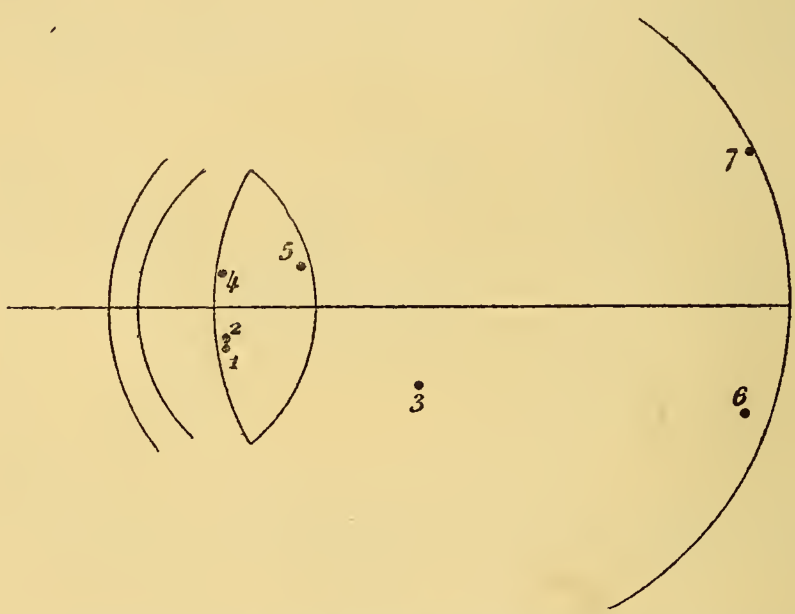


Fig. 28 B.—Positions of Seven Images in the Eye.
Object is assumed to be situated at 20 degrees below the visual line.

as they are often called, can be obtained from a normal eye using an ordinary candle or other luminous source as object. These three images, which are diagrammed in Fig. 29, are due to (1) the direct reflection from the cornea which, acting as a convex mirror, gives an inverted image of the flame situated apparently behind the cornea; (2) the image formed by reflection from the anterior surface of the crystalline lens which acts as a convex mirror giving an inverted image of the luminous object; since the radius of curvature of the anterior crystalline surface is greater than either the cornea or posterior surface of the lens this image will be greater than either of the other two and will be situated the farthest back into the eye of the three named images. This image is the most difficult of observation excepting that from the posterior surface of the cornea (Image No. 2, Fig. 28 B). Whatever procedure may be adopted for obtaining it, it is always more or less diffuse in appearance due to the fact that the index varies considerably in the superficial layers of the lens. To observe it one must look with care and at just the proper angle; this is usually obtained when the fixation direction of the person examined nearly bisects the angle between the eye of the observer and the luminous object. The image when located presents itself as a broad pale glow and changes position at the least motion of the observed eye. The light can be concentrated on the eye and by this operation the image will soon fill the entire pupil. The pupil frequently appears white and by using a magnifying glass small anatomical defects may be observed. Demicheri reports a case of variously colored zones in

Fig. 29.—Purkinje-Sanson Reflexes.

this image in a case of fairly mature cataract. Tscherning cites a case in which, with the crystalline lens apparently intact, the whole reflex was of an intense red color probably due to interference phenomena arising from the reflections at the finely-ribbed surface of the crystalline. (3) The reflection from the posterior surface of the crystalline; this surface is concave and hence gives an erect image situated somewhere between the two images formed by the two convex surfaces; as a matter of fact it will lie very nearly in the same plane as image No. I. This image (No. 3) is the smallest of the three for the reason that the greater the curvature the smaller the image size. This image usually offers no difficulties to the observer. The clinical value of the Purkinje phenomena lies in the fact that the presence and position of opacities, or absence of the same, may be learned by virtue of the presence or absence of certain of the images.

In Fig. 29, *C* represents the image formed by the cornea, *P* by the posterior lenticular surface and *A* by the anterior surface of the crystalline.

52. The optic systems which produce these images are quite complicated, but they can always be replaced by a single refracting surface which Tscherning designates as the apparent surface.

Let us make a study of the system involved in producing the image from the anterior lenticular surface. If one neglects the very weak refraction at the posterior surface of the cornea the rays experience, beside reflection, two refractions, the first on entrance and the second on emergence. This series of refractions and reflections can be replaced by a single reflection at the apparent surface. The position of this surface can be found by finding the position of the image of the real surface, seen through the cornea, by means of the relation

$$\frac{F_A}{f_1} + \frac{F_B}{f_2} = 1$$

The simplified eye, in the calculations which we have previously made, gives 24 mms. as the anterior focal length of the cornea and 32 mms. for its posterior focal length. The depth of the anterior chamber = $f_2 = 3.6$ mms. A substitution of these figures in the above formula gives as the position of the apparent surface, $f_1 = -3$ mms., i. e., three millimeters behind the real surface. It is then possible to find the position of the center of the apparent surface by finding in a similar manner the image of the center of the real surface seen through the cornea; this gives, when $f_1 = 13.5$ mms., the value of $f_2 = -17.5$ mms. The apparent surface then being at 3 mms. and its center at 17.5 mms., it performs the function of a convex mirror of 14.5 mms. radius placed 3 millimeters behind the cornea. Since the focal length of a curved mirror is equal to one-half its radius of curvature, therefore, in this case, the focal length will equal $\frac{14.5}{2} = 7.3$ mms. and the focal point will be at a distance equal to $7.3 + 3 = 10.3$ mms. Hence the Purkinje image from the anterior surface of the crystalline is formed at this point. Since the object used is generally at quite a distance from the eye, the images are formed very near to the catoptric foci of the apparent surfaces. The anterior corneal, the posterior corneal and posterior lenticular images by reflection lie nearly in the pupillary plane, while the image from the anterior crystalline surface is situated about 7 mms. behind this plane.

53. There are two false images referred to as the fifth and sixth images of Purkinje. The fifth is produced by an initial reflection at

the anterior surface of the crystalline lens and a second reflection at the anterior surface of the cornea. The rays (see Fig. 28 A) return toward the retina, hence these images are wholly subjective. The focus of the fifth image is near the posterior surface of the crystalline lens; hence the image of a distant object is formed at that surface. Before reaching the retina the rays are so dispersed that they are no longer visible. The focus of the sixth image, due to a first reflection at the posterior surface and a second reflection at the anterior surface of the cornea, is very nearly on the retina of an emmetrope: the image is generally easily observed subjectively. In a half-darkened room, with a fixation point at some distance away, we give the lamp a to-and-fro horizontal motion, moving it towards and away from the visual line. There will then be noticed on the other side of the visual line a pale image of the lamp. Some people see this image sufficiently distinctly to be able to say that the image appears inverted; the retinal image we know will be erect. The form of the image is most clearly discerned when the object passes below the visual line; the image then passes above and is seen with its apex directed downwards. This shows how fortunate it is that the "harmful" light is reduced to a minimum in the eye, since if it had any appreciable brilliancy all eyes would suffer from monocular diplopia. Calculations show that the sixth image is only about $1/40000$ of the brightness of the useful image.

54. *Point of fixation—Visual line—Optic axis—Angle alpha.* We have used the terms *fixation, visual axis* and *angle alpha* several times in the preceding pages on catoptric images. We shall now pass on to an application of the ophthalmophakometer to the determination of the angle alpha, hence it seems desirable to define the above terms at this point. When an eye *fixes* an object it does so, under normal conditions, in such a way as to place the image of the object fixed upon the fovea. The point fixed and the fovea are, therefore, conjugates. The fovea has an extent of 0.2 mm. to 0.4 mm. or subtends an angle of 0.75° to 1.50° at the posterior nodal point (16 millimeters from the retina). The diameter of the moon subtends at the nodal point of the eye approximately one-half a degree, hence when looking at the sky the fovea would cover an area of about three times the moon's diameter. It is easily possible to tell whether the right or left border of the moon is being fixed; in fact one can generally tell which of two points is being fixed as long as two can be distinctly and separately seen.

55. The *optic axis* is the central line of the globe connecting the geometrical center of the cornea with that of the fundus. It passes

through the center of the crystalline to a point near the inner margin of the macula lutea. According to the commonly accepted definition of the optic axis the anterior pole is the center of the cornea and the corresponding point on the fundus the posterior pole. In Fig. 30, *AB* represents the optic axis with *A* as the anterior and *B* as the posterior poles. We shall have occasion to discuss later on Savage's views as to the optic and visual axes; he believes, in brief, that the posterior pole of an eye should be considered the foveal fixation point with the anterior pole situated as chance may bring it where a line from the posterior point through the center of rotation of the eye cuts the cornea. An exact centering would demand that the four centers of curvature of the four ocular surfaces involved should lie on a straight line. A considerable number of eyes, which are functionally normal, show defects of centering; the most commonly en-

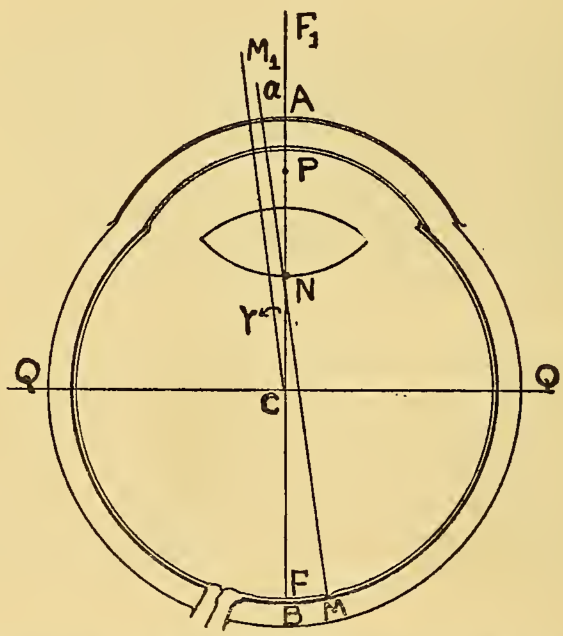


Fig. 30.—Diagram Illustrating Positions of Optic and Visual Axes and Geometrical Significance of Angles Alpha, Gamma and Kappa.

countered defect is that the center of the cornea is situated about a quarter of a millimeter below the axis of the crystalline lens. All have agreed, therefore, to call the optic axis the line which passes through the nodal points of the lens and that the optic system of the eye may be considered as centered around this line.

56. The *visual axis,* or line of vision, passes from the fovea through the nodal point *N* of the crystalline to the point of fixation. Strictly speaking it passes through the first nodal point and after passing into the vitreous proceeds to the fovea as if coming from the second nodal point. Since, in a normal eye, the nodal points are very near the

center of curvature of the anterior surface of the eye, little or no error is introduced by assuming an aphakic eye in which the visual line passes through the center of curvature of the anterior surface. The direction of the visual line is shown in Fig. 30 as MNM_1. It does not depend upon the position of the pupil. The visual line of the eye can be experimentally obtained by methods involving the use of the ophthalmophakometer to be discussed in succeeding paragraphs.

57. Since the fovea is not on the optic axis it follows that the visual and optic axes cannot coincide. The angle M_1NF_1, Fig. 30, formed by these two axes at the single reduced nodal point, N, of the eye is known as the *angle alpha;* the optic axis is on an average directed outward and downward from the visual axis by an amount of 5° to 7°. This is the definition of this angle as given by Donders, Tscherning, Howe and others. As we have seen, the anterior surface of the cornea, being flattened toward the periphery, may roughly be compared to an ellipsoid of revolution; certain authors designate as alpha the angle which the line of vision forms with the axis which passes through the summit of the corneal curve. Generally the axis of the cornea and optic axis coincide, so that these two definitions amount to the same thing. It seems best to retain as a definition of the angle alpha that laid down by Donders, to wit: the angle between the optic and visual axes.

The size of the angle alpha varies, therefore, with the distances MB and MN. If the distance MB is 1.25 mms. (Fig. 30) and that of MN is 15 mms., we find that

$$\sin a = \frac{\text{MB}}{\text{MN}} = .083 = \sin 5°.$$

This is approximately its value for an emmetropic eye; it varies however. In myopia the angle is less than in emmetropia, in fact may be reduced to zero or even be of such a value as to cause the anterior end of the visual axis to fall to the temporal side of the optic axis. In this case the angle is said to be negative. Extreme values may result from anatomical anomalies as to the position of the macula with respect to the posterior pole of the optic axis. In hyperopia, alpha is usually greater than 5° and may be as large as 8°—10°.

The *angle gamma* (γ) is frequently referred to in ophthalmic literature. It is the angle M_1CF_1 formed by the optic axis and the line of fixation at the center of curvature (see Fig. 30). This angle differs but slightly from angle alpha and the two may be considered as equal.

58. *The ophthalmophakometer.* Tscherning has devised an instru-

ment known as the ophthalmophakometer for minutely investigating the Purkinjean images and thereby experimentally determining the centering and decentering of internal surfaces in particular. The instrument consists essentially of a telescope which has a focal length of about eighty-five centimeters; the telescope is suitably mounted on a support. A copper arc is fitted around the axis of the telescope and bears a scale the zero of which coincides with the axis of the telescope. The radius of the arc is about eighty-five centimeters. The purpose of this long radius is to enable the telescope to be placed so far from the observed eye as to make possible the approximate focusing of the

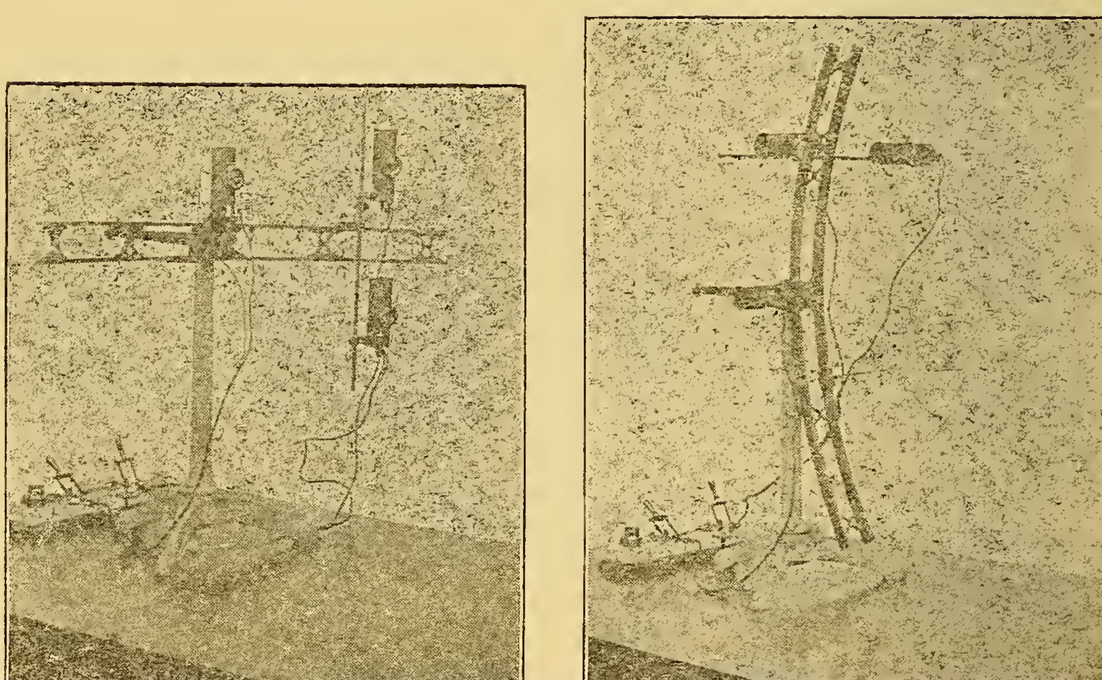


Fig. 31.—Ophthalmophakometer of Tscherning.
By permission of G. P. Putnam's Sons.

reflections from the cornea and the two surfaces of the crystalline at the same time. The arc can also be rotated about the axis by as much as thirty degrees. On the arc move several "cursors" or "carriers" which are fitted with electric lamps properly screened and carrying in front a convex lens for the purpose of concentrating the light on the eye. The carriers are so arranged that carrier *A* bears one lamp, carrier *B* bears an upright bar having two lamps and carrier *C* has an upright bar carrying a fixation object.

The instrument is shown with the arc horizontal in one of the diagrams of Fig. 31 and vertical in the second.

[The illustrations of Fig. 31 do not carry the letters *A*, *B* and *C*: however, the designation that carrier *A* bears one lamp, carrier *B* two lamps and carrier *C* the fixation object should suffice to prevent any confusion.]

59. *Measurement of the angle alpha.* The ophthalmophakometer of

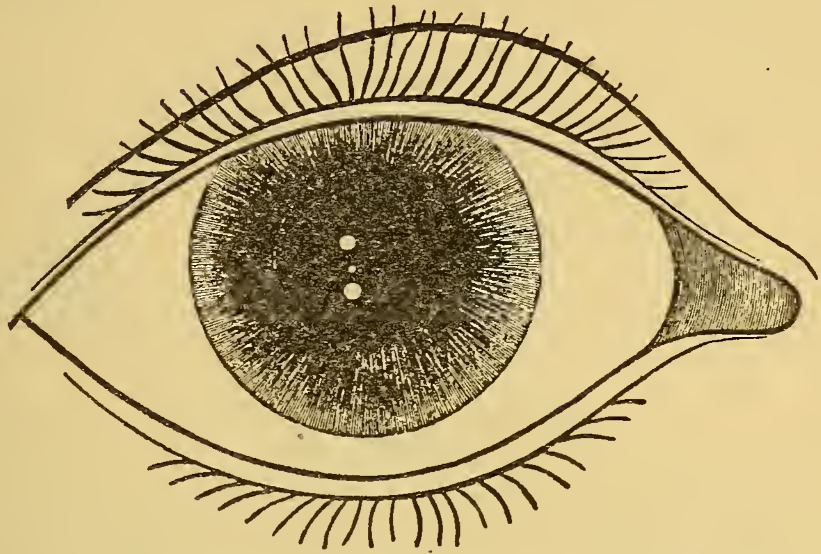


Fig. 32.—The Image of Purkinje observed with the Ophthalmophakometer when the Lens is in Alignment. (After Tscherning.)

Tscherning may be used for the purpose of determining the value of the *angle alpha.* The arc is placed horizontally and the cursor *B* at the zero point of the arc so that its lamps are in the same vertical

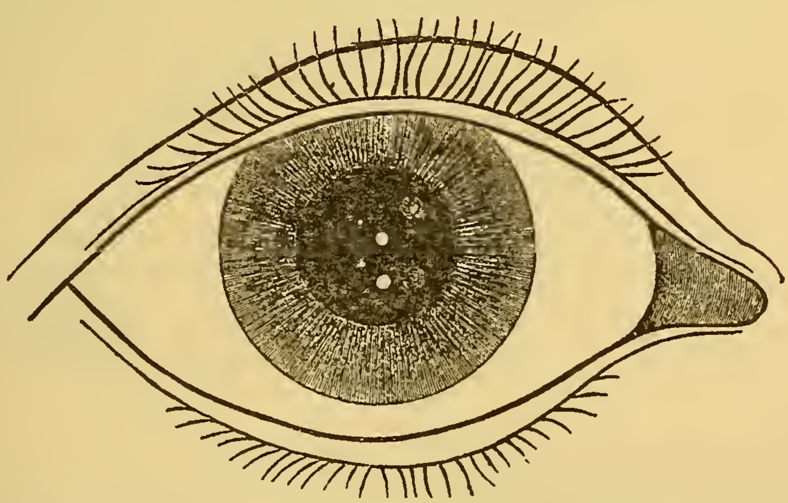


Fig. 33.—Images of Purkinje Observed with the Ophthalmophakometer when the Lens is not in Alignment.

Positions of the images when the observed person looks into the telescope.

plane as the middle of the telescopic objective and the patient is requested to fix the center of the objective. (The lamp and cursor *A*, Fig. 31, are removed in this experiment.) If the surfaces of the eye were all centered around the visual line there would be seen six images by reflection upon the same vertical line; strictly speaking there should be eight such images but those from the posterior surface of the cornea are not visible under the conditions specified. By regulating the dis-

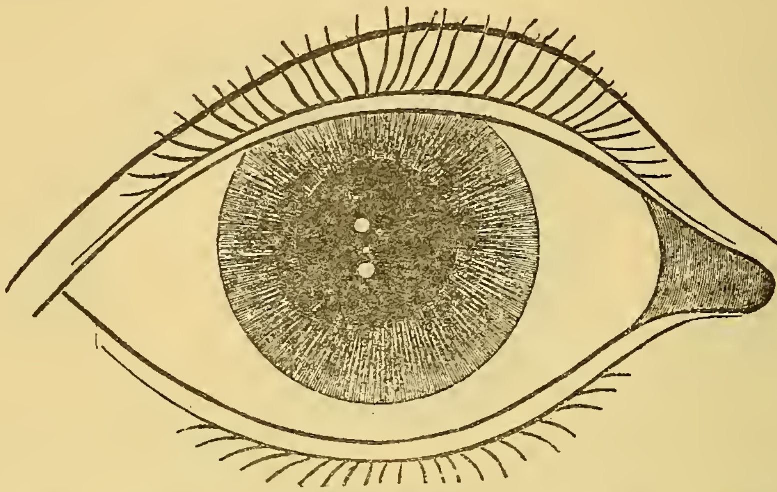


Fig. 34.—Images of Purkinje with the Ophthalmophakometer.

The two lamps B (Figure 31) are in the same vertical plane as the axis of the telescope and the observed person looks at 5.7° on the nasal side, so as to align the images. The optic axis of the eye coincides under these conditions with the axis of the telescope. (After Tscherning.)

tance between the lamps of carrier *B* it should be possible to superimpose the reflections of one of these lamps which come from the anterior and posterior lenticular surfaces upon the reflections from

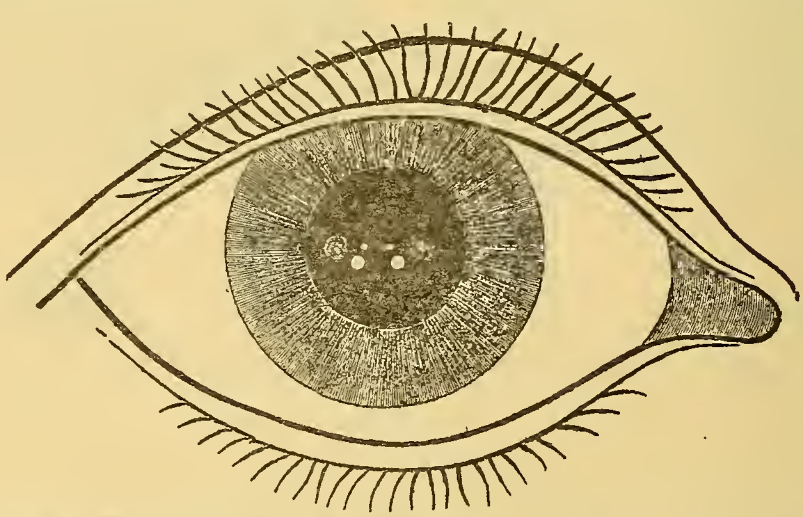


Fig. 35.—Defect of Centering. It is impossible to align the six images. (After Tscherning.)

the same surfaces due to the other lamp. Hence three images only, all in a vertical row or line, should be obtained if exact centering existed, as is shown in Fig. 32. Such a condition as shown in Fig. 32 has not been found to exist. The images are always seen as depicted in Fig. 33, with the corneal images (*C*) in the middle, those from the anterior surface of the crystalline (A) on one side and those from the posterior surface of the crystalline (P) on the other side. The patient is then requested to fix the bright ball at the center of the cursor C and this fixation point is slowly moved until the images are seen as shown in Fig. 34. The optic axis is then in the vertical plane passing through the axis of the telescope and the angular distance of the carrier C from the telescope indicates how much the visual axis deviates from the axis in the horizontal plane. This angle can be measured with considerable accuracy. This same method, with the arc vertical and the two lamps in a horizontal plane, gives a determination of the vertical deviation of the visual line. The optic axis is nearly always directed outwards from the visual line and most frequently downwards about 2° to 3°; sometimes, however, it is found in the same horizontal plane or deviated a little upwards.

It is often impossible to get the six images on a straight line. Two pairs can be aligned but the third remains outside. This occurs when the eye is not exactly centered, i. e., when the axis of the crystalline lens does not pass through the center of curvature of the cornea. We can nearly always establish slight defects of this kind, but generally they are negligible. When more considerable defects are found it is generally because the axis of the crystalline lens passes a little above the center of curvature of the cornea. Fig. 35 shows a defect of centering; it is impossible to align the six images.

60. *Ophthalmoscopic and perimetric methods of determining the angle alpha. The clinical value of the angle alpha.* Howe, in his *Muscles of the Eye,* gives as the clinical value of the determination of the angle alpha, the following:—

First. Two methods of measuring it can be made use of, with slight modification, to measure pathological deviations of the eye.

Second. The supposed divergence of some hyperopes can be shown to be only apparent.

Third. A large angle alpha may act as a predisposing cause of pathological deviations.

The easiest method of quickly estimating the size of the angle from the apparent position of the corneal reflex with reference to the center of the pupil is by the use of the ophthalmoscope. Maddox, in his *Ocular Muscles,* devotes a chapter to ophthalmoscopic corneal images and shows how they may be used for the determination of heterophoric and paretic conditions. A full discussion of this subject would lead us astray from our proper domain, however.

When the observed eye looks straight at the small ophthalmoscopic lamp, or at the opening in the center of the mirror, the visual axis normally passes through the inner side of the observed cornea to the fovea. If the angle be small or of zero value the reflex from the cornea appears in the center of the pupil. If the angle be large the reflex from the cornea will be close to the inner edge of the pupil. In myopes, when the angle alpha may be negative, the reflex may be seen at the outer or temporal edge of the pupil. One must be sure, however, that the pupil is central and normal. Fig. 36, taken from Maddox, shows the ophthalmoscopic reflections in emmetropic eyes; above, (a), when both eyes are looking at the center of the mirror and below, (b), with both eyes looking to the right. These diagrams show a symmetry of the corneal images owing to the angle alpha in the first case, but asymmetry of the images in the second case although the eyes are not squinting.

a.

b.

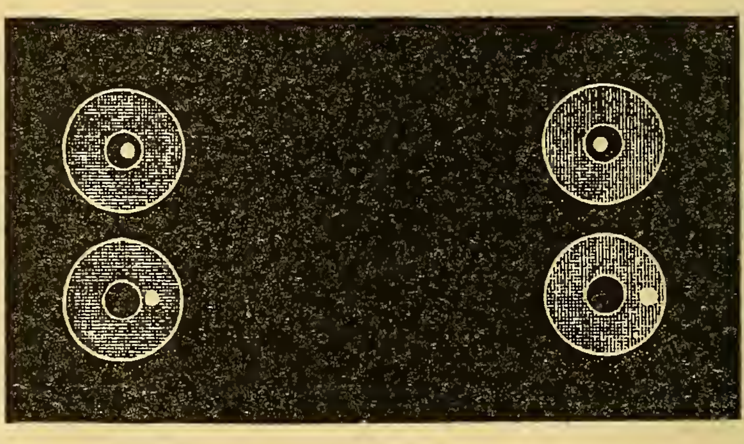


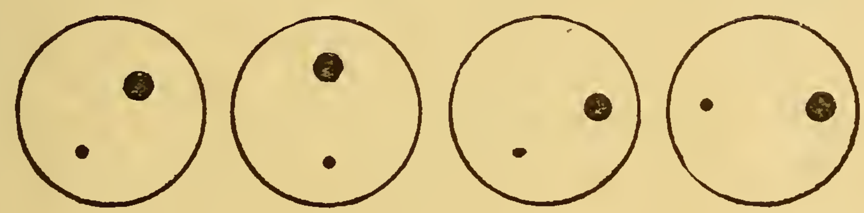
Fig. 36.—Ophthalmoscopic Corneal Reflections in Emmetropic Eyes. Above, with both eyes looking at the center of the mirror; below, with both eyes looking to the right, showing a symmetry of the corneal images owing to the angle alpha. (After Maddox.)

If one desires to decide whether the patient's squint is real or apparent, it is only necessary to flash the light on first one and then the other eye. If the corneal images occupy symmetrical positions in the two corneæ, no squint exists and the cause of the apparent squint will be made evident from the symmetrical inwards or outwards displacements of the reflexes. Marked unsymmetrical displacement shows the existence of real squint. The method is of special service with very young children, since deviations can be readily detected and the squinting eye located. It can be readily proven that when the corneal reflection of the lamp occupies the margin of a medium-sized pupil (say 3.5 mms.), the amount of squint is approximately 15° to 20°, while if it is situated at the sclero-corneal border it represents about 45° deviation. This affords a rough and ready method of estimating "fixation" conditions.

61. In the *perimetric* method, the right eye of the observed person

supposedly under examination for example, with the head adjusted in proper position in the usual manner, the patient is directed to look at the zero point of the arc. If then a small electric lamp or candle be placed at the zero point and the examiner, sitting in front, sights over this point, the corneal reflex may appear, for instance, to come from the inner or nasal side. If, with the observer still sighting along the zero degree line, the light is moved along the arc to the left of the patient and his eye follows the light, a point will be reached at which the corneal reflex will be in the center of the pupil. The number of degrees traversed is the measure of the angle alpha.

62. *Modification of the Javal ophthalmometer for estimating the position of the crystalline lens.* The axis of the crystalline lens does not in general coincide with the optic or the visual axis. According to Lucien Howe the lens usually faces temporal-ward in relation to the visual axis and its upper edge is generally tipped forward. Such



a *b* *c* *d*

Fig. 37.—Reflections from the Cornea and Posterior Capsula.

(A) When the lens is tipped outward (its usual position).
(B) When the lens is in vertical alignment.
(C) When the lens is tipped forward.
(D) When the lens is in horizontal alignment. (After Howe.)

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a tilting or malposition of the lens not only produces in and of itself a slight amount of astigmatism, but may produce conditions requiring considerable traction on the part of the ciliary muscle. The position of the lens may be quickly determined by the relative positions of the corneal and the posterior lenticular reflections, using a candle or other luminous source, as described in connection with Fig. 29. If the observer looks straight into the observed eye and, vice versa, the visual axes of both coinciding and a candle is held slightly above or below the observer's eye, he would see the reflection from the cornea and from the posterior surface of the lens in the same vertical line provided the axis of the lens coincided with its visual axis. But if the lens of the observed eye tilts outward or inward a little, the reflexes will be as shown in Fig. 37 (A). The lens usually faces temporal-ward, hence, if the observed eye is slowly rotated toward the median point, a position will be found when both images will be in a vertical line as in Fig. 37 (B). Fig. 37 (C) shows that the lens is tilted forward; the observed eye is then turned up or down slowly until the reflexes both stand in the same horizontal plane as in Fig. 37 (D). The measurements of the number of degrees the globe of the observed eye must be turned in any direction to give alignment of images are, of course, only approximate by such methods.

The Javal ophthalmometer may be easily modified to serve as a satisfactory ophthalmophakometer; the essential features of this method are due to Howe. The inner sheath which holds the prism should be removed and an arrangement, in the form of a slot and pin device, made so that the prisms may be quickly removed or inserted. A small electric light is placed about six centimeters below the center of the arc, turning when the arc turns. A conspicuous fixation point is attached horizontally to the top of one of the mires; a hat pin serves this purpose very well. In order to use this instrument as an ophthalmophakometer it is simply necessary to remove the prisms, light the small electric lamp and have the patient look at the fixation object or point placed above the barrel of the instrument as nearly at its axis as possible. The mires of the ophthalmometer are not in anywise serviceable as mires but do serve as a means of moving the fixation point away from the telescopic axis until the point is reached when the corneal and posterior capsular reflections are in line. This method also furnishes a ready means of finding the angle alpha.

63. *Determination of the positions of the internal surfaces.* In his *Physiologic Optics* Tscherning has given methods of using the ophthalmophakometer for determining the positions of the internal surfaces and also the centers of the internal surfaces. He says:—

"I take the anterior surface of the crystalline lens as an example and I suppose that we are making the measurement in a horizontal direction. It is useful to dilate the pupil.

"I place the arc of the instrument horizontally and I place also, as far away as possible from the telescope, the cursor A [see Fig. 31 of this text], the lamp of which must be sufficiently brilliant that the image of the surface to be measured may be quite visible. This done, I place the cursor *C*, which carries the mark of fixation, at a place such that the optic axis of the eye may bisect the angular distance between the telescope and *A*." [Note by C. Weiland, translator of Tscherning's *Physiologic Optics:* "If the eye is not centered we must replace the optic axis by the line passing through the center of curvature of the cornea and the center of the surface which we desire to measure. We find this line by aligning the corneal images with the images of the surface to be measured]. "It is necessary, therefore, to have previously measured the angle alpha. We then displace the cursor *B*, the lamps of which must be very feeble, so that we may see only the corneal images, until the crystalline image of *A* is exactly on the same vertical as the corneal images of *B*. Glancing at Fig. 38 (this article) it is easy to see that we now possess the elements necessary to calculate the distance of the anterior surface of the crystalline lens from the summit of the cornea, for the angle *c* is half the angular distance of *A* from the telescope, and the angle *d* is half of the angular distance of *B* from the telescope. Supposing that we knew the radius of the cornea R_1, which should have been measured previously, the triangle O_2C_1P [Fig. 38] gives us the relation

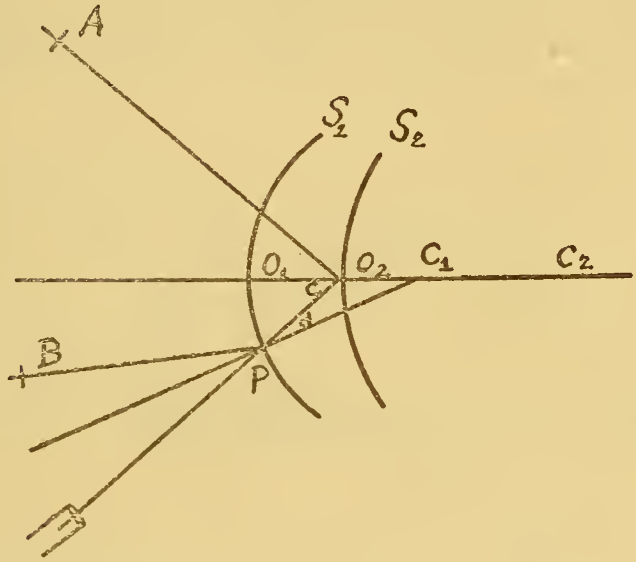


Fig. 38.—Method of Determining the Position of an Internal Surface of the Eye.

S_1, anterior surface of the cornea; C_1, its center; S_2, anterior surface of the crystalline lens; C_2, its center; C_1 C_2, optic axis of the eye. (After Tscherning.)

$$O_2C_1 = R_1 \frac{\sin d}{\sin c}$$

and we have for the distance looked for

$$O_1O_2 = R_1 - O_2C_1 = R_1 \left(1 - \frac{\sin d}{\sin c} \right) = R_1 \frac{\sin c - \sin d}{\sin c}.$$

If very great exactness is not desired the sines can be replaced by the arcs.

"Example:—Let the radius of the cornea be 7.98 mm., the distance of A from the telescope 28° nasal, that of B 16.8° nasal; we will have

$$O_1O_2 = 7.98\left(1 - \frac{\sin 8.4°}{\sin 14°}\right) = 3.16 \text{ mm.}$$ The apparent depth of the anterior chamber would, therefore, be 3.16 mm., whence we find the true value 3.73 mm. by placing in the formula $\frac{F_A}{f_1} + \frac{F_B}{f_2} = 1$, the values $F_A = 23.64$ mm., $F_B = 31.61$ mm. and $f_1 = -3.16$ mm.''

64. *Determination of the centers of the internal surfaces.* Tscherning writes:—''We place *A* above the telescope and we move *C* with the mark of fixation as far as possible from the telescope, but so that

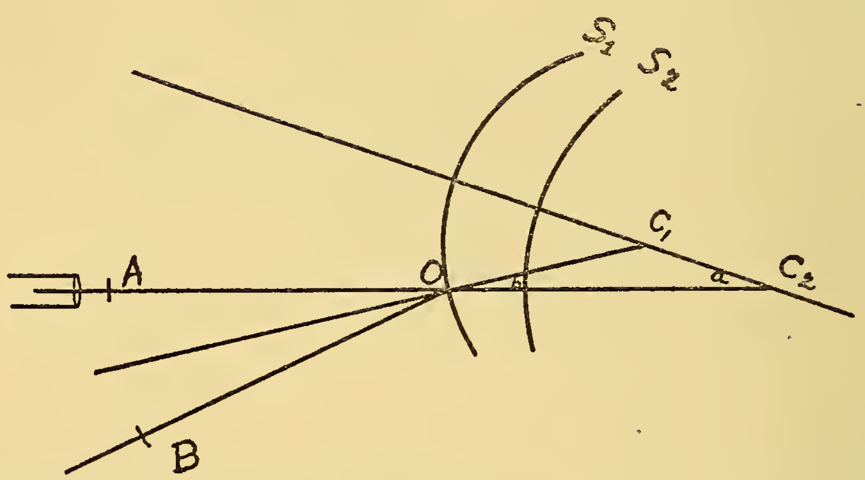


Fig. 39.—Method of Determination of the Position of an Internal Surface of the Eye. (After Tscherning.)

the image may not disappear behind the iris; then we displace *B* until the corneal images of its two lamps are on the same vertical line as the crystalline image of A.

''Under these conditions, the axis of the telescope is perpendicular to the apparent anterior surface of the crystalline lens.'' [Note by C. Weiland, translator: ''If we imagine the lamp placed at the center of the objective, the ray which meets the observer's eye would be reflected exactly on itself, which can take place only if it meets perpendicularly the apparent surface.'']. ''We find the angle *a*, Fig. 39, by adding (subtracting) the angle alpha (α) to the angular distance of *C* from the telescope. The angle *b* is half of the distance of *B* from the telescope; we have

$$C_2C_1 = R \frac{\sin b}{\sin a}$$

and the distance sought equal to

$$R_1\left(1 + \frac{\sin b}{\sin a}\right) = R_1\left(\frac{\sin a + \sin b}{\sin a}\right)$$

"Example:—In the same eye as before let alpha = 5.1°, the distance of *B* from the telescope 12.4° temporal and that of *C* from the telescope 9.9° nasal. We would then have the distance sought 7.98 $\left(1 + \frac{\sin 6.2°}{\sin 4.8°}\right)$ = 18.28 mm. and the apparent radius would be 18.28 mm. — 3.16 mm. = 15.12 mm. The position of the real center would be 13.78 mm." [Note by C. Weiland, translator:—"Considering that we have again obtained this apparent position with reference to the refraction of the cornea, we must, therefore, in the formula $\frac{F_A}{f_1} + \frac{F_B}{f_2} = 1$ put $F_A = 23.64$; $F_B = 31.61$ and $f_1 = -18.28$ mm.: this gives $f_2 = 13.78$ mms."] "The radius of the real surface is 13.78 mm. — 3.73 = 10.05 mms."

Ophthalmometry finds daily use in offices and clinics; the methods of measurement of the internal surfaces could hardly find such application. There are certainly differences between astigmatic findings as determined by the ophthalmometer, the retinoscope and subjectively by the trial case and these differences might possibly be explained if we knew more of the conditions present at the various internal surfaces. Tscherning has probably done more with his ophthalmophakometer than anyone else and frankly says that the methods and calculations, even when simplified or approximated, are too laborious, uncertain and complicated; likewise, it is not probable that the explanation of the differences between ophthalmometric and subjective astigmia would be discovered. These differences are possibly due to the fact that the peripheral parts of the cornea have an astigmatism different from that of the central parts which are measured with the ophthalmometer. But the ophthalmophakometer has served to throw much light upon the problem of the mechanism of accommodation.

PART TWO

MONOCULAR VISION AND METHODS OF OBJECTIVELY AND SUBJECTIVELY INVESTIGATING IT

VI. REFRACTIVE ANOMALIES

65. *Emmetropia and ametropia.* Two conditions must be fulfilled in order that the human camera may afford its possessor vision. The perceptive and receptive surfaces must be a mosaic, the individual parts of which can be stimulated by luminous rays, and this stimulation must be carried to the brain without affecting or interfering with the other parts of this surface. There must be a healthy, functioning retina to receive the image; but the dioptric apparatus must collect portions of the light emanating from an object and unite them as an image on this perceptive surface. Perfect vision is almost as impossible of definition as it is in actuality; certainly the prime requisites are perfect focus of image and perfect retinal reception and mental interpretation by the brain. Thus far the eye has been considered as an optical instrument possessed of various dioptric and catoptric media having certain properties, characteristics and constants; we have considered it in a general way (at least mathematically) as a perfect optical device. Such it would presumably be if we excluded chromatic and spherical aberrations which are, under normal conditions, of second-order effect ocularly considered, provided it fulfilled the criteria which have been deduced for the "reduced eye" in the earlier pages of this article. Yet it would not do to say that all eyes that measure, for example, just 23 mms. in their antero-posterior diameter are emmetropic, for while an eye may possess that length it may have a refractive system stronger or weaker than is consistent with its length. Again, the cornea, aqueous and lens may be possessed of such optical curves, indices, powers and what not as to cause parallel light to focus at a point 23 mms. behind the cornea, but the screen, the retina, may be ahead of or behind this point. From the standpoint of vision, therefore, all eyes may be included within two classes, *emmetropic* and *ametropic*. Emmetropia literally means an eye in measure, or an eye which has reached such a state of development that parallel rays of light will be focused upon the retina without any effort of accommodation: the static refractive power is proportional to the axial length of the globe. Ametropia means an eye "out of measure." An eye which is not emmetropic is ametropic; such an eye, in a *state of rest,* does not receive a distinct image of distant objects upon its retina.

There are two forms of ametropia—axial and curvature. In *axial* errors the dioptric apparatus refracts equally in all directions, but the retina of the eye, when at rest, is either closer to or farther away from the nodal point than the principal focus. In myopia this image is formed in front of, and in hyperopia behind, the retina. The first of these is commonly spoken of as the long eye, the second as the short or flat eye.

Curvature ametropia, in contradistinction to axial ametropia, is the condition due to the dioptric apparatus producing unequal refractive effects in different meridians, with the result that there is no focusing of all the rays at one point. It may be considered as that condition in which parallel rays of light entering an eye have two focal planes for two principal meridians usually at right angles to each other; this is commonly known as *astigmatism*. Strictly speaking the above limitation upon curvature ametropia should not be, because myopia and hyperopia may be due to curvature defects and in such a case as that reported by v. Reuss, in which a myope had a corneal curvature of 6.5 mm., there is no doubt but that a portion, at least, of the short-sightedness was due to curvature ametropia. But thus far experimentation has not disclosed a definite relation between refractive status and corneal curvatures and in a majority of cases myopia and hyperopia are due solely to an anomaly in the length of the eye.

There is in addition to axial and curvature changes, as causes of ametropia, the possibility of *indicial* anomalies. Up to the present time the only authentic cases in which anomalies of the indices have been established and which we have been able to find a record are those reported by Demicheri and designated by him as false lenticonus. The refraction at the middle of the pupil was myopic to an amount of 10 D. or more while the peripheral portions showed 3 to 4 D. of hyperopia. The cause of this marked change must, without doubt, be attributed to a diminution in the index of the peripheral layers of the crystalline lens. Such a change would produce a diminution in the refraction of the peripheral portions and greatly increase the central refraction.

66. *The far and near points in emmetropia and ametropia.* All definitions have been made to center around the meaning of the term *emmetropia*. This was stated to be that condition of the eye in which, in a state of repose, infinity and the retina were conjugate points. Infinity is then the fixed fiducial point in emmetropia; it is the far point or *punctum remotum*. One of the most important points to remember in visual optics is that the refractive power is a definite fixed quantity for any given pair of conjugate focal distances. If the

one conjugate is fixed, the other is nearer as the refracting power is greater, and farther away, in turn, as the power is smaller. The point of vision is always the conjugate focus of the retina, otherwise clear vision is impossible at that point. Whatever, then, may be the process and precise mechanism of accommodation the result is always a positive action causing an increased total refractive power of the eye. When the accommodation is totally relaxed the eye is in a condition of minimum refraction and is adjusted for its far point. When, however, the accommodation is totally exerted, the refractive condition of the eye is at its maximum and is adjusted for its near point or *punctum proximum*. The distance between the far and near points is termed the *range of accommodation* and is expressed in terms of linear measure. If the far point (R) is 100 cms. and the near point (P) is 10 cms., then $R - P = 90$ cms. is the range of accommodation. The quantity of accommodation possessed is termed the *amplitude of accommodation*. It is expressed in diopters, so that the amplitude is found from the equation

$$(1)\quad A_D = P_D - R_D \text{ (in diopters)}$$

This equation is frequently written (for example in Donders' *Accommodation and Refraction of the Eye*) as

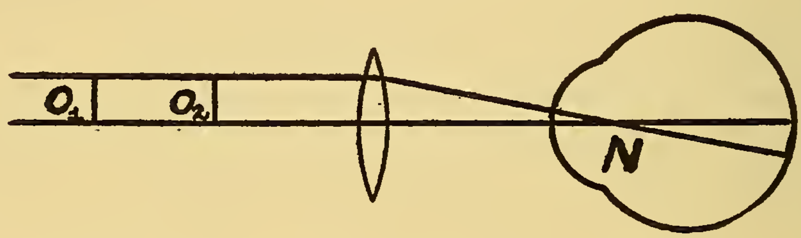
$$(2)\quad \frac{1}{A} = \frac{1}{P} - \frac{1}{R}$$

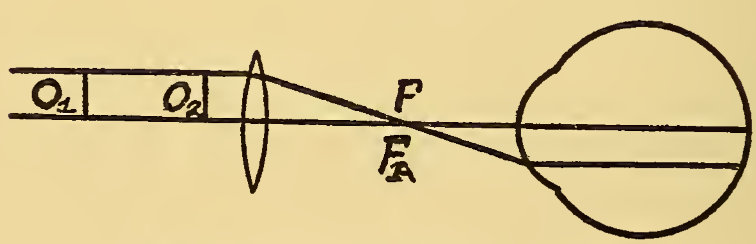
The two equations are identities, for in equation (2) P and R represent linear distances the reciprocals of which represent dioptric power; that is, if the far point is 25 cms. then $R_D = 4$ D.

It is, indeed, the distance of this farthest point of distinct vision from the eye, determined of course by clinical methods and standards, which affords the simplest and most rational method of defining and measuring the degree of ametropia. For one need no longer consider whether the axial length is too long or too short for the static refractive power or vice versa, but may define them as follows: The far point of an *emmetropic eye* is at infinity. In *hyperopia*, the static refraction is deficient so that only convergent light incident on the cornea will focus at the retina with the accommodation inactive; its remote point is, in other words, virtual, situated back of the eye and its distance is to be put down as negative. Thus a hyperope of 2 D. lacks two diopters of static power; he has only 48 D. of static power if we assume that an emmetrope possesses 50 D. power; hence in the above case of hyperopia light must converge to 50 cms. behind the cornea in order to be focused upon the retina. The punctum remotum in this case is designated as a negative distance. The extra 2 D. of power necessary to render the eye emmetropic must be supplied by a + 2 D. lens close to the cornea or by the exertion of 2 D. of accommodation. *Myopia* is that condition in which there is an excess of static power so that only light divergent from a near object can focus at the retina. Thus a myope of 3 D. has an excess of 3 D. static refraction and therefore light must diverge from a point 33.3 cms. in front of the eye in order to focus on the retina. The punctum remotum is, in this instance, positive and a — 3 D. lens placed near the cornea reduces the static refraction by 3 D. and hence renders it emmetropic. Parallel light refracted by the lens then apparently diverges from the 33.3 cm. point which is the far point in this particular case.

67. The determination of the far point can be done readily and with considerable accuracy; not so with the *near point,* however, which depends upon an effort of the patient and which may vary somewhat from day to day and with the patient's general health and nervous vitality. The commonest (and possibly most inaccurate) method is to employ No. 2 Jaeger type, which a person should normally, unless presbyopic, be able to read at 12 or 13 inches, and approach this toward the eye, its mate being covered, until the nearest point is reached at which it is clearly seen or can be read. This distance from the eye to the chart is the punctum proximum. In a case of emmetropia this distance, when reduced to diopters, gives the amplitude of accommodation. This method in ametropia, however, gives only the *apparent* or *available* and not the true amplitude. Likewise it may not be available in presbyopic conditions since the near point may have receded so far that fine print is not readable; in such a case, however, a plus or convex lens may be furnished to assist the patient and bring the near point to a measurable distance, finally deducting this lens from the dioptric value found. In ametropic cases the practitioner will be saved considerable time and trouble if he supplies the patient with the correction for the refractive error statically determined at infinity first of all, since then presumably the far point is at infinity or approaching it as closely as conditions, both pathological and non-pathological, will permit. The eye is then emmetropic (or as nearly in such a condition as circumstances will permit) and the procedure in determining the amplitude of accommodation is the same as that for emmetropia. This method, i. e., the nearest point at which fine print can be read, is open to objection on the basis that the normal reading distance is about 13 inches on an average and the function of accommodation, hence its amplitude, should be tested at this point and no

other. Incorrect accommodative amplitudes from near point determinations are obtained due, the writer believes, to two factors chiefly; (1) there is a more rapid increase of the visual angle than of the circles of diffusion, hence the person under test is able to read at a point nearer than that at which accommodation is still being proportionately enforced, and (2) the reduction in the size of the pupil, which in turn lessens the size of the circles of diffusion. These same reasons explain why, in cases of high hyperopia, small objects can be seen better, or fine type read, nearer the eye than at some distance from it, thus resembling a myopic condition.





Figs. 40, 41.—The Badal Optometer.

Showing the coincidence of the focal point of the lens with the nodal point of the eye.

Showing the coincidence of the focal point of the lens with the anterior focal point of the eye.

68. A great many optometers and punctuometers have been devised but most of them suffer from the errors enumerated above when operated at the near point. Badal's optometer has, in part at least, removed these objectionable features. If a lens, Fig. 40, is placed so that its focal point *F* coincides with the nodal point of the eye, a ray which is parallel to the axis before refraction passes after refraction by the lens through *N*, the nodal point, without further deviation no matter what the distance of the object may be. Hence the angle under which the image is formed is always the same and there is no sense of

change as the distance of the test type is increased or decreased. If the lens is placed so that F coincides with F_A, the anterior focal point of the eye, then a ray parallel to the principal axis of the lens, before refraction, is parallel to the principal axis of the eye after refraction. The refraction and apparent amplitude of accommodation of the eye are measured with equality of size of retinal image and the sense of nearness eliminated. A commonly used combination is a + 10 D. placed 115 mms. from the cornea (Fig. 41) or about 90 mms. for the conditions diagrammed in Fig. 40.

The Scheiner disc is a classic device for determining the nature and amount of ametropia, and also of determining the near point. It consists of an opaque disc with two very small apertures separated from each other by less than the diameter of the pupil of the eye under examination. Light from a constricted source at infinity enters the eye and if the eye is emmetropic the two images which are formed will coincide upon the retina and one image only will be seen. When ametropia exists two images will be seen and their location and character indicate the kind of error; in order to specify images it is customary to cover each aperture with a differently colored glass, such as red and green. If the eye is hyperopic, the two cones of light reach the retina before uniting; the right and left cones fall on the right and left sides of the retina respectively, forming two images which are uncrossed but which, according to the laws of projection, will be seen in space as crossed. In myopia the two images coincide in the vitreous and therefore fall upon the retina as crossed images; the projected images are then uncrossed. The punctum proximum is found by holding a thin object, such as a pin, at right angles to the plane of the apertures. It is then approached until a point is found at which the accommodation fails to give union of images and the object is seen double; the nearest point at which a single object only is seen is the near point.

69. A very satisfactory method of determining the *amplitude of accommodation* in practice is to employ *concave lenses,* each eye being tested separately. Briefly, No. 2 Jaeger type should be used at 13 inches; the patient should wear the full distance correction (particularly the cylindrical element, if the spherical member is omitted and afterward taken into the final accounting) and the maximum amount of concave lens power added, step-by-step but quickly, until the patient is unable to continue reading. Some allowance, best of all taken care of by using slightly higher numbers of Jaeger type such as No. 4 for example, should be made in cases of considerably reduced visual acuity and in old age. If full distance correction is worn, then the ampli-

tude of accommodation is quickly formed by adding the maximum concave lens value used (expressed of course as a positive quantity representing so much accommodative effort) to the three diopters of accommodation demanded when reading at 13 inches. For example, if No. 2 type is barely readable through a —4 D. lens at 13 inches, the eye being emmetropic, the amplitude of accommodation would be 7 diopters. This method has the objection that concave lenses cause a diminution in the apparent size of the object, hence this method will give a minimum rather than a maximum value of the monocular accommodation and will accordingly give values of about one-half to one diopter less than those given in the Donders' table of changes in the amplitude of accommodation with age.

The amplitude of accommodation, either monocularly or binocularly, may be obtained *objectively* after the manner illustrated in Figures 41 (B) and 41 (C) taken from Sheard's *"Dynamic Ocular Tests."* The essential optical principles underlying these tests are those involved in skiametry and the applications of the laws of conjugacy of foci. If the subject under test, either naturally or artificially rendered emmetropic for distance, should fix and read fine print at any close point, then optically the retina and the point fixed should be conjugate points. This is, however, the mathematical ideal condition, demanding and supplying one diopter of accommodative innervation for each diopter of lenticular change. If such ideal conditions hold, then the observer's nodal point, situated just back of the retinoscope and the card of letters attached to the side of the retinoscope, would be practically in the same plane and a neutral reflex or shadow would be skiascopically obtained. These conditions, however, assume the presence of a true or artificially produced emmetropia and a perfectly innervated and functioning ciliary and lenticular action. But in practice it will be found commonly that when the subject, wearing the full static corrections, reads monocularly letters—these letters may be arranged in a vertical row on a narrow, i. e., about an inch wide, card, which can be approached toward the eye from the nasal side, the operator following with the retinoscope from the temporal side—there is a hyperopic motion indicating that the optical conjugate to the retina is somewhat farther from the eye than the point apparently fixed or observed. If now the line of letters (or a pencil will serve the purpose for a rough test) is moved slowly by the subject toward his nose, while the observer keeps his retinoscope fixed in a given position, the operator will notice that the shadow changes from "with" to "neutral" to an "against" motion if a plane mirror is used. Hence the subject's *actual* point of optical conjugacy of retina with respect to the observer's nodal point may be made to change in such a way that the observer will be, in turn, inside of, just at, and outside of the point of optical conjugacy. In testing, therefore, for the near-point objectively, we proceed as follows: the patient draws the test object toward the eye from about the ten-inch point ordinarily. To the observer at thirteen inches the retinoscopic reflex will show, let us assume, an "against" or myopic condition indicating that he is outside of the optical ocular far-point dynamically considered. The practitioner then moves forward until he obtains the neutral shadow position. The test object is then to be carried still closer to the eye (blurred image as reported possibly by the patient makes no difference) and the nearest point of neutral shadow found and measured. This gives the *apparent* near-

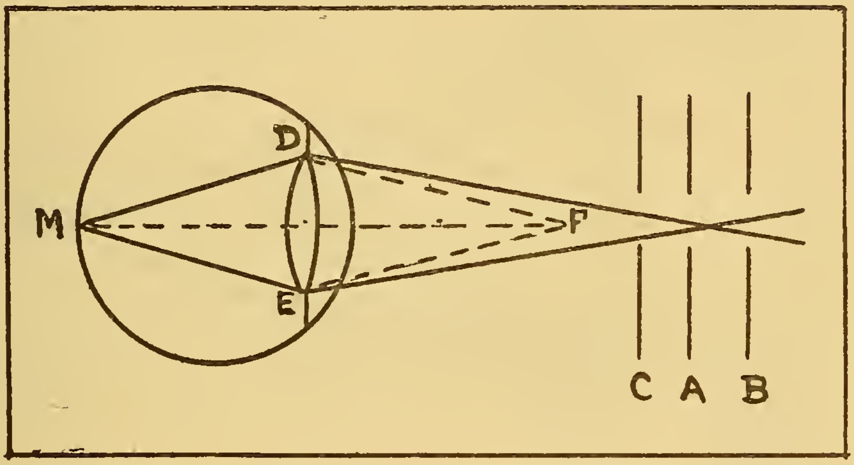


Fig. 41 B.—Illustrating the Optical Principles Involved in the Objective Monocular Test upon the Accommodative Amplitude. (From Sheard's *Dynamic Ocular Tests.*

point under whatever ocular conditions the test is made (ordinarily when wearing the distance correction) and from it the range and amplitude of accommodation are easily determined. It is to be noted that the distance *DA* and *not FD* (Fig. 41 B) is to be measured. In Fig. 41 (B) the point *A* is taken to illustrate the optical near-point of the eye, while *F* represents the point at which the eye may be endeavoring to look. The points *C, A* and *B* represent, therefore, positions of the observer with a plane mirror to obtain "with," "neutral" and "against" motions of skiascopic reflexes.

The element of convergence, i. e., binocular single vision, where possible, enters into the problem of the *binocular* amplitude of accommodation. Hence, marked differences in the values of the objectively obtained binocular and monocular near-points are nearly always indicative of disturbances in the extrinsic muscles, or their innervations, or the supplementary convergence. For the functions of accommodation and convergence ought to be so so-ordinated as to permit of binocular single vision and clearness or distinctness of vision. The *modus operandi* in the binocular accommodative amplitude test is the same as in the monocular test and involves the obtainance of the closest point of neutral shadow. Such binocular procedures throw considerable light upon the problem of the clinical importance of the difference between binocular and monocular near-points.

70. The accommodation has the same effect as a convex lens added to the eye. Let us suppose an eye devoid of all accommodative power;

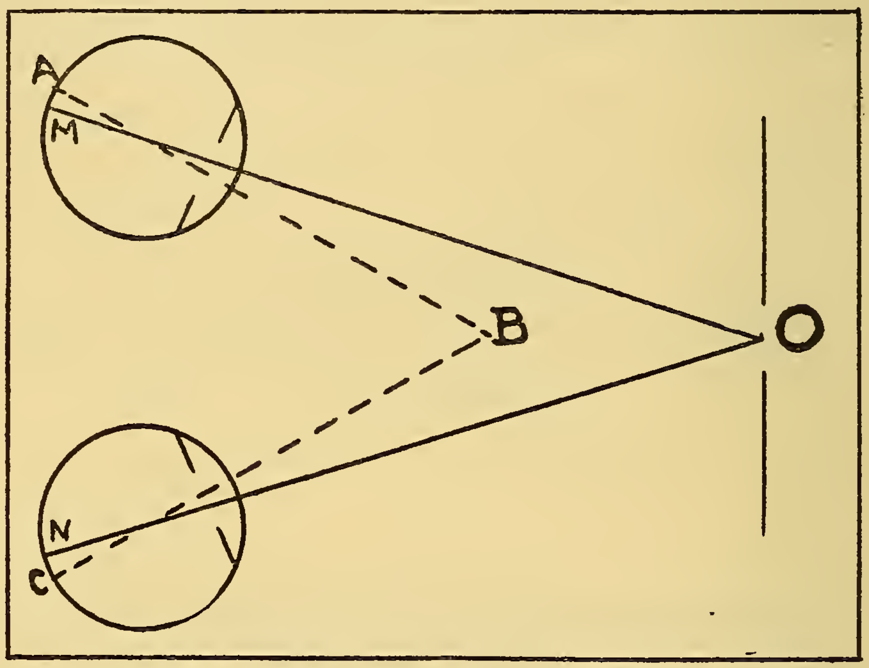


Fig. 41 C.—Illustrating the Optical Principles and Procedure in the Objective Binocular Tests upon the Accommodative Amplitude. (From Sheard's *Dynamic Ocular Tests.*

it will not be able to correctly focus anything inside its far point (R); for example, the object M, Fig. 42. To make vision at M possible, the divergence of rays coming from M must be diminished until they have such a direction as will make them appear to come from the point R to which the eye is adapted. If M is the punctum proximum of the eye, the lens which makes vision at the distance HM possible has, therefore, the same effect as the accommodation. The analogy between the effect of accommodation and a convex lens is all the more complete as the seat of the accommodation is in the crystalline lens; to be exactly comparable to the amplitude of accommodation, however, the lens ought to be placed within the eye. In reckoning its focal distance, then, we take as a starting point the cornea or, strictly speaking, the first principal point of the eye. In order, therefore, that the true ametropic error in any case be known the correcting lens must be near enough to the eye that the two may be regarded as united. A consideration of Fig. 43 will aid in making these mathematical statements clear and apparent. Let the distance *RH* from the far point to the eye equal 50 cms. If we place the correcting lens at a distance $OH = 10$ cms. from the eye then, since $RH - OH = OR$,

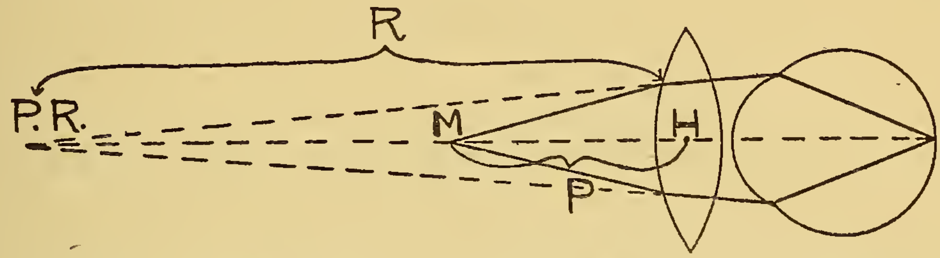


Fig. 42.—Illustrating the Optical Equivalence of Accommodation.

the focal length of the concave lens necessary to correct the real myopia of 2 D., when placed at the point *O*, will be $50 - 10 = 40$ cms. or -2.5 D. In practice the correcting lenses are generally placed at about 15 millimeters in front of the first principal point, or 13 millimeters in front of the cornea. If the punctum remotum of a certain eye be 125 mms. it would be said to be myopic 8 diopters. But the correcting lens, placed 15 mms. in front of the eye, will need a focal

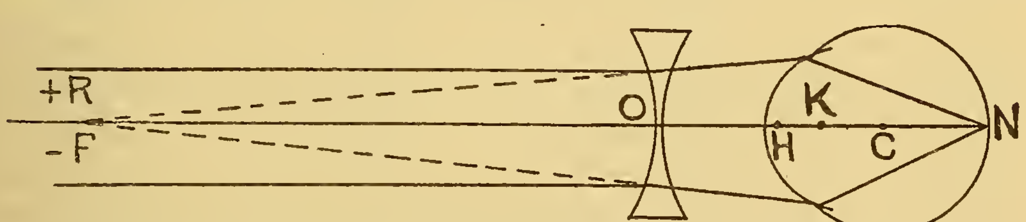


Fig. 43.—Diagram Illustrating the Optical Value of the Lens Needed to Correct a Specified Amount of Myopia; the Lens Being in Advance of the Cornea.

length of $125 - 15 = 110$ mms., or practically a -9 D. lens. This difference between the degree of myopia and the correcting lens becomes greater in proportion as the punctum remotum is nearer the eye. For example, a concave lens of 18 D. placed at a distance of 15 mms. corrects the myopia of a certain eye. The lens has a focal length of -55 mms., hence the punctum remotum of the eye is at $55 + 15 = 70$ mms. and this gives as the true myopic error $^{1000}/_{70} =$ 14.28 D.

71. In cases of hyperopia, however, since the far point is negative or virtual, the exact degree of hyperopia may be determined if allow-

ance is made for the distance between the glass and the eye. In fact, at whatever point a convex lens is placed in front of an eye, it is always possible to find one whose focus coincides with the far point of such an eye. It will cause parallel light to converge toward this point and will correct the hyperopia by enabling the eye to focus such rays upon the retina. Suppose an eye, Fig. 44, to have a punctum remotum at R such that $HR = 111$ mms. behind it. It has, therefore, a hyperopia of $^{1000}\!/_{111} = 9$ D. If we place the correcting lens immediately in contact with the cornea, practically at H, this lens must have a power equal to 9 D. But if we place it at 15 mms. farther from the far point, $-R$, it must then have a focal length equal to $OH + HR$, or $111 + 15$ mms. $= 126$ mms. representing a refractive power of 8 D. It is to be noted that from a strictly accurate, scientific standpoint all measurements should be made from the principal point H which is practically 2 mms. back of the cornea. When, for example, we refer

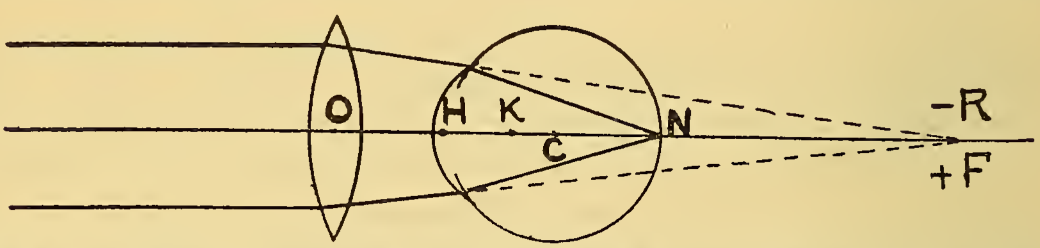


Fig. 44.—Diagram Illustrating the Optical Value of the Lens Needed to Correct a Specified Amount of Hyperopia, the Lens Being in Advance of the Cornea.

to a correcting lens placed 15 mms. from the cornea, the calculations should be made using 17 mms., the measurement of the distance from the principal point of the reduced eye to the position of the second principal plane of the correcting lens. Since the principal planes of a lens are dependent for their location upon the form and thickness of a lens (a subject which is properly treated under Ophthalmic Lenses and Prisms) our calculations are pertinent only to lenses which are thin bi-concave or bi-convex in form. The vertex refraction system, so-called, is the only correct method of applying lens corrections having the same power ophthalmologically, whatever the nature of the surfaces or thicknesses of the lens. This system, introduced we believe by von Rohr, of Jena, and proposed in his book, *Das Auge und die Brille,* is simplicity itself and is found from the back focus of a lens; it is independent of the shape of the lens and is based on the only factor which, in connection with the distance of the lens from the eye, is of significance in obtaining the correction. Two simple formulæ give the thin lens equivalent power of a thick lens and the vertex refraction thereof. These are:

$$(A)\quad D = D_1 + D_2 - \frac{d}{n} D_1 D_2 \text{ and}$$

$$(B)\quad D_V = D \times \frac{1}{1 - D_1 \frac{d}{n}}.$$

Equation (A) gives the dioptric power of a lens having dioptric curves of powers D_1 and D_2, of thickness d and index n. Equation (B) gives the vertex refraction D_V in terms of the dioptric thin lens power D and the power, D_1, of the side of the lens nearest the eye. The following brief table gives the vertex refraction of the correcting lens when placed at 13 mms. from the vertex of the cornea of an eye possessing the axial refraction stated in diopters.

Myopia		*Hyperopia*	
Axial Refraction in Diopters	Vertex Refraction of Correcting Lens	Axial Refraction in Diopters	Vertex Refraction of Correcting Lens
— 0.50	— 0.50	+ 0.50	+ 0.50
— 1.00	— 1.00	+ 1.00	+ 1.00
— 1.50	— 1.55	+ 1.50	+ 1.45
— 2.00	— 2.10	+ 2.00	+ 1.95
— 2.50	— 2.60	+ 2.50	+ 2.40
— 3.00	— 3.15	+ 3.00	+ 2.90
— 3.50	— 3.70	+ 3.50	+ 3.30
— 4.00	— 4.25	+ 4.00	+ 3.75
— 5.00	— 5.40	+ 5.00	+ 4.70
— 6.00	— 6.60	+ 6.00	+ 5.50
— 7.00	— 7.80	+ 7.00	+ 6.40
— 8.00	— 9.10	+ 8.00	+ 7.20
— 9.00	— 10.30	+ 9.00	+ 8.10
— 10.00	— 11.80	+ 10.00	+ 8.80
— 11.00	— 13.30	+ 11.00	+ 9.50
— 12.00	— 14.50	+ 12.00	+ 10.30
— 13.00	— 16.00	+ 13.00	+ 11.00
— 14.00	— 17.5	+ 14.00	+ 11.70
— 15.00	— 19.1	+ 15.00	+ 12.4

72. The terms far point and near point and methods for their determination have been briefly reviewed in the preceding paragraphs.

It is of importance to note once more that these measurements give the apparent or manifest refractive condition and the differences, dioptrically expressed, give the amplitude of accommodation. The far point can be, in turn, found from the value of the distance correction lens; in fact this is the general procedure in practice, after which the near point, with or without the distance correction as circumstances may demand, is found. Several illustrative examples will not be out of place. Suppose the near point is at 20 cms. (= 5 D.) and that the eye should be respectively hyperopic 2 D., emmetropic and myopic 2 D. The amplitude of accommodation is desired. Then in

Hyperopia 2 D, $A = P - R = 5 - (-2)\ D = 7\ D$
Emmetropia, $A = P - R = 5 - (0)\ D = 5\ D$
Myopia 2 D, $A = P - R = 5 - (+2)\ D = 3\ D.$

When, then, an ametropic condition is fully corrected, the near point shows the actual amplitude of accommodation provided we accept the validity of the ordinary near point tests. This means, for example, that if a hyperopia of 2 D. is corrected by a + 2 D. S., then the determination of the near point, the patient wearing his ametropic correction, gives the full available accommodation. If then the person under test wears full ametropic corrections he may be considered practically emmetropic refractively and accommodative tests, by near point or concave lens methods, give data upon the available amplitude. An uncorrected hyperope is under constant accommodative strain, an emmetrope exercises accommodation inside of infinity while a myope does not accommodate outside of his far point, which point is always at a finite distance. An emmetrope, therefore, whose near point is at 12.5 cms. has a range of accommodation of 8 D. Suppose a hyperope has the same near point, what is his accommodative amplitude? Since his far point is negative, the accommodation will need to act to make the image due to parallel light focus upon the retina and then proceed to focus from infinity down to the near point. The far point must, therefore, be known. Suppose he is a hyperope of 3 D. The near point is assumed to be at 12.5 cms. This person is possessed of 8 + 3 = 11 D. of accommodative power. If the far point is within infinity, at 50 cms. for instance, while the near point is at 8 cms., the eye will accommodate only within the 50 and 8 cm. points; this gives as an amplitude of accommodation 12.5 — 2 D. = 10.5 D.

73. The *reserve accommodation* is the difference between the total accommodation available in diopters and that needed at any specified point, usually the reading distance of about 13 inches. If a person under test develops 6 diopters of accommodation in toto, while in

daily practice his nearest work is at 20 inches, thus requiring normally the exertion of 2 D. of accommodation, the reserve would amount to 4 D. If this person should read at 13 inches, requiring 3 D. of accommodative action, the reserve at this point would amount to 3 D. It has been generally agreed that, for comfortable and satisfactory working conditions, at least *one-third of the total available accommodation must be kept in reserve.* When such a reserve is not found present, whether due to age or disease, such as diphtheria, scarlet fever and measles, which temporarily or permanently impair the accommodation, the condition may be classed as one of presbyopia, premature presbyopia or subnormal accommodation.

The term *presbyopia* means ''old-age sight''; by strict definition such is the case and it can be defined as that condition in which, on account of increased years, additional convex power is required for near vision. It may be defined, however, as that condition in which the near point has receded beyond 22 cms. or 9 inches or when the amplitude of accommodation is below 4.5 D. In a normal condition the strength of the Mueller's muscle, to which the accommodative changes are to be chiefly if not wholly attributed, varies with the age of the patient until advancing years cause it to lose practically all power. The recession of the near point is due, then, to a weakening of the Mueller's muscle and to a general loss of lens elasticity. The refractive power of the eye at rest does not change much, according to Donders, until the age of 55 years. The positive punctum remotum recedes from the eye and R_D becomes less; the negative punctum remotum of the hyperope comes nearer to the eye or $-R_D$ increases. The emmetrope commences to get hyperopic (acquired hyperopia and presumably not an axial or curvature condition at all), the myope notices a decrease in his myopia proportionate with the recession of the far point and the hyperope experiences an increase in his hyperopia. This decrease in the static refractive power of an eye is independent of the nature of the refraction. It affects the hyperope or myope in the same degree as the emmetrope.

Fig. 45, following Donders, shows the course of accommodation in an emmetropic eye. The figures at the top indicate the age; those at the side the amount of accommodation and the near point (P.P.) in centimeters; the oblique line *PP* represents the course of the punctum proximum and the horizontal line *RR* that of the punctum remotum. This diagram presents what may be considered a fair average of accommodative amplitudes for an emmetrope; it is not, of course, to be implicitly relied upon but serves rather to give an average value. The values given in the accompanying figure are probably too high; other determinations and sets of curves have been given by Risley, Duane, Jackson and others. In the curve *RR,* shown in Fig. 45, it will be noticed that the line begins to deviate downwards at an age point of 50 to 55 years; this indicates that the refraction begins to diminish, i. e., an emmetrope becomes hyperopic. In the case of myopia the curve *RR* is exactly the same as for emmetropia; the diminution of static refraction is the same; only the position of the punctum remotum in the two cases is not identical. The curve *RR* is bodily shifted into the positive portion of the diagram (since myopia represents an excess of refractive power optically considered) by an amount equal

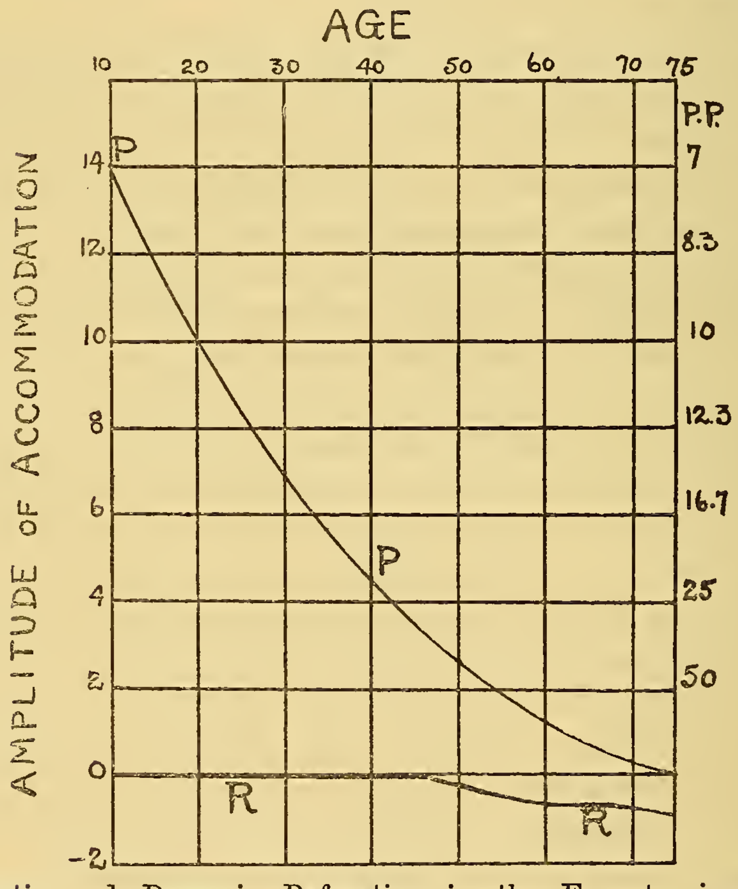


Fig. 45.—Static and Dynamic Refraction in the Emmetropic Eye. (After Donders.)

to the static myopia. The curve will then follow parallel to the line *RR* for emmetropia in everywise; the myopic *RR* curve may not, however, if the myopia is sufficiently great, ever cross over into the negative portion of the diagram. A myope of 1 D. will become emmetropic for distance at approximately 65 to 70 years according to these curves. In the case of hyperopia the entire curve *RR* will be below the zero line since hyperopia may be optically regarded as a deficit of refractive power. The decrease in refraction which is due to advancing age is added to the original hyperopia and increases it.

Likewise, as a general proposition, the change in the amplitude of accommodation is the same whatever be the refractive condition of the eye. Emmetropia and ametropia are alike subject to these laws governing the range of accommodative amplitude at different periods of life. If then the amplitudes of accommodation are practically independent of ametropic condition, for ametropia one has only to displace the zero line or the *RR* line to the condition of static refraction found and to bear in mind the ordinates in order to obtain the values along the *PP* curve. For example, assume a myopia of 6 D.; the curve *RR* would therefore have its zero point changed to + 6 and it would terminate on the right of the diagram (Fig. 45) between + 4 and + 3 D.; the curve *PP* would commence at 14 + 6 = 20 divisions above the zero line and would descend as in emmetropia. But while the range of accommodation (amplitude) is equal for different conditions of the static refraction of an eye, the positions of the near and far points are evidently not the same.

74. If the proportion of the total accommodation that can be comfortably and economically exerted be taken as one-half, then the *presbyopic correction*, D_P, in diopters can be found from the expression

$$D_P = D_R - \frac{A}{2}$$

in which D_R represents the dioptric value of the desired reading distance and A the amplitude of accommodation *supposedly* possessed at a given age. The amplitude of accommodation in presbyopia can be found by adding arbitrarily sufficient convex lens power as to permit of the patient's reading normal type. A determination of the far and near reading points will give a fairly accurate estimation of the range of reading amplitude in presbyopic cases. To illustrate; a person over forty years of age is under observation; the static refraction having been determined and supplied it is found that he cannot read No. 2 or No. 3 Jaeger type (V = 0.50 D. to 0.62 D.) at the normal reading distance. A + 1 D. lens is supplied with which he can read all near test-types; the nearest point at which he can read No. 1 or No. 2 Jaeger type is 10 inches = 4 D. and the farthest is 20 inches = 2 D. The range of amplitude of accommodation as determined from the near and far reading points is, then, 2 D. According to Donders' table he should be about 50 years of age. If this person reads and does close work at 16 inches or 40 cms. there will be demanded an accommodative equivalent of 2.5 D. Being provided with a + 1 D. S. and possessing 2 D. of amplitude, his accommodative resources will be sufficient theoretically, but practically they will not be, in general, since at least a good third of the total accommodation should be kept in reserve. Hence, approximately a + 1.50 D. S. or + 1.75 D. S. should be supplied.

When presbyopia is corrected the eyes are rendered artificially myopic by the convex lenses given and the range of accommodation is limited as in myopia. Suppose an emmetrope, or an ametrope made emmetropic by lenses, has 2 D. amplitude of accommodation; his punctum remotum is at infinity and his punctum proximum is at 50 cms. In order to correct the presbyopia present suppose + 1.5 D. lenses be given for 40 cms.; then the range of vision is between the artificial far point which is at $\frac{100}{1.5} = 66$ cms. and the punctum proximum which is at $\frac{100}{2 + 1.5} = 28$ cms. With the presbyopic correction given for 40 cms. he can see as far away as 66 cms. by a relaxation of all accommodative effort and as near as 28 cms. by exerting all his accommodation. Expressed in diopters, the far point is represented by the correcting convex lens and the near point by the convex lens plus the accommodative amplitude; in this case, then, $A = P - R = 3.5 - 1.5$ D. $= 2$ D. When the amplitude is greatly depleted the range of accommodation with the reading (or presbyopic) correction is very restricted. When the presbyopia is incipient the range is fairly large but becomes smaller as the natural accommodation is replaced by the artificial accommodation of convex lenses until, when it is all artificial, the amplitude becomes zero, the near and far reading points coincide and the presbyope can see clearly at one distance only with his correcting lenses.

75. *Aphakia.* Aphakia is due to absence of the crystalline lens and is a refractive anomaly of the eye usually due to artificial causes, i. e., extraction of the lens on account of cataract or dislocation of the crystalline from traumatism, but it is rarely congenital. When the crystalline lens, which has a power of about 16 D. *in situ,* is removed there is produced in an originally emmetropic eye a high degree of hyperopia. This defect can be corrected by a convex lens of less than + 16 D. power however. Accommodative action is nil; a few cases are on record where a patient has apparently been able to see distance and read ordinary type with the same lens, but in these cases the pupils invariably resembled narrow slits; with such a pupil and looking

obliquely downward through a strong convex lens it would be possible to read fairly small type since the peripheral portions of a high convex lens are more powerful dioptrically than the central portions, hence affording added lenticular assistance in reading.

When the crystalline lens is extracted the eye is reduced to an optical system of one refracting medium, the cornea, which then forms with the aqueous and vitreous a uniform medium of index approximating 1.33. If the length of the reduced eye be taken as 24 mms. and the radius of the cornea as 8 mms., then the anterior focal length, F_1, of the cornea is 24 mms. and the posterior focal length, F_2, is 32 mms. Hence the image in such an eye, devoid of the crystalline lens or its equivalent refractive power, tends to be formed considerably back of the retina and a clear retinal image can only be obtained by the use of a high power convex lens. This retinal image will, likewise, be considerably enlarged. The formula for conjugate foci of a single refracting surface is

$$\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1$$

In this case $F_1 = 24$ mms., $F_2 = 32$ mms. and $f_2 =$ the length of the eye globe $= 24$ mms. By substitution we have $f_1 = -72$ mms. The eye has, therefore, a far point equal to 72 mms. behind the cornea or $^{1000}/_{72} = 14$ D. of hyperopia. If the correcting lens is put 15 mms. in front of the cornea, the focal length of the correcting lens should be $72 + 15$ mms. $= 87$ mms. which represents a dioptral power of 11.5 D.

It would be incorrect to apply this numerical solution to all aphakic conditions; the original ametropic condition must be considered. The mathematical expression which we have just used is, however, applicable to the calculation of f_1 under any condition of length of globe, f_2. The lens correcting an aphakic eye in which the previously existing ametropia was known can be calculated in the same manner as for emmetropia. For example, suppose a myopia of 6 diopters existent before operation. Axially this eye is 2 mms. longer than the normal eye; hence $f_2 = 26$ mms. and $F_1 = 24$ mms. and $F_2 = 32$ mms. as before. Therefore, $\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1$ gives $f_1 = -104$ mms (behind the cornea); the correcting lens, being placed at 15 mms. in front of the cornea, must have a focal length of $104 + 15$ mms. or practically a dioptric power of + 8 D. An operation involving the cornea affects its general refracting power as well as causing astigmatism. Also, if the normal eye is 25 mms. in length and the corneal radius is 7.5 mms., the correcting lens in aphakia for an eye originally emmetropic would be weaker than 11.5 D.; calculation gives about 10 D. The following table was calculated by Dr. Stadfeldt.

Before operation	H = 7 D.	H = 5	H = 3	H = 1	E (= 0)
After operation	H = 15 D.	H = 13.8	H = 12.5	H = 11.3	H = 10.6
	M = 1	M = 3	M = 5	M = 7	M = 9
	H = 10.1	H = 8.9	H = 7.8	H = 6.6	H = 5.5
	M = 11	M = 13	M = 15	M = 17	M = 19
	H = 4.4	H = 3.4	H = 2.3	H = 1.3	H = 0.2
	M = 21	M = 23	M = 25		
	M = 0.8	M = 1.8	M = 2.7		

Pflueger, in his published results on a series of measurements before and after operation, concludes that the above values are approximately correct. His results show:—

Before operation—

M = 10 D. M = 11 M = 12 M = 14 M = 16 M = 18 M = 22

After operation—

H = 5 D. H = 5.5 H = 3.5 H = 3.5 H = 2.5 M = 2 M = 2

In order that an eye may be emmetropic after extraction of the crystalline it must previously have been myopic about 18 diopters. An approximate calculation as to the correcting lens is to add half the original correcting lens to 11 D. for the aphakic correction.

76. One or two points of optical procedure are worthy of note. The usual correction for aphakia following operation is a sphero-cylinder. These corrections are made up, in common usage, as a sphere on one side and a cylinder on the side nearest to the eye. This virtually amounts to a lens of plano-convex form while trial case lenses are double convex. The optical centers and principal planes being differently situated the sphero-cylindrical correction may not be quite equivalent to the trial case findings. Of greater importance, however, is the accurate record of the distance of the trial case lenses from the cornea; several instruments, such as the Wessely keratometer (Bausch & Lomb), are on the market by means of which such measurements may be made. The subjective findings in the trial frame can thus be reduced to their equivalent vertex refraction and the proper correction produced for any distance of the lens from the eye provided the

original distances of the trial lenses are known. The influence of the distance from the cornea upon the power of a sphero-cylindrical lens ocularly considered is considerable. An eye is corrected, for example, by a + 10 D. S. ⊃ + 4 cyl. at 15 mms. from the cornea. The lens has a focal length of 100 mms. in one meridian and 71 mms. in the other. The far point of the eye in one meridian is, therefore, 100 — 15 mms. = 85 mms., corresponding to a dioptric value of 11.75 D., and in the other meridian a far point of 71 — 15 = 56 mms. or 17.9 D. The astigmatism is really then 6 D. (approximately) instead of 4 D. This calculation elicits the statement that the ophthalmometric determinations give, with slight error, the true astigmatism of the eye considered as residing in the anterior corneal surface. In a case such as that just described, assuming that the astigmatism is not too irregular to be suitable for measurement with an ophthalmometer, the ophthalmometric reading would be higher than subjective tests would indicate. In the case of simple cylinders the same statement holds; a subjectively found convex cylinder of 6 D. corresponds to a true astigmatic error of about 6.5 D.; a concave cylinder of 6 D., on the other hand, with one of 5.5 D. Again, suppose the correction + 10 D. S. ⊃ + 4 cyl. was found at 15 mms. Suppose this is optically ground and furnished the patient so that it stands 12 mms. from the cornea. Our calculations then give 100 — 12 mms. = 88 mms. or 11.4 D. and in the other meridian 71 — 12 mms. = 59 mms. or 16.9 D. and the astigmatic difference is 5.5 D. The original correction showed a real astigmatism of 6 D.; hence this same correction worn at 12 mms. instead of 15 mms. would under-correct the astigmatism by 0.5 D. These are important practical points.

VII. CIRCLES OF DIFFUSION

77. When the image of a distant point object is received upon a screen and this, in turn, is moved to and fro, one position will be found in which the image is most distinct. In other positions of the screen there will be found a luminous spot of the same shape as that of the aperture in front of the lens and which changes its size and its brightness as the screen is advanced. This luminous spot is known as the *circle of diffusion*. The same phenomena occur in any lens system and the eye is no exception. But inasmuch as the retina is fixed in position the luminous object point must be moved. The pupil, being normally circular, will cause a round image of diffusion. If an object of finite size is employed and the image is formed in front of or behind the retina, then each point of the object produces on the retina a circle of diffusion which is overlapped by the next circle, except near

the edges of the diffuse image in toto. Around the contour of the object there is also formed a border which is equal to half the diameter of the circle of diffusion and the intensity of which diminishes toward the periphery. The size of the circle of diffusion can be calculated from the relation

$$r = \frac{pd}{d + f} \quad \text{(I)}$$

in which *p* represents the diameter of the pupil of exit, *f* its distance from the retina and *d* the distance of the distinct image from the retina, while *r* represents the diameter of the circle of diffusion. Of course the quantity *d* may be an additive or subtractive term in the denominator of the above fraction depending upon the refractive

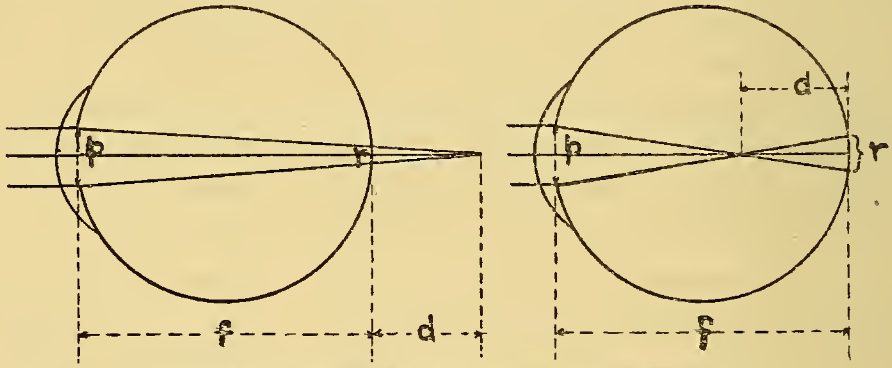


Fig. 46.—Size of the Circles of Diffusion.

anomaly. For hyperopia $r = \frac{pd}{d + f}$ and for myopia $r = \frac{pd}{d - f}$.

Fig. 46 illustrates the hyperopic and myopic conditions. The above relations can be deduced from it by the similarity of triangles. These effects have important bearing upon the size and clearness of retinal images under different ametropic conditions: the "blurriness" in astigmatism is explained by the overlapping or diffusion circles. If the pupil becomes smaller the diffusion circles are decreased. Contraction of the pupil, through the sphincter, is the normal accompaniment of accommodation: it is probable that the relatively high visual acuity of hyperopes may be accounted for in part by the reduction in the size of the diffusion circles through pupillary contraction. Especially is this true at the punctum proximum. The pupillary diameter decreases in general with age; this, according to Sulzer, explains why the belief has arisen that myopia decreases with age. The experiments of Bertrand have shown that in emmetropia a 3 mm.

pupillary diameter gives the best visual acuity; for 0.5 D. myopia a pupil of 2.5 mm.; 1 D. myopia, 1.75 mm.; 2 D. myopia, 1 mm.; 9 D. myopia, 0.5 mm. One, therefore, readily comprehends why ametropes and presbyopes prefer strong or powerful illumination since, by a contraction of the pupil under such influences, there is produced a considerable improvement in visual acuity. For a myopia of 2 D. with a pupil of 4.5 mms. a letter of 45 mms. size can be read at a distance of 5 meters; with a pupil of 1 mm. such a myope can read a small letter of 6 mms., that is to say, about 8 times smaller than the first sized letter, placed at the same distance. According to Bertrand a myope of 4 D., whose pupillary diameter can be reduced to 1 mm., will have the effects of his ametropic condition neutralized. The influence of the pupillary diameter upon the size of the circles of diffusion explains, in part, the great differences in the visual acuity of ametropes of the same degree.

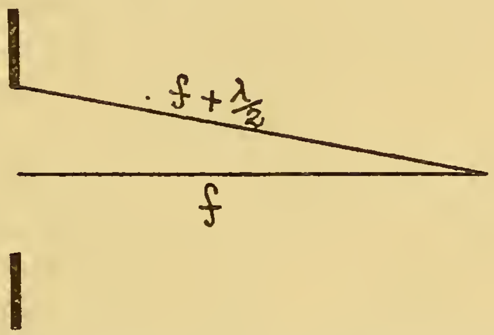


Fig. 47.—Illustrative of the Condition of Improvement of Vision by Small Apertures.

78. The improvement of vision produced by looking through an opening smaller than the pupil, as for example a pin-hole or a *stenopaic slit,* is due to the diminution of the circles of diffusion. This is why myopes see better at a distance and why in a great many instances vision is considerably improved by the subject due to the formation of a natural slit by narrowing the palpebral fissure; this is particularly true in myopic astigmatism. Such an opening can also be used as a magnifying device; an object can be moved very close to the eye and a large retinal image thus obtained; there is, of course, a loss of image brightness at the same time since the same total luminosity is distributed over a larger retinal area. There is a limit to the smallness of the size of the stenopaic opening, however. It is a theorem of physical optics that the maximum concentration of light at the center of an image is obtained when the axial and marginal rays coming through the hole differ in length by one-half a wave-length of the light employed. This is illustrated in Fig. 47, where the ray coming

from the edge of the hole is represented as $\frac{\lambda}{2}$ longer than the central ray *f*, λ being the wave-length of the incident light. If *r* is the radius of the hole, then it follows that

$$r^2 = \left(f + \frac{\lambda}{2}\right)^2 - f^2$$

$$= \lambda \cdot f \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(II)}$$

when $\frac{\lambda^2}{4}$ is negligible. Hence the limiting value physically is given by $r = \sqrt{\lambda \cdot f}$. If a luminous point which is distinctly seen is looked at through a very small opening it will become enlarged into a small luminous surface surrounded with bright rings. The effect of *diffraction*, according to Tscherning, begins to play a part with an aperture of the pupil or of the stenopaic opening of about 2 millimeters. This explains why very small apertures should be avoided in retinoscopes used for skiascopic investigations of refractive errors. A too strong illumination should also be shunned since pupillary contractions thereby result which are not present when the patient is under normal and work-a-day conditions. Likewise these statements should serve as a warning against the use of too narrow stenopaic openings in the determination of refractive errors. In passing it may be stated that such openings improve the visual acuity not only because of a reduction of diffusion circles but also because they eliminate spherical aberration effects, a subject yet to be discussed.

79. The diffraction caused by a circular screen of radius *p* gives rise to concentric dark rings. The first minimum, as proven by Neumann, occurs when the angle of diffraction is such that

$$\sin \Phi = 0.61 \frac{\lambda}{p} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(III)}$$

From equation II, $r = \sqrt{\lambda \cdot f}$ or δ, the diameter, equals $2\sqrt{\lambda \cdot f}$. Since the arc divided by the angle in radians is always equal to the radius, then, from equation III,

$$\delta = \frac{\lambda}{\Phi} = \frac{p}{0.61} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(IV)}$$

since $\delta^2 = 4\lambda \cdot f$, then $\delta \cdot \delta = 4\lambda \cdot f$ and a substitution of the value of δ just deduced gives the equation

$$\delta = 2.440 \frac{\lambda \cdot f}{p}$$

If, then, p is the diameter of the aperture of the refracting system, f the position of the image produced by the same, λ the wave-length of light, the diameter of the mean diffusion circle, δ, is given by the expression

$$\delta = 2.440 \frac{\lambda \cdot f}{p} \ldots\ldots\ldots\ldots\ldots\ldots (V).$$

Taking λ as $\frac{1}{2000}$ mm. as the average wave-length and f of the eye equal to 20 mms., expression (V) reduces to

$$\delta = 0.0244 \frac{1}{p}.$$

If, then, the smallest pupillary diameter is taken as 2 mms., $\delta = 0.0122$ mm. This size of the diffusion circle corresponds to a visual angle

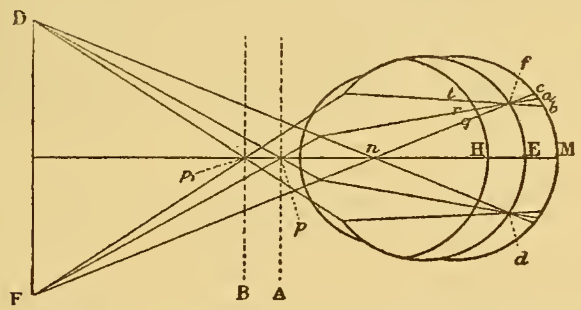


Fig. 48.—Effect of a Stenopaic Opening.

of 2 minutes and 6 seconds. Since the minimum visual angle separating two points is usually taken as 1 minute, it can be readily seen that the diffraction produced by very narrow pupils must influence the acuteness of vision.

The effects of a narrow stenopaic opening upon the size of retinal images in various refractive conditions can be seen from an inspection of Fig. 48. Let *DF* be a line object and *df* its retinal image which we construct by tracing the straight lines passing through the nodal point *n*. *H* indicates the position of the retina in an hyperopic eye, *E* of an emmetropic and *M* of a myopic eye. Place in front of the eye a screen *A,* perforated at the point *P,* situated on the axis. The only rays proceeding from F which can, under these conditions, enter the eye is the ray *FP,* with the result that, in the hyperopic eye, the image is not formed at *q* but at *r.* The image of *DF* is then correspondingly enlarged and it will be found that the magnifying effects will be increased if the screen *A* is moved farther from the eye. In the myopic eye the image is not formed at *a* but rather at *b,* with the slit at *A,* with the result that the retinal images are minimized.

VIII. CHROMATIC ABERRATION

80. A point source of white light on the axis of a single lens never gives rise to an image at a single point. There are two effects which vitiate this much to be desired result, namely, *chromatic* and *spherical aberrations.* The latter of these defects finds its counterpart in the eye and has to do with distortion, unequal magnification and curvatures of the image. Chromatic aberration of a lens arises from the consideration that every lens is fundamentally a composite prism and every prism has an angle of refraction, an angle of deviation, a resolving power and a dispersive power. This last property is its ability to produce from composite light a spectrum or analysis of its constituent parts. A lens, therefore, if one considers the central portion only, causes a series of colored images or points to be formed on the axis, the blue end of the spectrum being nearer the lens. This is represented in Fig. 49. If a single lens is used to form an image on a screen it will be impossible for the various colored images to be simultaneously in focus. The ordinary lens formula is

$$\frac{1}{F} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

wherein *F* and *n* represent the average focus and index respectively for yellow (D line) light. Writing the above equation, in turn, for F_r and F_b, where *r* represents red (C line) and *b* blue (G line), we have by subtraction

$$\frac{1}{F_b} - \frac{1}{F_r} = \frac{n_b - n_r}{n-1} \cdot \frac{1}{F}$$

$$\text{or} \quad F_r - F_b = \frac{n_b - n_r}{n-1} \cdot F.$$

Hence the chromatic aberration for parallel rays is equal to the mean focal length of the lens multiplied by the dispersive power of the substance of which the lens is composed. *Achromatism* may be obtained under certain conditions with two or more lenses; if the lenses are in contact achromatism exists when

$$\frac{\omega_1}{F_1} + \frac{\omega_2}{F_2} = 0,$$

an equation which states that the dispersive powers (ω_1 and ω_2) are inversely proportional to the focal lengths. This equation also shows that one lens of the combination is, of necessity, a concave or minus dioptric power member when the two lenses making up the combination are in contact.

81. The eye is not an achromatic combination although, in everyday life, the chromatic aberration is not noticeable. That it does possess this error can be shown by the following simple experiment: look through a pinhole at the line of separation of a roof against a bright sky. Slowly raise the pinhole, which will allow light to enter the peripheral portions of the pupillary area. The sky just above the roof will appear of a reddish color. A flame, when thus looked at, will appear blue in the upper portion and red in the lower part. This is explicable remembering that retinal images are inverted and that light when incident at the upper edges of the lens will have its blue component deviated downward more than its red component. A ready experimental test is to color a printed page in alternate red and blue indigo or cobalt blue and attempt to read; the constant change of focus demanded is small but extremely annoying. Another method of rendering the chromatic phenomena of the eye easily visible consists in eliminating the middle of the pupil by means of a narrow ribbon of black paper. A white line on a black background, placed inside of the far point, will then appear double; the inner borders of the two lines will appear blue and the outside edges red. If the object is placed outside of the punctum remotum these phenomena will be reversed; also, if the colors of object and background are interchanged the order of colors will be reversed. Or if one half of the pupil be occluded by a card, the eye is converted into a strong prismo-sphere and the colored fringes will fall upon the macula and be seen as such when a small white body on a black background is viewed. If the lower half of the pupil be covered the exposed half of the eye acts as a prism base down so that a blue-violet fringe is seen at the upper and a red-orange fringe at the lower edge of a luminous body.

We frequently observe very striking phenomena due to ocular chromatic aberration by fixing black objects on a white ground placed at a distance for which the eye cannot accommodate itself. When looked at toward the sky, the slits of the optometer of Young present very vivid colorings.

The chromatic aberration increases with the diameter of the pupil. It is useful, therefore, in studying it to make use of mydriatics.

82. Young made the first estimates upon the chromatic aberration of the eye, while Fraunhofer made the first reliable measurements on the difference in the focal lengths of the eye for the extreme spectral colors. He observed a prismatic spectrum through an achromatic telescope the eye-piece of which carried a cross-hair. Fraunhofer noticed that he had to move the ocular nearer the cross-hairs, for clear vision, when observing the violet portions in contradistinction to the red regions. By fixing an external point with one eye he so adjusted the eye-piece as to give equal distinctness of the cross-hair and object in two spectral regions. The optical constants of the eye-piece being known, the corresponding visual distances could be found. He discovered from these researches that an eye which sees, without accommodation or practically at infinity, an object of color corresponding to the spectral line *C* (between orange and red) cannot see this object clearly, with the same accommodative status, in light of wave-length corresponding to line *G* (between green and blue) unless it is some 18 to 24 inches nearer the eye. Helmholtz modified the experiment somewhat and allowed monochromatic light to pass through a very small opening in a screen; he then determined the greatest distance at which this opening still remained of pin-point form. The greatest visual distance in red light was 8 feet, in violet 1.5 feet and in the extreme violet bordering on the ultraviolet about 1 foot. These data show that Fraunhofer found about 1.5 to 3 D. and Helmholtz 1.8 D. as the chromatic aberration of the eye.

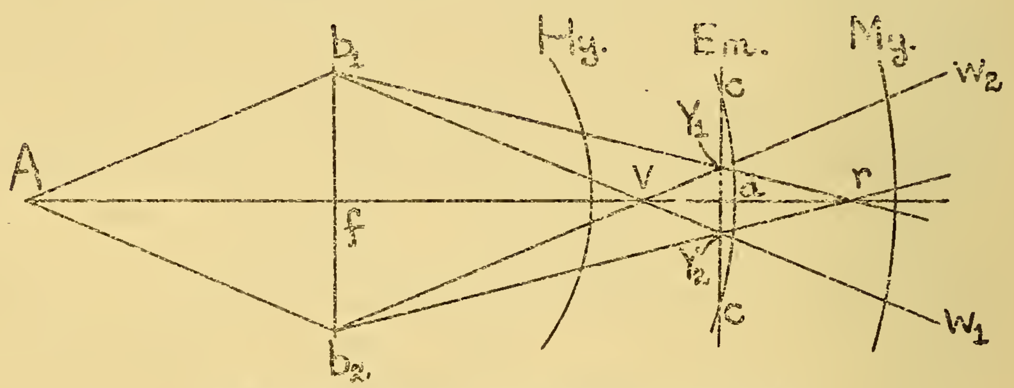


Fig. 49.—Chromatic Aberration of a Lens.

A luminous point seen through a prism gives a linear spectrum. If the experiment is made, however, the whole of the spectrum is not at one and the same time distinctly seen. If the luminous point is at a considerable distance, the emmetropic eye will see the red extremity of the spectrum as a distinct band or ribbon (line) while the blue extremity will be enlarged, fuzzy or diffuse. If the luminous source be approached, the eye still not accommodating, a distance will be found such that the blue end of the spectrum will be sharp and the red portion, in turn, indistinct. A determination of the far point for each end of the spectrum will give a measure of the degree of chromatic aberration.

83. The *cobalt glass test* has been and is still at times used in the determination of refractive conditions. It is based in operation upon the chromatic aberration of the eye. If a distant point light source is viewed through the glass, which transmits only the extreme portions of the spectrum, and the eye is emmetropic, one composite color effect only will be noticed. If the eye is hyperopic, the far point is back of the retina and the conditions shown at *Hy* in Fig. 49 will be present; that is to say, a blue center surrounded by a red halo. If, in turn, the myopic condition *My* (Fig. 49) is investigated just the reverse positions of the color phenomena will occur. If, then, the luminous object is inside the near point it will be seen as blue surrounded by a red halo; if the luminous point is beyond the far point there will be, on the contrary, a red center surrounded with blue.

84. Recently Nutting has given a precise method for determining the axial focal lengths. The monochromatic rays from a Nernst or mercury lamp pass through a slit and then through a moveable achromatic lens, of 20 cms. focal length, to the eye. The image of the slit formed by the lens serves as a test object. The accommodation is fixed by means of a glass plate placed at 45 degrees between the eye and the lens, reflecting the image of a side object, such as a dark letter or distant tree trunk, at the desired distance. A shift of the lens by 1 cm. corresponded, in Nutting's experimentation, to 0.01 mm. shift in focal point at the retina. The results of seven observers are shown in Fig. 50. The axial error curve for pure water is given for comparison. In the most luminous part of the spectrum, from $\lambda = 5200$ to $\lambda = 6600$ Angstroms, all eyes tested showed less variation in focus than an equivalent eye of pure water would have.

Since the refractive indices of the optical media of the eye do not differ greatly from the index of water, the above comparison of experimental results with the human eye to the "pure water" eye seems legitimate. The refractive indices for water for various wave lengths may be taken as follows: for red light (line C) 1.331705, for violet light (line G) 1.341285. Applying the methods for calculating reduced eyes to the Listing eye, having the radius of its refracting surface equal to 5.125 mms. and using the above indices, the focal lengths can be calculated as 20.574 mms. for the red and 20.140 mms. for the blue. This gives a chromatic aberration of 0.434 mm. which corresponds to practically 1.1 D. Or, if an eye is accommodated for infinity for the red so that the retina is at the focal point of the red rays, then the focus for the blue rays lies some 0.434 mm. in front of it, with the result that this eye must accommodate for a 26-foot distance

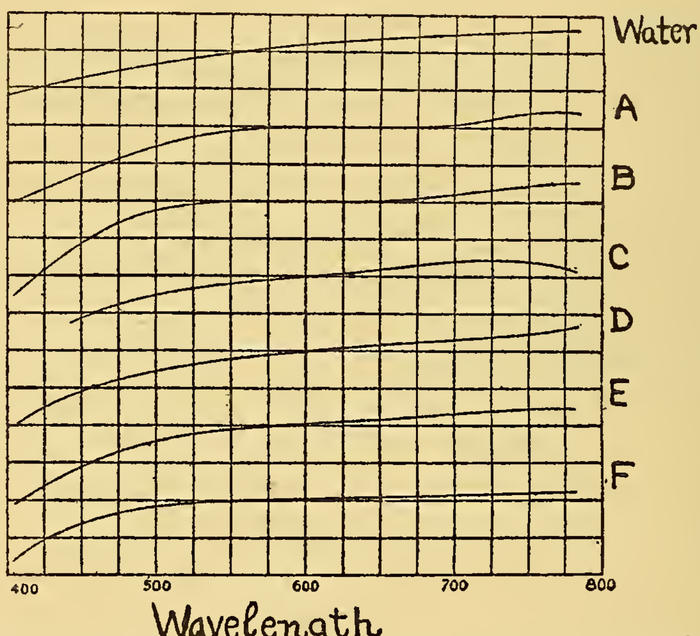


Fig. 50.—Curves of Nutting on the Axial Focal Lengths of the Eyes of Various Observers for Different Wave-lengths of Light.

in order to bring the blue in focus upon the retina. Fraunhofer found in his own eyes 18 to 24 feet. Helmholtz obtained similar results. Matthiessen calculated from his researches on this subject about 0.58 to 0.62 mm. instead of Nutting's value of 0.434 mm. The results obtained by different observers following various experimental methods indicate that the human eye approaches very closely in its chromatic aberration to an eye of distilled water but that it probably has a greater dispersion.

85. While the eye is not achromatic, yet when an object is at such a distance that it can be seen distinctly we do not see it surrounded with colored borders. An explanation can be given somewhat as follows:—Let A (Fig. 49) be a luminous object which sends the cone Ab_1b_2 into the eye. After refraction, chromatic aberration occurs;

the red rays form the cone b_1rb_2 and the violet rays the cone b_1vb_2 and the eye accommodates itself in such a way that the retina is between the two foci, placed so that the red and blue diffusion circles overlap at y_1y_2. The yellow and the green portions of the spectrum, which are intermediate between red and blue, and which are the most useful visually, are therefore concentrated at the middle of the diffusion circle when they coincide with a portion of the red and of the blue. The peripheral parts of the red and violet form a purple border all the way around but this border is very narrow, since the region y_1y_2 is specified and known as the *circle of least confusion*. Being formed by the extreme spectral rays, which are of little service visually, this border is too weak to be perceived. In order to calculate the size of this circle, y_1y_2, we need to consider the eye as a Listing's reduced eye of water. From Fig. 49 it follows that

$$\frac{y_1y_2}{b_1b_2} = \frac{dr}{fr} = \frac{dv}{fv}$$

$$\text{Therefore,}\quad y_1y_2 \cdot fr = b_1b_2 \cdot dr$$
$$\text{and}\quad y_1y_2 \cdot fv = b_1b_2 \cdot dv.$$

By addition there follows the equation

$$y_1y_2\,[fr + fv] = b_1b_2\,[dr + dv]$$
$$= b_1b_2\,[fr - fv]$$

$$\text{Hence}\quad y_1y_2 = b_1b_2\,\frac{fr - fv}{fr + fv}.$$

If b_1b_2 is placed equal to the average diameter of the normal pupil, i. e., 4 mms., and the determinations of the focal lengths of the reduced "water" eye used, in which $fr = 20.574$ mms. and $fv = 20.140$ mms., then $y_1y_2 = 0.0426$ mm. A calculation similar to that made by Helmholtz (*Physiologische Optik,* I Band, edition 1909, page 111) for the Listing's eye shows that the size of this dispersion circle is the same as that which is produced when a luminous point is situated at 1.5 meter from the eye adjusted for infinity. Such a variation in the accommodation should produce a decided inexactness of image. In order to explain why the chromatic dispersion in the eye produces no noticeable inequality of images in spite of the inequality of the circles of least confusion one must consider not only the size but also the distribution of light in these circles. This question of brightness distribution Helmholtz has discussed in a masterly manner. His mathematical calculations lead to a curve of the shape shown in Fig. 51. The distances from the center are plotted along the horizontal or "x" line; the brightness along the vertical or "y" line. The line *ab* corresponds to the brightness of the middle of the surface; *c* corresponds to the position of its edge; the dotted line *adc* shows the brightness distribution of a very sharp, clear image. The limits of the dispersion circle from *c* are *b* and *g*. The curve shows clearly that the brightness curve falls off extremely rapidly at the point *f* and that the portion of the curve *fg* and area of luminosity *fgc* are fairly negligible.

Clearness or sharpness of vision is not apparently produced by the correction of chromatic aberration. An eye can be corrected for these errors by means of a concave lens of flint in the same manner as a convex lens of crown glass can be achromatized. The dispersive

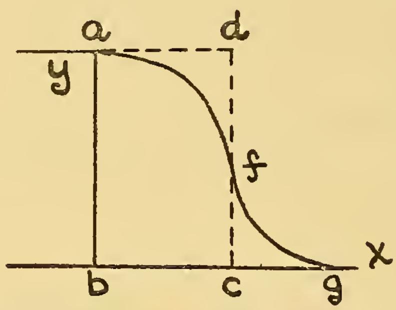


Fig. 51.—Illustrating Brightness Distribution. (After Helmholtz.)

power of glass is about one-third that of the eye; since the refracting power of the eye is approximately 60 diopters there would be needed a concave flint lens of about 20 D. A myope of 20 D., fitted with a flint glass lens, would have his ametropic and chromatic defects simultaneously corrected. An emmetrope would need in addition a second achromatic convex system in order to optically neutralize the concave lens correcting the chromatic aberration. This could be done by means of a lens of crown and flint components so calculated that their combined dioptric powers would be zero for yellow, about + 0.5 D. for red and — 1 D. for blue light. To accomplish this the crown glass component would need to be about + 66 D. of refractive efficiency (v) equal to 55 and the flint component — 66 D. of refractive efficiency $v = 27.5$. Although the effects of chromatic aberration are quite negligible, an ordinary convex lens tends to increase and a concave lens to decrease the natural chromatism of the eye before which it is placed.

IX. SPHERICAL ABERRATION

86. *Caustic caused by refraction at a curved surface and the phenomenon of spherical aberration as exhibited by a lens.* In Fig. 52 let *AMP* be a spherical surface of which *C* is the center of curvature, *P* the pole and *PC,* produced, the axis. Air and glass are the media. The paths of the various refracted rays due to the incident parallel rays may be found according to the law of refraction. It will be seen that the rays refracted near the pole cut the axis and each other at a point *F*. On the other hand, rays refracted at the surface remote from the pole cut the axis at points nearer to the surface than *F*. The more remote the point of incidence the nearer the point at which the refracted rays cut the axis. This phenomenon is termed *spherical aberration.* Rays refracted at neighboring points of the surface somewhat remote from *P* intersect each other before reaching the axis.

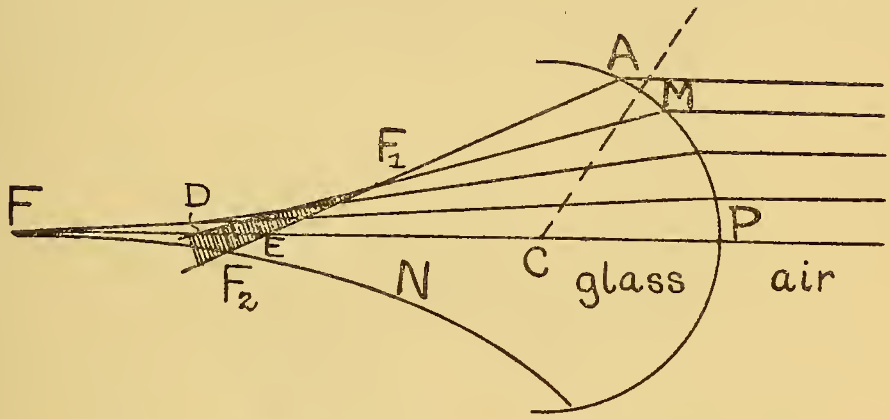


Fig. 52.—Focal Lines Formed by Refraction at a Spherical Surface.

Each point of intersection is a sort of focal point; the curve joining them is termed the *caustic curve;* its form is indicated in the lower portion of the diagram as *FN*. Taking any two isolated rays at *A* and *M* remote from the axis, together with the corresponding refracted rays *AE* and *MD,* we see that these rays cut each other at F_1. Supposing everything rotated through a small angle to give a surface instead of a cross-sectional view, it will be found that a parallel pencil incident at *AM* gives rise to an astigmatic pencil passing through two perpendicular lines at F_1 and F_2 respectively. These lines at F_1 and F_2 are known as the *first* and *second focal lines.* Somewhere between F_1 and F_2 the section of the refracted pencil is approximately circular and is known as the *circle of least confusion.* It can be seen, then, that when the aperture is not very small the rays do not, after refraction, meet at a point; the peripheral portions are more refracting than the central ones. The degree of aberration increases as the square of the aperture of the lens and as the cube of its refracting power. It likewise depends upon the distance of the object and the form of the lens. A lens, for example, having a central power of 20 D., when examined at a point 15 mms. from the lenticular center shows the following results:—crossed lens, 21.1 D.; plano-convex with the convex surface in front, 22.3 D.; bi-convex, 23.6 D. and plano-convex, with the plane surface toward the incident light, 23.8 D.

87. Schiener used a two-apertured opaque screen in his classic method of determining the near point and refractive condition of the

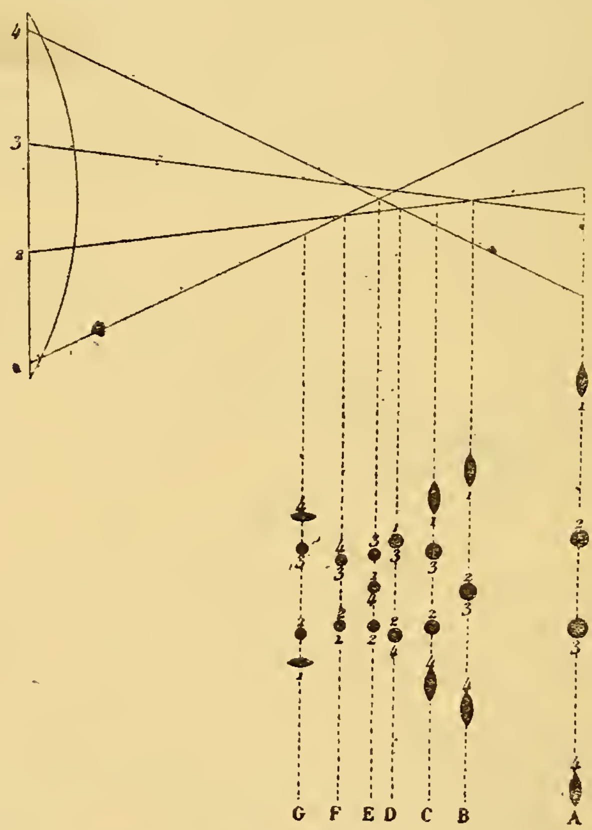


Fig. 53.—The Spherical Aberration of a Lens.

eye. This same device can be used to determine the aberration values and forms of image due to a lens. Four apertures may be used. These are equidistant; two are near the center and two are situated at the peripheral regions of the lens. The lens is illuminated with a broad light source at a considerable distance from it; a screen is used to receive the images and is placed in various positions with respect to the lens. Four images will be formed as shown in Fig. 53; if the screen is outside the principal focus for the central apertures the images will be inverted (A, Fig. 53); the central images will be circular and the peripheral images, in turn, spots elongated in the vertical direction due to eccentric refraction. By moving the screen nearer

(B), the two central images may be made to coincide; this gives the principal focal point of the near-central portion of the lens. Advancing the screen still nearer to the lens a position (E) will be found in which the peripheral region images will blend, hence giving the focal point due to the border portions of the lens. Passing on still nearer to the lens, the four images appear as shown in (G); they are now received on the screen, however, in such positions as put them in juxtaposition with the apertures which give rise to them. The peripheral spots are now elongated in the horizontal direction, however. To determine the degree of aberration it is necessary to measure the position of the focus of the central portions and again of the peripheral portions and take the difference between these distances expressed in diopters. If the circles of diffusion are to be examined no perforated screen is to be employed; as long as the screen is situated beyond the focus the light is concentrated at the middle of the circle and the brightness diminishes rapidly at the periphery; the distribu-

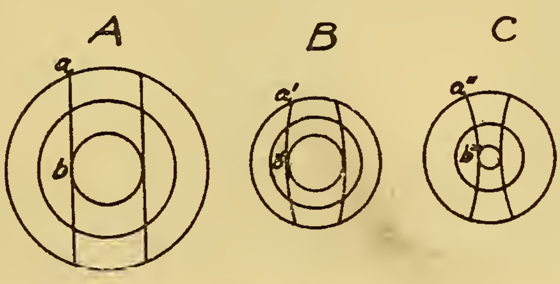


Fig. 54.—Deformation of the Shadows of a Needle by a Lens.

tion of luminosity is similar to that shown in Fig. 51. When the screen is inside the focus, however, there will be found a luminous disc surrounded by a more brilliant circle.

88. Images will be distorted or deformed because of spherical aberration. Likewise, shadows produced by placing an object, such as a needle, against the surface of a lens, will exhibit similar effects upon a screen; the shadows are visible in the circle of diffusion. These effects are illustrated in Fig. 54. The shadow is straight only when the needle coincides with a diameter of the lens, otherwise it is curved. In Fig. 54, A, is shown the needle in contact with the lens. If the screen is between the focus and the lens the shadow will be concave toward the center (Fig. 54, B) and when placed outside the focus it will be convex toward the center (C). These results are explicable by reference to Fig. 53, which shows that after refraction the corresponding zones of the circle of diffusion diminish in width toward the periphery when the screen is situated between the focus and the lens, while they increase in turn toward the periphery beyond the

focal point. The concentric circles shown in Fig. 54 represent these corresponding zones; two needles are shown and represented by the heavy and dotted lines in order to indicate the direction of the curvature of the shadow and the position relative to the object which gives rise to it. An over-corrected lens exhibits all these phenomena but in the reverse order; an aplanatic lens is free from all of them and the shadow of the needle will remain straight in all positions of the screen.

89. *The aberration of the human eye. Young's researches.* The great savant, Young, prepared a series of experiments which conclusively prove that the eye is not aplanatic. (1) A myopic eye, or one which is mechanically made so by the addition of convex lens power, sees a distant luminous point as a circle of diffusion with its bright-

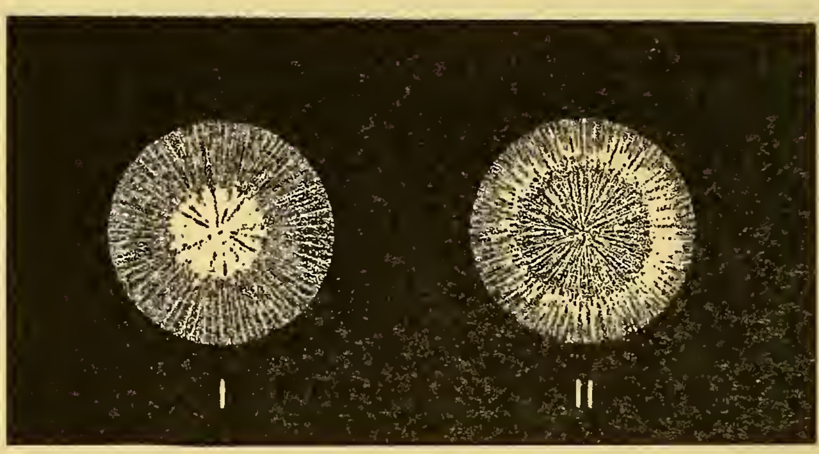


Fig. 55.—Distribution of the Light of the Circle of Diffusion in an Eye with Strong Aberration. (After Antonelli.)
In I the luminous point is beyond and in II within the focus.

ness concentrated at its center if the eye has positive spherical aberration. If the aberration is over-corrected, or if the luminous point is inside the far point, the peripheral regions will be the more luminous: an eye exhibiting such a condition is spoken of as possessing negative spherical aberration. These two conditions of under- and over-corrected or positive and negative aberrations may be seen objectively very often when an eye is examined by the methods peculiar to skiascopy, especially if the pupil is naturally or artificially dilated. The distribution of light in the circle of diffusion of an eye possessing strong aberration is shown in Fig. 55; in (I), with + 4.5 D. S., and exhibiting positive spherical aberration and in (II), with — 7 D. S. showing that the aberration is overcorrected. (2) Placing a needle in front of the eye made myopic during the performance of the test given in (1), the shadow of the needle will be seen in the circle of diffusion. If the shadows remain straight at all points there is then no appreciable or perceptible aberration; if it is concave toward the periphery ordinary aberration is indicated but if it is concave toward the center there is evidence of an overcorrected aberration. If an eye is made hyperopic by means of a strong concave lens the observed phenomena are just the reverse of those given above if the accommodation is not exerted. This experiment can be performed with the needle in different meridians and proof obtained of the fact that the aberration is not always the same in the different directions.

90. *Experiments of Volkmann.* Volkmann applied the method of Scheiner, described in connection with Fig. 53, to determine the aberration of the eye. He used four openings in the positions indicated in Fig. 56 (A). Looking at a pin placed beyond the punctum remotum through these openings, four pins are seen as in Fig. 56 (B) *a*.

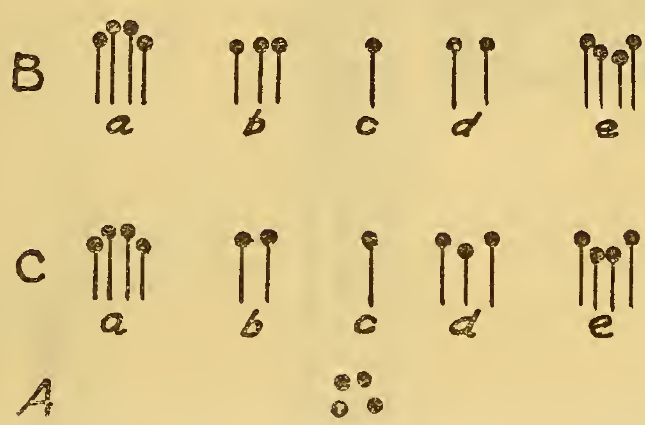


Fig. 56.—Illustrative of the Experiment of Volkmann.

By moving closer to the pin he observed the series of changes diagrammed in (B) in the order shown from *a* to *e;* these results are accounted for when they are compared with those shown in Fig. 53 illustrating the various images obtained by a lens exhibiting spherical aberration. In the position *b* the point is exactly at the far point of the central portions of the pupil since the two central images are united into one, It is still, however, beyond the focus of the peripheral portions since two peripheral images appear. In (C) are shown the observed order, number and relative positions of the images of the pin as seen by an eye over-corrected aberrationally. In the position *d,* Fig. 56 (C), the pin is at the far point of the central portions and within the peripheral far point. Tscherning remarks that it is probable that these latter results are due to accommodation because it is rare to find overcorrected aberration in an eye in a state of repose.

During accommodation the aberration is, as a general rule, overcorrected. This effect may be shown in a very simple manner as described by Edser. Look with one eye, the other being occluded, at the upper edge of a printed square (or a sheet of paper) placed just beyond the shortest distance of distinct vision. Cover the eye slowly from below by means of a cover-card; just before the edge of the object viewed vanishes it will be seen to sink. The upper edge of the figure or sheet of paper lies on the optic axis of the eye. The rays from it, traversing the middle of the pupil, form an image at the point where the optic axis meets the retina. Those traversing the upper edge of the pupil are insufficiently deviated and thus form an image above the true one; the interpretation of objects in space by the law of projection causes the object viewed to appear to sink.

91. *The optometer of Thomas Young.* The optometer of Thomas Young,—of which Tscherning says: "It appears to me to be one of

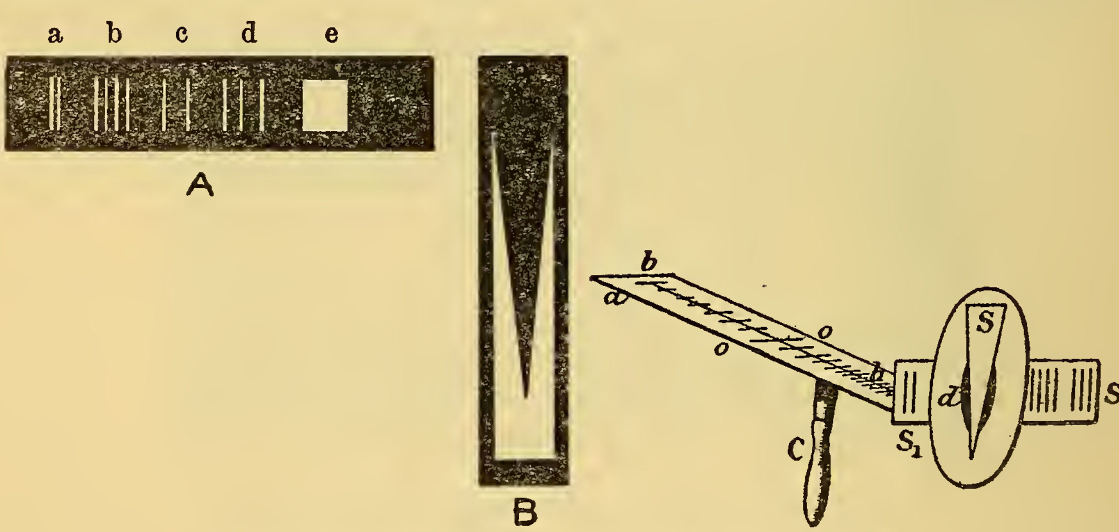


Fig. 57.—The Rules of the Optometer of Young.

the most important instruments for the study of physiologic optics,"—enables us to measure spherical aberration directly. It is in the form of a little rule carrying a fine white line on a black background on one side. The observer looks along this line through a + 10 D. lens. In front of the lens is placed a small horizontal rule, free to move and carrying different groups of slits. Two slits in this rule will act like the openings in the experiments of Scheiner. Each point of the line appears double except that which is seen distinctly; in this manner it may be used to determine the far point of an eye by placing a small cursor at the point where the observer sees the lines intersect. The rule carries various sets of slits differing in number and position thereby permitting the determination of the refraction at different parts of the pupillary space. One may also make use of a second form of rule,—placed vertical and having an M-opening thus producing a pointed metal triangle at the center of this M-shaped opening,—

to eliminate a greater or smaller portion of the middle of the pupil. These two instruments are shown in Fig. 57. The horizontal slide carrying the slits is as *A* and the vertical slide with the M-opening as *B*. The portions which are cut away or are open in these instruments are printed in white in Fig. 57.

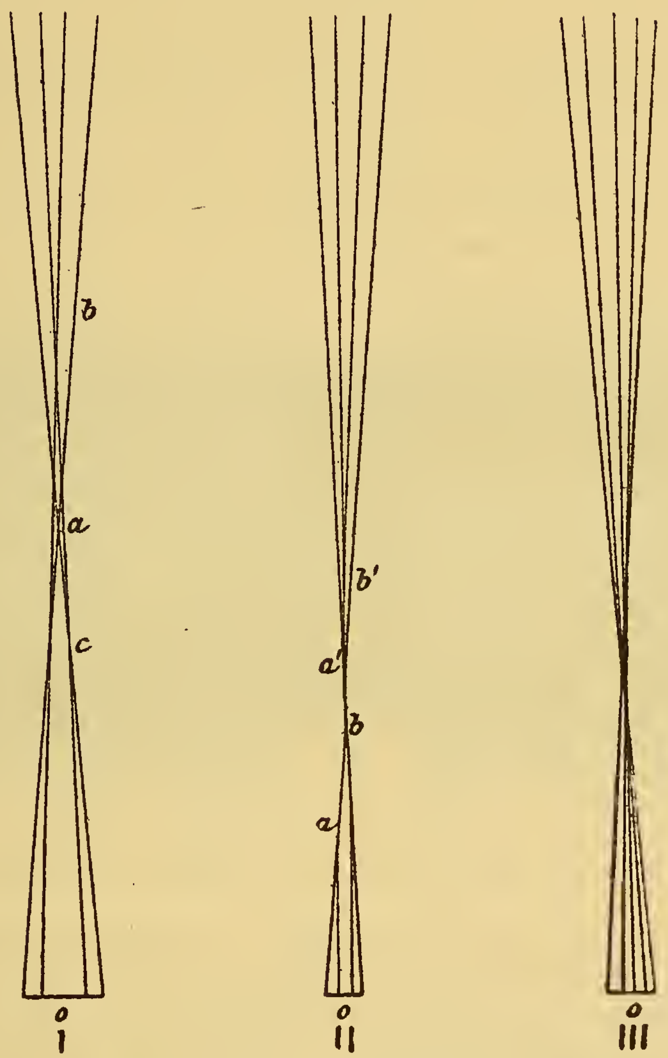


Fig. 58.—The Appearance of the Line of the Optometer of Young.

Seen through four slits by one eye with strong spherical aberration. O, position of the eye; a (a′) far point of the peripheral parts; b (b′) far point of the central parts.

This instrument does not afford a satisfactory device for the examination of refractive errors because it is extremely difficult for an inexperienced observer to use it without bringing into play his accommodation. If one can control his accommodation this instrument gives a means of measuring simultaneously both the refraction and amplitude of accommodation since the near point may be determined in the same manner as that described for the far point. This device of

Young does permit, however, of a direct measurement of aberration. In the horizontal rule (Fig. 57 A) there are placed at *a* two narrow slits very close together. The central refraction of an eye may thereby be determined by finding the point of intersection of the two apparent lines present when the observer looks at the single line along the auxiliary rule. The two apparent lines must be equally distinct; when this is the case the slits are practically at the center of the pupil. The quadrangular opening *e* is then slipped in front of the lens and the slide *B* carrying the triangular shaped piece is slowly lowered, thus cutting off a greater and greater portion of the middle of the pupil. Two lines will then be seen which separate farther and farther until one of the lines disappears. The difference between the measurement made at this point and that obtained by the central refraction

I II

Fig. 59.—Deformity of the Shadows in an Eye with Strong Spherical Aberration. (After Antonelli.)
I, in a state of repose; II, during accommodation.

using the two narrow slits measures the maximum degree of aberration. Young made these two measurements at the same time by employing four slits as shown at *b* in Fig. 57 A. This method is much better but is more troublesome. With the four slits a corresponding number of lines will be seen as in Fig. 58. If there is spherical aberration the central lines will intersect at a point *b* farther away than the peripheral lines which meet at *a*.

92. *The aberroscope of Tscherning.* Tscherning constructed a simple instrument known as the aberroscope for the subjective determination of aberration. It consists of a plano-convex lens, suitably mounted on a handle, on the plano side of which is cut a mesh of small squares. A distant luminous point is viewed through this lens, situated some 10 to 24 centimeters from the eye, and the observations made upon the character of the lines seen, i. e., as to whether they are curved or not. Most persons show a certain degree of positive aberra-

tion which corresponds closely to the toric form of the optic part of the cornea. Fig. 59 (I) shows the deformity of the shadows in an eye with strong aberration and when in a state of repose; part II of this figure shows that this aberration is largely corrected during the act of accommodation. Fig. 60 indicates that the aberration is over-corrected toward the borders and Fig. 61 shows aberration over-corrected in the entire pupillary space.

93. *Method of Stadfeldt and Tscherning.* Stadfeldt and Tscherning applied the method, devised by Foucault to examine his telescopes, in order to determine the aberration of the dead crystalline lens. Foucault's method, which may be applied to the study of the aberrations of any lens system, is essentially as follows. A luminous point is

Fig. 60.—Aberration Overcorrected Toward the Borders.

Fig. 61.—Aberration Overcorrected Everywhere.

placed a little beyond the principal focus of the lens to be investigated. The image will, therefore, be found at a considerable distance from this lens; the observer places himself beyond this image so that his eye is in the luminous pencil on the axis of the lens. By slowly approaching the lens the eye will see as luminous those portions of the lens which send rays to it. If the lens is aplanatic all rays will meet at the focal point and the whole lens will appear luminous to an eye situated at the focus; at other positions only a small central portion will be luminous since the other rays do not enter the eye. If the lens has spherical aberration a series of phenomena will be seen which are explicable by means of the constructions given in Figs. 52 and 53. When sufficiently far from the lens a small central spot only will be visible which increases in diameter as the focus is approached when it obtains its maximum size; this is shown in Fig. 62 (a). Approaching still nearer, a luminous ring becomes visible, separated from the central part by a narrow, dark zone as shown in Fig. 62 (b). As the lens is approached this ring dilates more and more and becomes farther removed from the central spot which, in turn, contracts in size thus causing the dark zone to become enlarged. On reaching a certain point the ring extends to the border of the lens and finally disappears as shown in Fig. 62 (e). These phenomena are intensified if observed through a narrow aperture in front of the eye. An examination of Fig. 53 in conjunction with the drawings of Fig. 62 shows clearly that if the eye, considered as having a pupil reduced to a point, should be placed at *E* (Fig. 53), rays 1 and 4 would enter and the borders of the lens would appear luminous, while rays 2 and 3 would pass outside the pupil, thus producing the dark zone. A small luminous area will, however, always appear at the center since the axial rays enter the eye. The dioptral difference between the place where the ring first appears and where it disappears will measure the

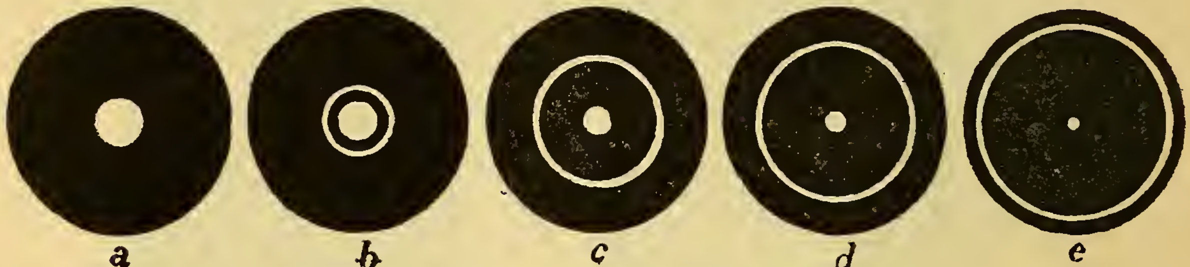


Fig. 62.—Phenomena of Spherical Aberration.

spherical aberration. Any lens system which is overcorrected will exhibit these phenomena in the reverse order and therefore the eye must be removed from, rather than approached toward, the lens in order to see the ring.

94. In order to study the aberration of the crystalline, Stadfeldt used the device shown in Fig. 63. The crystalline lens, removed from an eye in its capsule and with the zonula, was fixed in a ring which was, in turn, placed in a small receptacle filled with serum and enclosed front and back with glass plates and placed upon a stand, shown at *A*, moveable upon a graduated scale at *ED*. The lens *C* served to concentrate light upon a very small aperture in *BD*. The crystalline was observed by means of a telescope at *K*, and the diameter of the aberration ring, corresponding to a given distance between *A* and *BD*, determined by means of an ocular micrometer. After making the necessary reductions, the distance of the luminous point to the crystalline gives the focal distance of the zone which appears luminous. The series of changes diagrammed in Fig. 62 and discussed in connection

therewith occur in these investigations on the crystalline. When the condition shown in Fig. 62 (a) arises, i. e., when the aberration circle and the central luminous spot become one, the luminous point is at the principal focus for the central portion. The focus of the central portion can be obtained very accurately by removing the plate *BD* and substituting a microscope of low magnifying power in the tube *K*. An object placed at a considerable distance is first selected and the line of sight directed toward it; then, by displacing the carrier *A*, there is put in focus, first of all, the image of the distant object formed by the crystalline lens and then, in turn, the posterior surface of the lens itself. The difference between the two positions of *A* enables the

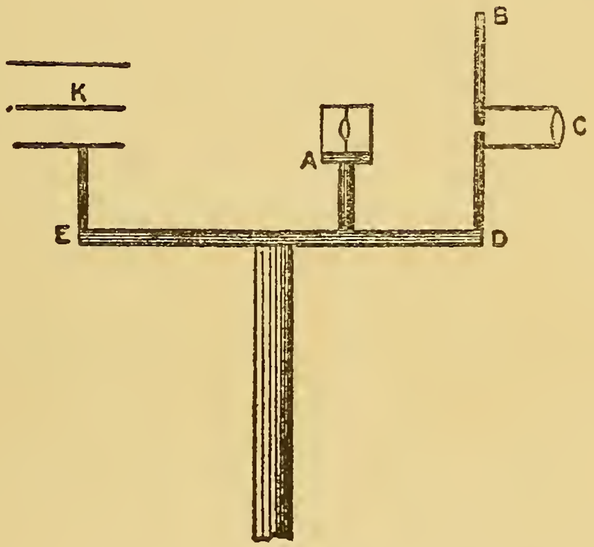


Fig. 63.—Instrument of Stadfeldt for Measuring the Aberration of the Crystalline Lens.

focal length of the lens to be calculated. Some of the various results obtained by Stadfeldt are given in the following table:

Distance from the central axis	Focal Lengths				
	I	II	III	IV	V
0 mm.	48.6 mms.	59.4	55.8	47.8	50.4
2	48.6	59.4	55.8	47.8	53.0
2.5	51.7	66.1	63.2	60.8	59.7
3	51.7	66.1	63.2	60.8	63.7
3.5	46.3	66.1	63.2	60.8	73.1
4	41	63.4	55.2	55.5	
4.3		56.8	47.2		

From these results we see that the central part of the crystalline lens of about 4 millimeters diameter (from 0 to 2 mm. in the above table) is very nearly aplanatic, that the aberration in the region comprised between 2 and 3 millimeters radius is overcorrected by an average of about 2 diopters and that the aberration from there on is undercorrected and falls off rapidly in the extreme peripheral regions. The decrease of the refraction in the paracentral zone must be attributed to a diminution in the index of refraction toward the periphery, while the increase of the refraction at the border must be due to the greater curvatures encountered there. The average central area refraction is 18 diopters, that of the paracentral zone 16 diopters anl that of the extreme peripheral portions, which play little part in general in vision, about 20 diopters.

95. In a general way spherical aberration increases with the angle of incidence. The anterior surface of the crystalline will produce little aberration since the rays incident upon it, as previously refracted by the cornea, will fall upon it nearly normally. The two surfaces chiefly instrumental in causing these errors are the cornea and the posterior surface of the lens, since the paths of the incident rays upon these surfaces are such as to make the angles most unfavorable. Tscherning has calculated the various aberrations for the three surfaces in a ''large eye'' and ''small eye'' as he terms them. The large eye has a radius of the cornea equal to 8.5 mms. and anterior and posterior lenticular surface radii of 12 mms. and 6 mms.; the corresponding quantities in the small eye are 7, 8 and 5 mms. respectively. The results as tabulated are:—

Diameter of Pupil	Large Eye			Small Eye		
(Apparent)	4mm.	6mm.	8mm.	4mm.	6mm.	8mm.
Aberration, cornea	1.0D	2.2D	4.6D	1.6D	4.1D	9.7D
Anterior crystalline		0.1	0.4	0.2	0.3	0.8
Posterior crystalline	1.0	2.5	5.5	1.9	5.0	11.7
Whole system	1.6	3.8	8.2	2.8	7.2	17.3

The aberration increases as the square of the aperture and inversely as the cube of the focal distance. It follows then that the spherical aberration will be more pronounced in eyes of small dimensions.

96. Exaggerated or large aberrational effects are found in keratoconus. The diagrams given in Fig. 64 are taken from the work of Cordiale. The abscissæ show the distances from the visual line in millimeters, while the ordinates give the aberration in diopters.

97. Although aberration may be quite pronounced it does not appear to injure the visual acuity much as long as it remains regular. The reason for this appears to be that the smallest diameter portion of the

cone is not used by patients. We know that at the focal point for the central rays, a luminous point has the form of a point surrounded by a feebly luminous halo. If the object observed has low luminous intensity this halo will be too weak to be seen and the image becomes good; if the luminosity is high the pupil excludes, by its contraction, the peripheral portions of the optical system in such a manner as to practically destroy the halo.

Aberration is a factor, however, which makes less accurate or impos-

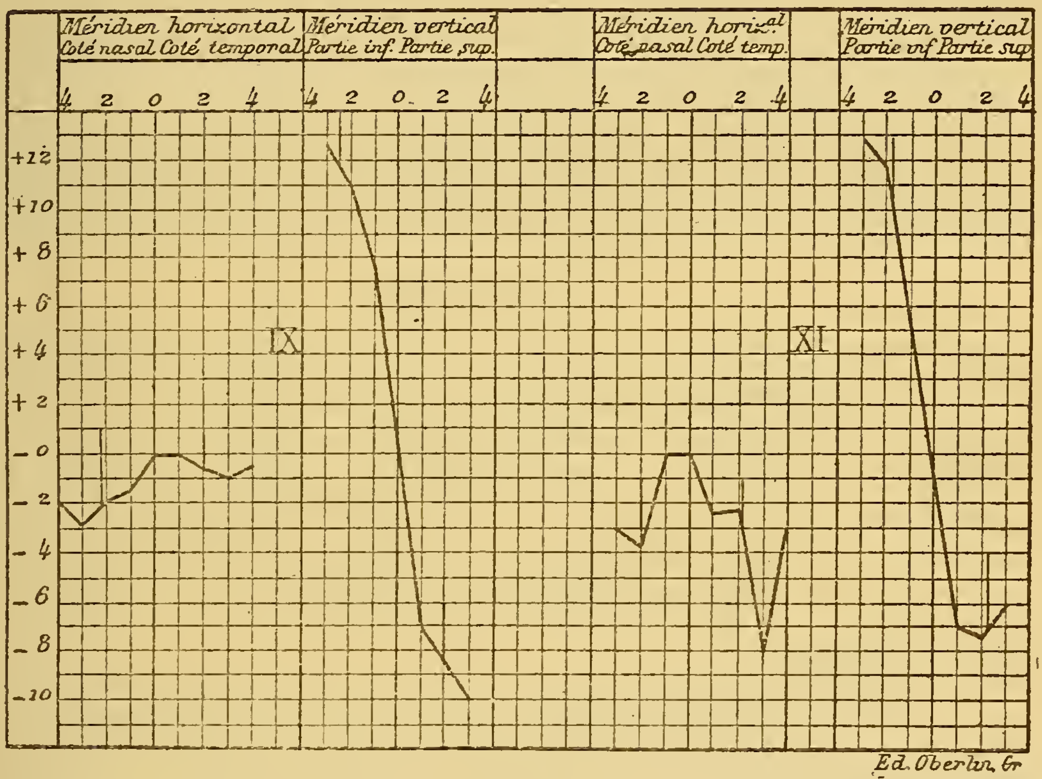


Fig. 64.—Curves Showing the Spherical Aberration in Two Cases of Keratoconus. (After Cordiale.)

The abscissæ indicate the distances in millimeters from the visual line; the ordinates the aberration in diopters.

sible the exact determination of the refractive condition of an eye especially if the aberration is not regular. We know that a section of the caustic (the most luminous part of the cone) has the form of an arrow and that it is the point of this arrow, formed at the focus by the rays centrally refracted, which in general serves for vision. As this is very pointed it follows that exact determination of the refraction cannot be made. The spherical aberration acts, in this respect, as a narrow diaphragm; it is difficult to determine accurately the focal length of a lens which is highly diaphragmed. Because of this form of caustic, eyes possessing a strong aberration can still have visual acuity of the highest order. If the aberration is absent, that is to say if it is approximately corrected, it will serve as another source of uncertainty in the determination of the refraction. This is apparently paradoxical; it would appear that such an eye should be practically aplanatic. Such indeed it would be, but another factor enters into the question from the refractive standpoint and that is the size of the pupil. The central portion of the pupil will then, with aberration annulled, lose its superiority, for its focus practically coincides with the foci of the portions more remote from the axis. As a result, therefore, since the focus is somewhat dependent upon the aperture of the system and since the pupil is of varying size, it follows that the refractive condition must be variable and uncertain. It is stated that an emmetrope "by day" will become a myope of approximately one diopter "on the approach of night." ("Emmétrope le jour il devenait myope de environ une dioptrie à l'approche de la nuit.")

X. ASTIGMATISM

98. *General considerations.* It has been previously stated that elementary beams whose rays have but a small inclination to the axis and which proceed from points either on or close to the axis may be brought to a point focus. The beam may be said, therefore, to be *homocentric* in the image space. We shall now consider what will occur when the elementary beam has an inclination to the axis or when the curvatures of the reflecting or refracting surface are different in various meridians.

If a ray of light proceed from a point P off the axis it will not be homocentric in the image space. If a plane elementary beam whose rays in the image space are normal to a certain element e_1 of a line of curvature, then an image will be formed. The image will be located at the center of curvature of this element e_1 since its normals intersect at that point. Since every such element of curvature in a curved surface is intersected at right angles by some other element, e_2, of another line of curvature, a second elementary beam will exist which also produces an image but the positions of the two images do not in general coincide because the curvatures of e_1 and e_2 are usually not identical.

Let *a-b-c-d*, in Fig. 65, represent the four intersections of the four lines of curvature which bound an element of the surface under consideration. Let the curves *ab* and *cd* be horizontal and *ac* and *bd* be vertical. Let the normals at the points *a* and *b* intersect at *a, b;* those at *c* and *d* at *c, d.* Since the arcs *ab* and *cd* have practically the same curvature the points *a, b* and *c, d* lie at the same distance from the surface *a-b-c-d*. Hence the line l_1 is perpendicular to the ray *S* which passes through the middle of *a-b-d-c* and is normal to it. The normals to any horizontal line of curvature intersect at some point of l_1. Likewise the normals to any vertical line of curvature intersect at some point along l_2 which must be horizontal and at right angles to *S*. These two lines are known as the two focal lines of the beam and the difference between them is called the astigmatic difference. The term *astigmatism* means "without a point or without focus," i. e., an object point cannot give rise to an image point but rather to lines separated by an interval. It is a phenomenon which produces effects similar to those due to spherical aberration; the causes which are operative in the two cases are, however, in general dissimilar, since astigmatic effects are essentially due to curvature changes while spherical aberration arises from eccentric refraction.

Astigmatic images must in general be formed when the elementary

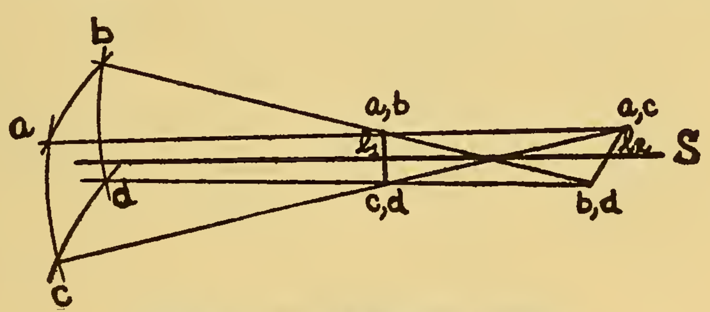


Fig. 65.—An Astigmatic System.

refracting or reflecting surface has two different curvatures. Thus cylindrical lenses, for example, show marked astigmatism.

Reflection or refraction at a spherical surface also renders a homocentric elementary beam astigmatic when the incidence is oblique. In order to consider the case more fully, let the object point *P*, the center of the sphere *C* and the point *A* in which the principal ray of the elementary beam emitted by *P* strikes the spherical surface lie in the plane of Fig. 66. Let the line *PA* be represented by f_1 and the line AP_2 by f_2. Then, since

$$\triangle PAP_2 = \triangle PAC + \triangle CAP_2$$

it follows that

$$f_1 f_2 \sin(\Phi_1 - \Phi_2) = f_1 r \sin \Phi_1 + f_2 r \sin \Phi_2$$

in which Φ_1 and Φ_2 represent the angles of incidence and refraction respectively, and r denotes the radius of the sphere. Since by the law of refraction $n_1 \sin \Phi_1 = n_2 \sin \Phi_2$, it follows from the above equation that

$$\frac{n_1}{f_1} + \frac{n_2}{f_2} = \frac{n_2 \cos \Phi_1 - n_2 \cos \Phi_2}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (I)$$

It will be noted that this equation differs from the fundamental equation of refraction at a spherical surface by the introduction of trigonometric functions of the angles of incidence and refraction.

It is evident that all rays from *P* which have the same angle of inclination *a* with the axis must, after refraction, cross the axis at the same point P_2. This is known as the *sagittal beam* with a focal point at P_2.

But a meridianal beam, which is one whose rays all lie in the plane *PAC,* has a different focal point, P_1. Let *PB* be a ray very close to *PA* and let its angle of inclination be slightly greater than that of *PA,*

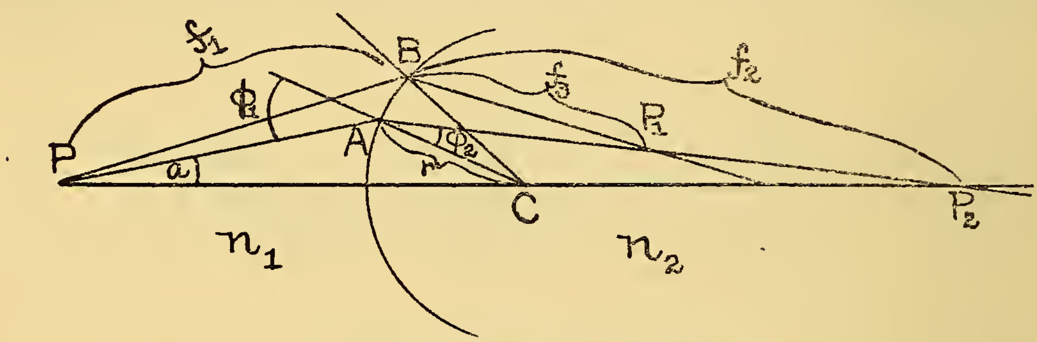


Fig. 66.—Astigmatic Images.

the latter being represented by *a*. A mathematical calculation of some length (unless calculus is employed) will give the result

$$\frac{n_1 \cos^2 \Phi_1}{f_1} + \frac{n_2 \cos^2 \Phi_2}{f_3} = \frac{n_1 \cos \Phi_1 - n_2 \cos \Phi_2}{r} \quad \ldots . (II)$$

From equations I and II there are obtained different values of f_2 and f_3 corresponding to the same value of f_1. This means that *P* is imaged astigmatically. This astigmatic difference is greater the larger the obliquity of the incident beam.

The equations for a reflecting spherical surface, which are of value in the deductions relative to the astigmatic errors of various zones of the cornea, can be deduced from I and II by substituting in them $n_1 = 1$, $n_2 = -1$, i. e., $\Phi_1 = -\Phi_2$. Thus for this case

$$\frac{1}{f_1} - \frac{1}{f_2} = -\frac{2 \cos \Phi_1}{r}$$

$$\text{and} \quad \frac{1}{f_1} - \frac{1}{f_3} = -\frac{2}{r \cos \Phi_1}$$

and by subtraction,

$$\frac{1}{f_2} - \frac{1}{f_3} = \frac{2}{r}\left(\frac{1}{\cos \Phi_1} - \cos \Phi_1\right)$$

$$\text{or} \quad \frac{f_3 - f_2}{f_3 f_2} = \frac{2}{r} \sin \Phi_1 \cdot \tan \Phi_1 \ldots\ldots\ldots\ldots\ldots\ldots \text{(III)}$$

99. *The interval of Sturm.* If a screen is placed near but behind a convex sphero-cylindrical lens, such for example as + 3 D. S. ⊃ + 3 cyl. ax. 90, it will be found that the light from a small brilliant source of light at some distance in front of the lens (the writer uses a small

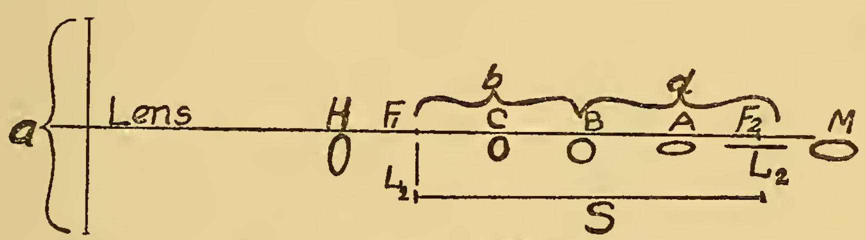


Fig. 67.—The Interval of Sturm.
Circles of diffusion and focal lines of a regularly astigmatic system.

high candle-power incandescent lamp with frosted bulb at 20 feet) will be thrown as a luminous patch on the screen. If now the screen is withdrawn from the lens to a distance of 16.66 cms., which is the focal length of the combined sphere and cylinder, a vertical line will be formed at F_1, Fig. 67. As the screen is moved back still farther from the lens this vertical line gradually changes into a prolate oval at *C,* into a circle at *B,* into an oblate oval at *A* and finally into a straight horizontal line at F_2. The screen is then at the distance corresponding to the focal length of the sphere only; in the illustration given it is at 33.3 cms. The distance between the two foci, F_1 and F_2, of the two sharply defined lines is known as the astigmatic interval or *interval of Sturm.* As the screen is carried still farther from the lens the image formed takes the shape of an ever-increasing horizontal ellipse. The two focal lines are at the focal distances of the two principal meridians and their lengths, represented by L_1 and L_2, are proportional to the diameter, *a,* of the aperture. A series of relations can be deduced connecting the following quantities:—

F_1 = focus of the first principal meridian.
F_2 = focus of the second principal meridian.
S = interval of Sturm.
L_1 = length of first meridianal line.
L_2 = length of second meridianal line.
D_1 = dioptric value of first principal meridian distance.
D_2 = dioptric value of second principal meridian distance.
B = size of circle of confusion.
b = distance from first focus to circle of confusion B.
d = distance from second focus to circle B.
a = aperture of the lens.
p = distance of B behind the lens.

These relations are:—

(1) $S = F_1 - F_2$

(2) $L_1 \cdot F_2 = L_2 \cdot F_1$

(3) $L_1 = \frac{aS}{F_2} = \frac{aS}{D_1}$

(4) $L_2 = \frac{aS}{F_1} = \frac{aS}{D_2}$

(5) $\frac{d}{b} = \frac{L_1}{L_2} = \frac{F_1}{F_2} = \frac{D_2}{D_1}$

(6) $d = \frac{SF_1}{F_1 + F_2} = \frac{SD_2}{D_1 + D_2}$

(7) $b = \frac{SF_2}{F_1 + F_2} = \frac{SD_1}{D_1 + D_2}$

(8) $d + b = S$

(9) $B = \frac{dL_2}{S} = \frac{bL_1}{S}$

(10) $p = \frac{2F_1F_2}{F_1 + F_2} = \frac{200}{D_1 + D_2}$

Assuming that the average eye has a posterior refractive power of 45 D., and further assuming that the refractive errors are due to curvature, let us calculate the various quantities whose theoretical values have just been written in the case that an eye is myopic 2 D. and 5 D. respectively in the two principal meridians. The aperture of the pupil (a) will be taken as 3.7 mms. lying in the principal refracting plane. The dioptric refracting powers in the two meridians will be, therefore, 47 D. and 50 D. respectively corresponding to focal lengths of 21.5 mms. and 20 mms. When these values are substituted in the foregoing formulæ it will be found that $S = 1.5$ mm., $L_1 = 0.258$ mm., $L_2 = 0.2775$ mm., $\frac{d}{b} = \frac{20}{21.5} = 0.93$ mm. (app.), $d = 0.723$ mm., $b =$

0.7791 mm., B = 0.134 mm., p = 20.723 mms. These calculations are of interest since they evidence: (1) the axial focal lengths corresponding to various refractive conditions, showing approximately 3 D. of error to a change of length of the globe of 1 mm., (2) in astigmatic cases the ratio of the length of horizontal to vertical focal lines is never equal to unity, which means that vertical and horizontal objects of the same size, as for example a square, can never give correspondingly equal retinal images, and (3) if the pupil is larger or smaller L_1 and L_2, as well as B, vary in proportion, hence the larger the pupillary aperture the greater will be the lengths of the focal lines and the size of the circle of least confusion.

100. The following table gives data for the lengths of the posterior and anterior focal lines expressed in millimeters corresponding to an eye of 22 mms. antero-posterior diameter when the degree of astigmatism is expressed in diopters and the diameter of the pupil in mms.

Diameter of Pupil (mm.)	0.1D Anterior Focal line	0.1D Posterior Focal line	0.5D Anterior Focal line	0.5D Posterior Focal line	1D Anterior Focal line	1D Posterior Focal line
1	0.001361	0.001364	0.006781	0.006818	0.01345	0.013636
2	0.002732	0.002728	0.01356	0.01363	0.0269	0.02727
3	0.004083	0.004092	0.02034	0.0245	0.04035	0.04091
4	0.005444	0.005456	0.02712	0.02727	0.0538	0.0545
5	0.00680	0.00682	0.03392	0.03409	0.06725	0.06818
6	0.008166	0.008184	0.04069	0.0409	0.0807	0.08202
7	0.009537	0.009548	0.04733	0.0473	0.0942	0.09545
8	0.010888	0.01091	0.05425	0.0545	0.1076	0.10909

For an astigmatism of 0.1 diopter and for a pupillary aperture of 4 millimeters the length of the focal line is 0.0054 mm. as is shown in the above table. This length of line is very nearly double the diameter of a macular cone. It is thus seen that even a tenth to an eighth of a diopter of astigmatism should diminish the visual acuity when the size of the pupil is about the average, i. e., 4 mms. When the error is 1 diopter, with an average pupil, the lengths of the two focal lines are 0.023 mm. and 0.025 mm. and each of these covers approximately ten distinct retinal elements. A diopter of astigmatism will, therefore, have very distinct influence upon the visual acuity. A cursory inspection of the dimensions of the focal lines and their dimensions relative to the perceptive elements of the retina shows that astigmatism diminishes the acuity when it is as low as a tenth of a diopter. It would, therefore, appear that Donders placed his limit of 1 diopter for physiologic astigmatism considerably too high. It is in fact still a disputed question as to when astigmatism should be considered pathologic in contradistinction to physiologic; the limits have been placed at 0.5 D. to 1.5 D. The foregoing table and similar ones which may be quickly calculated to the first order of accuracy by simply multiplying the values given for one diopter by the number of diopters of astigmatic error present, show clearly that the focal lines vary decidedly with the pupillary diameter; hence it is readily conceivable that a larger amount of astigmatism in a particular eye may still give better visual acuity or show less improvement in vision with cylindrical lenses than an eye which has a lesser amount but a larger pupil. A pupillary contraction, therefore, which accompanies accommodation will considerably reduce the errors due to astigmatic images by diminishing the lengths of the focal lines. When the size of the pupil is reduced from 6 mms. to 2 mms. the lengths of the focal lines are reduced in the ratio of 3 to 1 for the two conditions. This explains why astigmates see better under strong illumination. It may with profit be pointed out that, with pupils dilated under the influence of cycloplegics or when examined subjectively in very much darkened rooms, the central astigmatism is replaced by the peripheral astigmatism in the determination of the astigmatic correction and the astigmatism is thereby determined for a portion of the refracting system which does not play a part in ordinary vision when the pupil possesses its normal diameter. This same remark may be made in reference to the correction of axial ametropic conditions, for the circles of diffusion due to large pupils considerably affect the visual acuity, as was pointed out in the section on *Diffused Circles*. It is pertinent, therefore, to call attention to the desirability of obtaining a record of the average pupil in any case under uniform and moderate illumination and to then proceed to the refraction of the eye, either with or without the use of cycloplegics as the practitioner sees fit, screened by an iris diaphragm set a trifle larger than this average or normal pupil size.

101. Fig. 67 gives us further information of everyday value in ophthalmic practice. Suppose that in an eye the first focal line is vertical and the second horizontal and that a single luminous point is viewed. Then the shape of the image on the retina depends upon the position of the retina in the refracted astigmatic pencil (accommodation being eliminated) as follows:—

Position of the Retina	Class of Astigmatism	Retinal Image
At M	Compound myopic	Horizontal ellipse
At F_2	Simple myopic	Horizontal line
At C	Mixed	Horizontal ellipse
At B	Mixed	A circle
At A	Mixed	Vertical ellipse
At F_1	Simple hyperopic	Vertical line
At H	Compound hyperopic	Ellipse

If the retina is in the second focal plane, a horizontal line object is seen distinctly although slightly extended, but a vertical line will be seen blurred. In order that a vertical line be seen clearly the retina must be in the first focal plane. When the retina is between the two focal planes both vertical and horizontal lines will be blurred, the horizontal lines being expanded vertically and the vertical horizontally. Oblique lines will be confused for any position of the retina in such an eye. Hence, in an astigmatic eye the perception of a line is good if the direction of that line corresponds to the direction of the focal line which is at the retina. This explains why astigmatism may be subjectively determined by means of the radiating lines of the clock-dial and other similar tests, and why some of the lines, or chart letters for that matter, are seen more distinctly or sharply than others. Suppose the two principal meridians to be horizontal and vertical. If the retina is situated at or near the horizontal focal line, the confusion discs at the retina correspond in direction to the horizontal retinal image itself so that the edges of the horizontal portions of a test-object are seen sharply and clearly. But the horizontal confusion lines are at right angles to the vertical focal line which will, therefore, cause the vertical portions of the object to appear blurred. If, in turn, the retina is at or near the vertical focal line, the vertical part of the object would be most sharply seen. These conditions are represented in Figs. 68 and 69; Fig. 68 represents the condition in which the retina is at the horizontal focal line. It is a well known principle of optics that the refracting power of a cylinder lies at right angles to its axis; from this it follows that the image of a vertical object is itself vertical, but the power meridian which gave rise to it must have been horizontal. The reverse is the case when a horizontal object is imaged. An astigmatic eye has, therefore, two meridians of power, usually at right angles to each other, and the power meridian which produces any focal line must be at right angles to this focal line. If, for example, in Fig. 67 the retina is assumed to be at F_2 horizontal lines will then be distinctly seen, since the horizontal focal line lies upon the retina at this point; this indicates that the curvature in the vertical meridian of the eye is correct. The focal line at F_1, situated in front of the retina, will be seen blurred and of the form shown in Fig. 69 due to the excessive curvature in the horizontal meridian of the eye. This can be alleviated by reducing the power in the horizontal meridian and would be accomplished in the case in hand by adding a concave cylinder with its axis vertical.

102. The following chief points may be noted:—

(a) The clearest and most indistinct lines of the chart correspond to the focal lines of the eye.

(b) The focal lines of the eye are at right angles to the meridians of which they are the respective foci.

(c) The clearest chart lines correspond in direction to the most ametropic power meridian of the eye; hence the power of the cylinder is needed in that meridian.

(d) The most indistinct lines of the chart correspond to the emmetropic or nearest emmetropic meridian of the eye; hence the rule, as commonly stated, is that the axis of the correcting cylinder should be placed in a direction corresponding to the least distinct (or blurred) lines. Thus, for an eye corrected by a — 3 cylinder axis 180°, we see:—(1) the horizontal meridian is emmetropic and the vertical meridian is myopic, (2) the horizontal focal line is in front

Figs. 68 and 69.—Figure 68 Represents the Condition in Which the Retina is at the Horizontal Focal Line: Figure 69 the Condition When the Retina is at the Vertical Focal Line.

of the retina, the vertical focal line is at the retina, (3) the vertical chart lines are seen clearly, the horizontals are blurred and (4) the vertical meridian of this eye requires concave power, i. e., less power than it possesses; the horizontal meridian requires no change in power; the axis of the correcting concave cylinder should be placed along the horizontal or 180° line. It is outside the purview of an article such as this to enter into the various methods and means of testing for astigmatism; the paragraphs penned above and involving some points in the subjective method have been inserted in the hope of making clear the relations between the visual perceptions of test objects, power meridians and directions of axes of correcting cylinders.

103. The foregoing statements also serve to explain why it is that a luminous point when viewed through a Maddox rod (which is simply a very high power cylinder) placed vertically appears as a horizontal streak and, vice versa, with the rod horizontal, a vertical streak of light is obtained. The addition of power to the vertical

meridian of the eye obtained by placing the Maddox rod with its axis horizontal makes the eye artificially myopic in the vertical meridian; the horizontal meridian is unaffected. Assuming for illustrative purposes that the eye is initially emmetropic, it will be seen that the vertical meridian, with the rod placed horizontally, is rendered myopic while the horizontal meridian is left emmetropic. The vertical meridian of the eye under these conditions will, therefore, cause light to converge much more in the vertical direction than does the horizontal meridian in a horizontal direction. The result will be, then, that the retina will receive a vertical ribbon or band of light which, by the laws of projection, will be seen in space as an inverted, upright ribbon or streak of light.

In summary, it may be said that the first focal line is at the focus of the meridian of greatest refraction; it is parallel to the meridian of least refraction; the second focal line is at the focus of the meridian of least refraction and parallel to the meridian of greatest refraction. The diffusion spots are everywhere elliptical except at one point of the interval of Sturm where the luminous spot becomes circular.

104. *Astigmatism of the human eye.* This defect was discovered by Thomas Young in 1801. He used his optometer and measured his astigmatism as the difference in refraction of the two meridians. He had 1.7 D. of astigmatism against the rule. The astronomer Airy was probably the first to correct astigmatism by a cylindrical lens (1827). The invention of the ophthalmometer of Helmholtz and the subsequent measurements of Knapp and Donders drew attention to the prevalency of this defect of the eye. Since that time greater refinements have been made in subjective and objective methods of measuring refractive anomalies and greater accuracy together with the elimination of various errors have been introduced into the mechanical side of ophthalmic lenses. This has meant the correction of astigmatic errors of amounts which were formerly neglected but which, as all practitioners realize, are often highly important in asthenopic conditions. It is rare to find an eye completely free from astigmatism. The chief seat of astigmatism is in the anterior surface of the cornea. Under ordinary circumstances it is the form of the anterior surface that determines the amount and character of the astigmia: the examination of this surface, therefore, plays an important part in the search for astigmatism. A deformity of one of the internal surfaces of the eye has but little influence relatively since there is but little difference in the indices of the internal media of the eye. The refraction of a curved surface separating two media is expressed, as we have shown under

the *Dioptrics of the Eye*, by the relation $\frac{1000\ (n_2 - n_1)}{R}$; that is, for the cornea by $\frac{337.5}{R}$ and for one of the internal surfaces by $\frac{60}{R}$. The same deformity, therefore, existing internally, as might occur at the anterior surface of the cornea, would produce an effect some five to six times less.

105. *Corneal astigmatism.* The corneal astigmatism is measured by means of the ophthalmometer. The essential principles and underlying mathematics of this instrument have been discussed in the paragraphs devoted to the *Catoptrics of the Eye.* In using an ophthalmometer of the usual type one focusses first of all the ocular for the spider thread and then puts the whole instrument in focus for the eye under examination. The images of mires are put in contact in one meridian and its position and power as indicated on the attached drum are read. The instrument is then turned 90° and a similar procedure instituted and the difference indicates the ophthalmometrically determined astigmatism.

Astigmatism ''with the rule'' is a term often used and signifies that the meridian of greatest curvature does not differ much from the vertical. It is usually specified as lying between the limits of 45 degrees on each side of the vertical. Astigmatism ''against the rule,'' sometimes designated as ''inverse'' or ''perverse'' astigmatism, indicates that the horizontal meridian has the greater refraction. Various tables of statistics showing the percentage of different classes of astigmatism may be found in numerous treatises upon the eye: in general, these tables show about 80 per cent. of all astigmatism as being ''with the rule'' and not in excess of 5 to 10 per cent. ''against the rule.'' This frequency of astigmatism with the rule, demanding for the correction thereof plus cylinders at axis 90° or minus cylinders at axis 180° or within 45° thereof, thus indicating that the vertical meridian possesses the greater refractive power, may be accounted for by reason of the anatomical structure of the eyeball and of the orbit, and by the action of the lids and the insertion and operation of the extra-ocular muscles.

106. Fig. 70 is a presentation of the curvatures of a centrally non-astigmatic eye at various points of the cornea. After having measured with the aid of the ophthalmometer the refraction of the two principal meridians at the point of intersection of the visual line with the

cornea, the eye is caused to fix points 5°, 10° . . . . . . to 30° in the temporal, nasal, superior and inferior directions. The curvatures are measured in the two principal meridians with the eye deviated through these various angles 5° at a time. By taking into account the spherical aberration (since the calculations are to be made for incident rays parallel to the visual line), the astigmatism of these various 5° zones of the cornea which are concentric with the point of intersection of the visual axis and the cornea can be determined.

Fig. 70.—Cornea Without Central Astigmatism but Having Peripheral Astigmatism Against the Rule.

The results of a large number of such experiments upon both astigmatic and non-astigmatic eyes show:

(1) The peripheral portions of the cornea possessing no central astigmatism show an "inverse" or "against the rule" astigmatism.

(2) The peripheral portions of an astigmatic cornea showing weak "with the rule" conditions will indicate either no astigmatism or weak astigmatism "against the rule."

(3) When the central portion shows astigmatism "against the rule," the peripheral portions will show the same except to a higher degree.

(4) The peripheral portions of a cornea having astigmatism "with the rule" of moderate or high amount are oft-times much more astigmatic than the center, and again at times less astigmatic.

The differences between the central and peripheral portions of the cornea are bound up with spherical aberration and with the condition that either the meridian of greatest or least curvature flattens out very rapidly as one passes from the center to the corneal periphery.

107. *Astigmatism by incidence.* It is known that the various refracting surfaces of the eye are not generally accurately centered. Defects due to this lack of exact centering are generally small in that they produce little astigmatic error except in cases of pathological

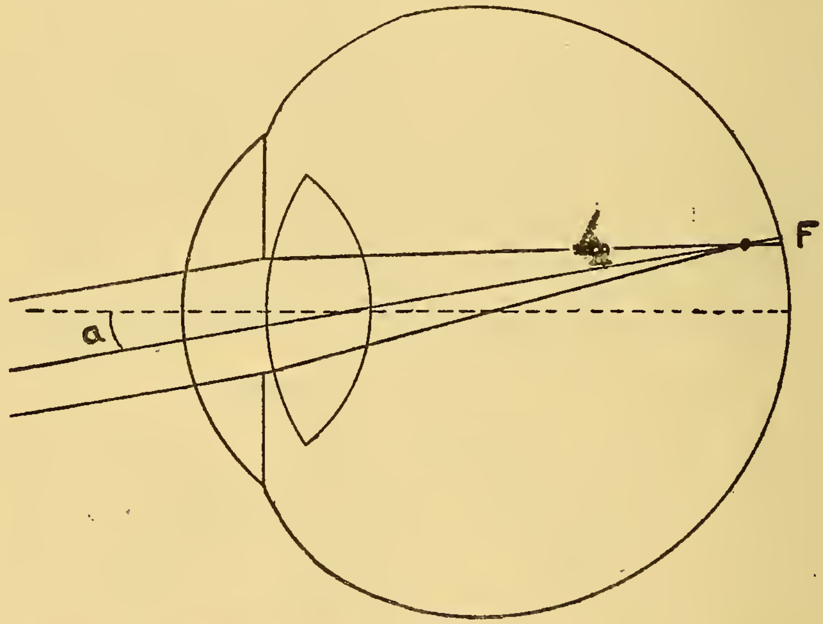


Fig. 71.—Astigmatism by Virtue of the Angle Alpha.

luxation of the lens. The pupil is ordinarily exactly centered with respect to the axis of the system; but the object "fixed" is not upon this line. This condition we have previously discussed and designated as the *angle alpha* which has ordinarily a value of about 5° but may be greater or, in some cases, negative in value. Even though the incident beam along the optic axis should be devoid of astigmatism, it will not be true of the beam emanating from the object and passing along the visual axis. This is illustrated in Fig. 71 which is a horizontal cross-section of the right eye and shows the astigmatism due to incidence by virtue of the angle alpha. The optic axis is deviated outwardly and downwardly with respect to the visual line. Since the meridian passing through the axis is most powerful refractively, the astigmatism produced by the angle alpha is inverse or against the rule, the meridian of greatest refraction now being horizontal and slightly inclined temporal-ward. If D denotes the refractive power of the meridian of greatest refractivity and i the angle of incidence, the degree of astigmatism is equal to $D(1 - \cos^2 i)$. The values of the astigmatism in diopters corresponding to various values of the angle alpha calculated for an average eye are as follows:

	Diopter 1°	3°	5°	7°	8°	9°	10°
Corneal astigmatism.......	0.02	0.13	0.35	0.66	0.86	1.11	1.35
Lenticular astigmatism.....	0.01	0.05	0.14	0.26	0.35	0.44	0.54
Total astigmatism.........	0.03	0.18	0.49	0.92	1.21	1.55	1.89

By virtue of this angle alpha an inverse astigmatism arises which is, on the average, about one-half to three-quarters of a diopter but may reach values as high as 2 D.

108. *Astigmatism due to lens obliquity.* Another form of astigmatism due to incidence may be designated as astigmatism due to lens obliquity. It can be mathematically demonstrated that when a spherical lens is rotated about any diameter there will be produced by this obliquity of the spherical lens a slightly stronger sphere coupled with a cylinder whose axis corresponds to the axis of rotation. This condition is referred to under the name of *oblique centric refraction.* The formulæ for the cylindrical effect of oblique sphericals are usually given in a complex form (see for instance their development in A. S. Percival's *Optics,* pages 270-281), but the simple relations which follow in the next sentence may be found in Laurance's *General and Practical Optics.* If F represent the focal length of the lens, and F_1 and F_2 indicate the effective focal lengths of the meridians of greatest and least power, while a represents the angle of incidence, it can be shown that

$$F_1 = F_2 \cos^2 a$$

$$F_2 = \frac{F(3 - \sin^2 a)}{3}.$$

Thus, for example, if the crystalline lens *in situ* has a power of 16.66 D. and it should be tilted about the horizontal axis and at right angles to the axis of the ocular system by an angular amount of 10°, then F will be found to be 6 cms., F_2 to be 5.938 cms. and F_1 to be 5.758 cms. or approximately $D_1 = 17.36$ diopters and $D_2 = 16.82$ diopters. This is equivalent as a sphero-cylinder to 16.82 D. S. ⊃ + 0.50 cyl. ax. 180 practically. We have, therefore, by the tilting of the crystalline lens by an angular amount of 10 degrees produced an increased spherical power of about $\frac{1}{6}$ D. coupled with a $\frac{1}{2}$ D. cylinder. Savage, in the chapter on the muscles of the ciliary body in his book on *Ophthalmic Myology,* discusses at some length the probable functions of the Mueller's muscle and the Bowman's muscle. He presents excellent reasons for believing that the former is concerned largely with the accommodative changes while the Bowman's muscle, which consists of the meridianal fibers, is actively concerned in the placing or holding in position of the lens. It does not appear improbable, therefore, that in the process of the development of an eye the lens may assume its mathematically correct position; but if it does not the Bowman's muscle, under the guidance of the retinal sensations, may come to its assistance. It may also happen that a corneal astigmatism may be in part or wholly offset by a lenticular condition or tilting of the lens such as that just discussed through the agency of the action of the individual fibers of Bowman's muscle.

Savage cites his own personal case in the emphasis of two points:—(1) there was a lenticular astigmatism that almost completely neutralized the corneal astigmatism for a considerable number of years, the final full corneal astigmatic correction, as originally determined by the ophthalmometer, being eventually given and worn and (2) the power that affected the neutralizing lenticular astigmatism was not suspended by the repeated use of mydriatics. The logical conclusion seems to be that the lenticular astigmatism was produced by fibers of Bowman's muscle. "If the tilting of the lens by contraction of a single portion of Bowman's muscle is not the cause of the lenticular astigmatism, the simultaneous and equal action of two opposite parts of Bowman's muscle, by making these the corresponding parts of the zonula, could so compress the part of the lens intervening as to increase its refractive power, thus effecting lenticular astigmatism."

109. *Astigmatic accommodation.* It is pertinent to discuss in this connection the question of astigmatic accommodation. Dobrowolsky first expressed the idea that astigmatic persons could partly correct their defect through an irregular contraction of the ciliary muscle, thus producing a deformity of the crystalline lens in the opposite direction. G. Martin, Vacher, Clarke and others have attributed to astigmatic accommodation a train of pathological conditions, including keratitis and cataract. Eriksen, Sulzer and George Bull do not admit astigmatic accommodation. The basis for the belief in astigmatic accommodation lies in the change of the astigmatism observed on instilling atropine. The phenomenon is doubtless due in part to the differences in the astigmatic conditions at the peripheral and central zones of the pupil, a topic which has been discussed in preceding paragraphs.

Again, suppose the diameter of the pupil is brought from 4 mms. to 8 mms. The area increase in one condition over the other is about 40 sq. mm., or triple the pupillary area of the original 4 mm. pupil. Thus much more light enters through these peripheral parts and, as Tscherning says, "It is not surprising that this fact greatly influences the answers of the patient." Likewise the researches of Savage indicate that atropine has little or no effect in suspending lenticular astigmatism and that this power could not have been derived through the fibers of the third nerve supplying the ciliary. On the other hand, Lucien Howe (*Muscles of the Eye*) points out several facts indicating that such contraction of the ciliary process does occur. These are briefly as follows: (1) It is entirely possible from the anatomical arrangement. (2) Measurements of contraction in other muscles show that there is frequently a difference in degree of tension in different fibers. (3) It is probable that part of the filaments which go to the ciliary muscle may be in a normal condition, while others may be partially paretic or insufficient or, again, over-active, thus producing irregular action on the zonula. (4) Subjectively no astigmatism may be evidenced, the vision being 20/20 easily, while all objective tests show the presence of a decided astigmatism, indicating an astigmatic accommodation. (5) The clinical experience that the correction of an objective astigmatic condition lessens discomfort and ocular headaches, although no improvement of the vision may result.

110. *Frequency of astigmatism.* The frequency of low amounts of astigmatism against the rule, commonly found as plus cylinders axes 180° or close thereto, in persons over 40 years of age on, cannot have escaped the notice of practitioners. Faehndrich has given us the following curves showing the relative frequency of "with" and "against the rule" astigmatism with age. These results are shown in Fig. 72. From 40 years on throughout the presbyopic period astigmatism against the rule is found in increasing percentages, reaching about 80 per cent. at 70 years of age. The writer believes that this inverse astigmatism is, in the majority of cases, a lenticular condition or condition of lens situation due to increasing obliquity of the lens with weakened powers of the Bowman muscle because of increasing years and because of their possible abnormal development in certain directions due to visual habits. It is to be remembered that the gaze is rarely fixed upon objects in the primary isogonal or straight-away position or inclined upwardly but rather is it inclined downwardly in all the ordinary demands upon vision. This means a rotation of the eyeball slightly downward for a greater portion of one's working hours and therefore a slight tension, presumably through the medium

of Bowman's muscle, to keep the crystalline properly placed; that is, to prevent its assuming a partially perpendicular position rather than a properly centered one with respect to the remainder of the system. It seems plausible, then, to believe that visual habits should cause a development of small astigmatic errors against the rule due to tilting of the lens. It is possible, also, that the supposed decrease of myopia with increase of years beyond the fiftieth year may be explained not only by the contraction of the pupil with age and by the acquisition of hyperopia presumably due to increased density of the cortical layers of the crystalline lens, but in many cases there may be the additional factor of the increased spherical power produced by lens obliquity.

We are not, however, to suppose from these remarks that the seat of this astigmatism against the rule may not be in the posterior surface

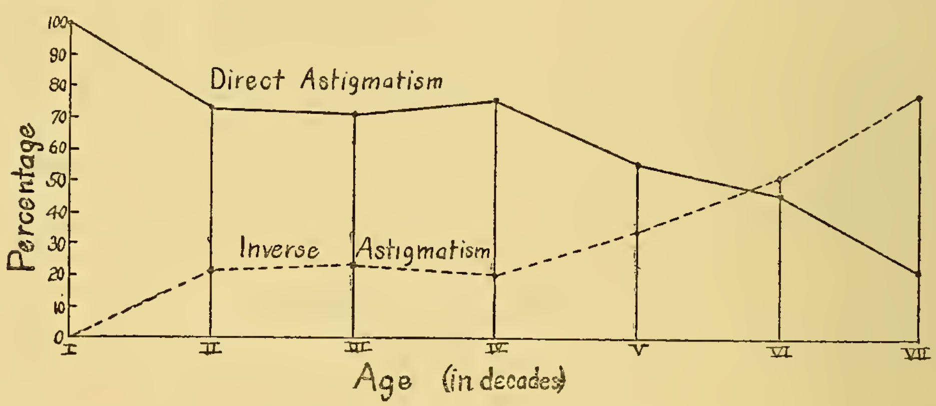


Fig. 72.—Relative Frequency of "With" and "Against" the Rule Astigmatism According to Age. (After Faehndrich.)

of the cornea. In fact it seems possible to successfully and rapidly locate the seat of such errors. For if the ophthalmometer indicates no astigmia and the subjective and skiametric methods show its presence, the skiametric shadow being practically uniform in its motion in any specified meridian and that, too, in various portions of the pupil, we may ordinarily attribute the defect to the posterior corneal or anterior crystalline curvatures. But if retinoscopically, the examination being made along the visual axis, the skiametric reflex divides into two portions, one of which indicates hyperopic and the other myopic corrections, thereby exhibiting the so-called *"scissor movement,"* one may consider the crystalline lens obliquity as being one of the factors, to say the least, if not the only one to be considered.

111. *Astigmatism due to the forms of the ocular surfaces.* It has been found experimentally that about 70 per cent. of all corneas show an astigmatism varying from 0.5 D. to 1 D.; this is, as a rule,

the degree of astigmatism given by the anterior corneal surface. But little is known of the *posterior surface of the cornea.* This surface appears in general to possess a much greater curvature in the vertical than in the horizontal meridian. It is a deformity analogous to that which, if lying in the anterior corneal surface, would produce astigmatism with the rule. Since the posterior corneal surface is concave, i. e., acts like a concave lens, it may often happen that this difference in curvatures produces an ''against the rule'' astigmatism of a quarter to a half diopter. This doubtless explains why—in many cases having subjective astigmatism—we find no ophthalmometric astigmia.

The measurements of three eyes representing (I) astigmatism with the rule, (II) against the rule and (III) practically nil, as made by Tscherning with his ophthalmophakometer, are inserted at this point; the symbol *''d''* represents direct astigmatism while *''i''* indicates the inverse condition.

Anterior Surface of Cornea	I	II	III
Horizontal radius (mms.)......	7.98	7.78	8.29
Vertical radius	7.69	7.90	8.33
Astigmatism (diopters)	2.4(*d*)	0.8(*i*)	0.22(*i*)
Posterior Surface of Cornea			
Horizontal radius (mms.)......	6.2	5.7	6.2
Vertical radius	5.0	5.1	5.9
Astigmatism (diopters)	—0.6(*d*)	—0.6(*d*)	—0.2(*d*)

Tscherning says:—''Although we manifestly cannot draw general conclusions from the measurements of three eyes, I wish, however, to direct attention to some of these results. We observe in the first place that the vertical meridian of the posterior surface of the cornea presents a more pronounced curvature than the horizontal meridian. This condition is repeated in the three eyes to which I here refer, as well for the first, the anterior surface of which presents astigmatism with the rule, as for the other two in which it presents astigmatism against the rule. I have also met the same deformity in other eyes which I have measured, so much so that there is reason to believe that the condition is general.''

112. Tscherning, Stadfeldt and Awerbach have made measurements upon the crystalline lens using the ophthalmophakometer. The vertical and horizontal meridians were measured but no attempts were made to actually determine the principal meridians. The (+) sign indicates astigmatism with the rule and the (—) sign shows inverse

astigmia in the following selected portions of data obtained from some sixteen different crystalline lenses examined by these experimenters.

Crystalline Astigmatism in Diopters

Anterior surface.	+ 0.1	+ 0.7	+ 0.5	+ 0.3	+ 0.8	+ 0.3	+ 0.5	+ 0.8
Posterior surface	— 0.1	— 1.8	— 0.3	+ 0.2	— 1.1	— 1.2	— 1.5	+ 1.4
Total	0	— 1.1	+ 0.2	+ 0.5	— 0.3	— 0.9	— 1.0	+ 2.2

The anterior surface of the crystalline in all cases examined showed a direct astigmatism while the posterior surface often exhibited the inverse type. Judging from the limited data at hand it appears that the crystalline surfaces are more spherical in form than the cornea. We can express the refractive power of a spherical surface by $\frac{1000\,(n-1)}{R}$. For the cornea the factor $1000\,(n-1)$ has a value of 337.5 and for the crystalline it is approximately 74. A certain deformation of the cornea will, therefore, produce an effect four or five times as great as the same deformation in one of the crystalline surfaces. The astigmia of the normal cornea being about 0.5 D. to 0.75 D., the crystalline surfaces will not, therefore, show more than 0.1 D. to 0.2 D. astigmatism if they are as regular as the corneal surfaces. The results of Awerbach, Stadfeldt and Tscherning in a general way support this conclusion.

113. *Post-operative astigmatism.* An examination of a cornea soon after a cataract operation shows a large amount of astigmatism against the rule. This is sometimes as high as 15 diopters. The vertical meridian is considerably flattened, probably due to the interposition of an exudation between the folds of the incision; the phenomenon is more pronounced if there is a hernia of the iris. This astigmatism gradually diminishes and generally reaches a final value of 1 to 2.5 diopters.

Post-operative astigmatism is due, according to Treutler, both to vertical flattening and to increase in the horizontal curve. In some cases the astigmatism, found a month after operation, persists or is even increased: in other cases it diminishes somewhat and disappears in a few months. This reduction may be attributed to readjustments of wound surfaces; in sclero-corneal sections it may be that the closer growth of epithelium interposes a wedge between the edges of the wound. Jackson found that in fifteen per cent of cases a permanent degree of astigmatism was reached within two months of operation: in about twenty-five per cent regressive changes continued after more than three months. Rollet, in 150 cases, found that five months after the operation twenty-five per cent exhibited no marked astigmatism, while the remainder had an average amount of 2.57 diopters. A year or more after the extraction there was either a complete disappearance of the astigmatism or a small amount developed at right angles to the original directions.

114. *Keratoconus and irregular astigmatism.* The highest degrees of corneal astigmatism are met with in conical cornea, excepting post-operative results following immediately after cataract extraction. The apex of the cone does not in general coincide with the visual line. This gives rise to a strong astigmatism the direction of which varies. An ophthalmometric examination shows the images of the mires in irregular forms and often so confused that they cannot be brought into line. When the curvatures of the central and peripheral zones of the cornea are markedly different yet symmetrical, strong spherical aberration rather than irregular astigmatism is essentially produced. Irregular corneal astigmatism is generally considered to be a result of wounds or ulcers, although it may be congenital or spontaneously acquired. There is no *complete* correction for irregular astigmatism, although vision may be aided by spheres or cylinders or their combination. Aid can also be given by stenopaic spectacles in the form of slits or small apertures. The only true remedy would consist of a cell of water, the liquid being held in contact with the cornea by means of a thin spherical shell. Irregular lenticular astigmatism may be caused by iridic adhesions thus producing irregularities of the lens capsule. It may also result from change of density, refractive index, or a deformity of shape or position may exist congenitally or as the result of a condition such as incipient cataract. It may, again, be caused by differences in the refractive indices of the vitreous, possibly due to the presence of sugar.

115. A particularly interesting type of irregular astigmatism is one which arises from the circumstance that the two principal meridians of the cornea are not at right angles to each other, or practically so. Such cases demand as correcting lenses a combination of two cylinders at oblique axes which can be transposed over into equivalent sphero-cylinders by the methods of Donders, Jackson, Weiland, Prentice or Sheard. Such cases can perhaps be most successfully handled by the employment of a 1 mm. stenopaic slit and determining thereby the meridians or positions of the slit giving best and poorest vision. Each meridian having been corrected in turn to give as nearly 20/20 vision as possible, the complete findings are inserted in the trial frame after making the calculations for the equivalent sphero-cylinder

and the axis of the cylinder determined either mathematically or by subjective testing only.

It is to be said by way of introduction that prescriptions involving cylinders crossed at oblique axes are rarely encountered, and it may likewise be stated that such corrections, when found, are often due to lack of skill and technique on the part of the practitioner; they do exist, however, and when they do must be classed as cases of peculiar irregular astigmatism due to corneal defects (ectasia corneæ), such as conical cornea or displacement or turning of the lens (ectopia lentis).

It is not feasible within the limited space of this article to develop the mathematical theory of the dioptric formulæ for combinations of cylinders with axes at any angular deviation. This has been admirably done by C. F. Prentice, M. E., in his work on *Ophthalmic Lenses* and in the section on Lenses found in *The American Encyclopedia of Ophthalmology* and again by the writer of this article in a brief and possibly much simplified form in the *Physical Review* in 1914. In succinct form the equations as developed by the writer are:

(1) $X = d_1 \cos^2 \theta_1 + d_2 \cos^2 \theta_2$,

(2) $Y = d_1 \sin^2 \theta_1 + d_2 \sin^2 \theta_2$,

(3) $A + B = X + Y$,

(4) $A \cdot B = d_1 d_2 \sin^2 \gamma$.

(5) $(B - A) \cos 2\delta = Y - X$.

The symbols have the following significances:

d_1 = dioptric power of first cylinder of the oblique-angled combination.

d_2 = dioptric power of second cylinder of the oblique-angled combination.

A = dioptric power of first cylinder of cross-cylinder combination.

B = dioptric power of second cylinder of cross-cylinder angled combination.

X = total dioptric power in the horizontal direction due to the two members of the oblique axis combination.

Y = total dioptric power in the vertical direction due to the two members of the oblique axis combination.

γ = angle between axis of the oblique combination.

δ = angle which one member of the right-angled equivalent combination makes with the horizontal line, 0° — 180° line.

θ_1 = angle which first member of oblique combination makes with the 0° — 180° line.

θ_2 = angle which second member of oblique combination makes with the 0° — 180° line.

Fig. 73(A) is inserted in order to aid the reader of this treatise in mentally placing the various angles and powers involved. No importance is to be attached to the actual dimensional values of the geometrical functions involved in this diagram; the drawing is inserted solely for illustrative purposes.

In the solution of equations (3) and (4) it will be found that there are two numerical values which satisfy A and likewise two satisfying B. When these results are substituted in equation (5), the angle δ will be found to have a positive value when one set of values of A and B is used and a negative or minus value, algebraically considered, when the second set of values of A and B are used. A general rule

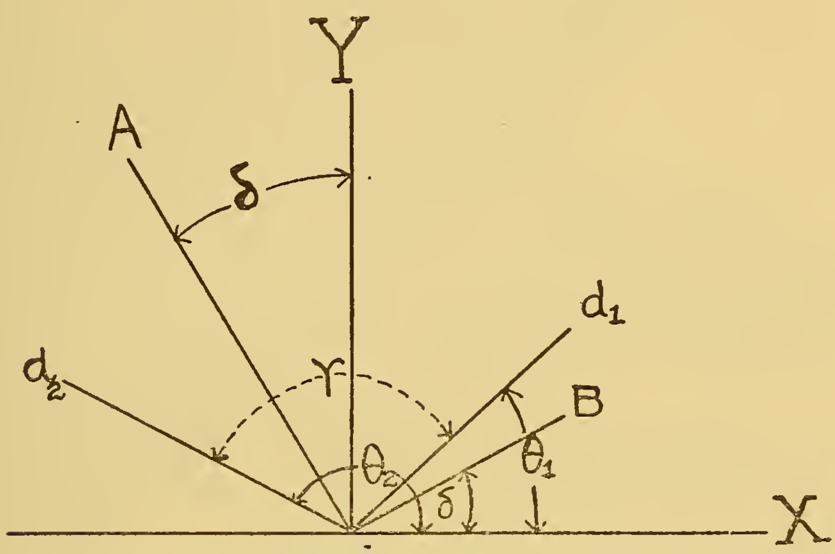


Fig. 73.—(A). Explanatory of Symbols Used in the Theoretical Development of the Equivalence of Cylinders Crossed at Oblique Axes.

relative to the angles at which the two members of the equivalent cross-cylinders are to be placed may be formulated as follows:—

If the solution of the equation $(B - A) \cos 2\delta = Y - X$ gives a positive value to the angle δ, then the cylindrical value of A used in the solution of this equation is to lie with its axis at the angular position indicated by δ; if the solution of the equation gives a negative value to the angle δ, cylinder B lies at that angle.

In addition, complications will be avoided when the two cylinders at oblique axes have different signs if they are transposed first of all into an equivalent sphere combined with the two cylinders which should now have the same algebraic signs but one of the axes changed by 90°.

An illustrative case will show the operative and mathematical procedure. O. D., stenopaic slit used; slit position at 55°, vision sharpest;

slit position 165, vision poorest. + 2 D. S., slit at 55°, gave V = 8/10 and — 4 D. S., slit at 165, produced V = 8/10. The prescription as a double cylinder at oblique axis is, then, + 2 cyl. ax. 145° ⊂ — 4.0 cyl. ax. 75°. The angles θ_1, θ_2 and γ are known; they are 145°, 75°

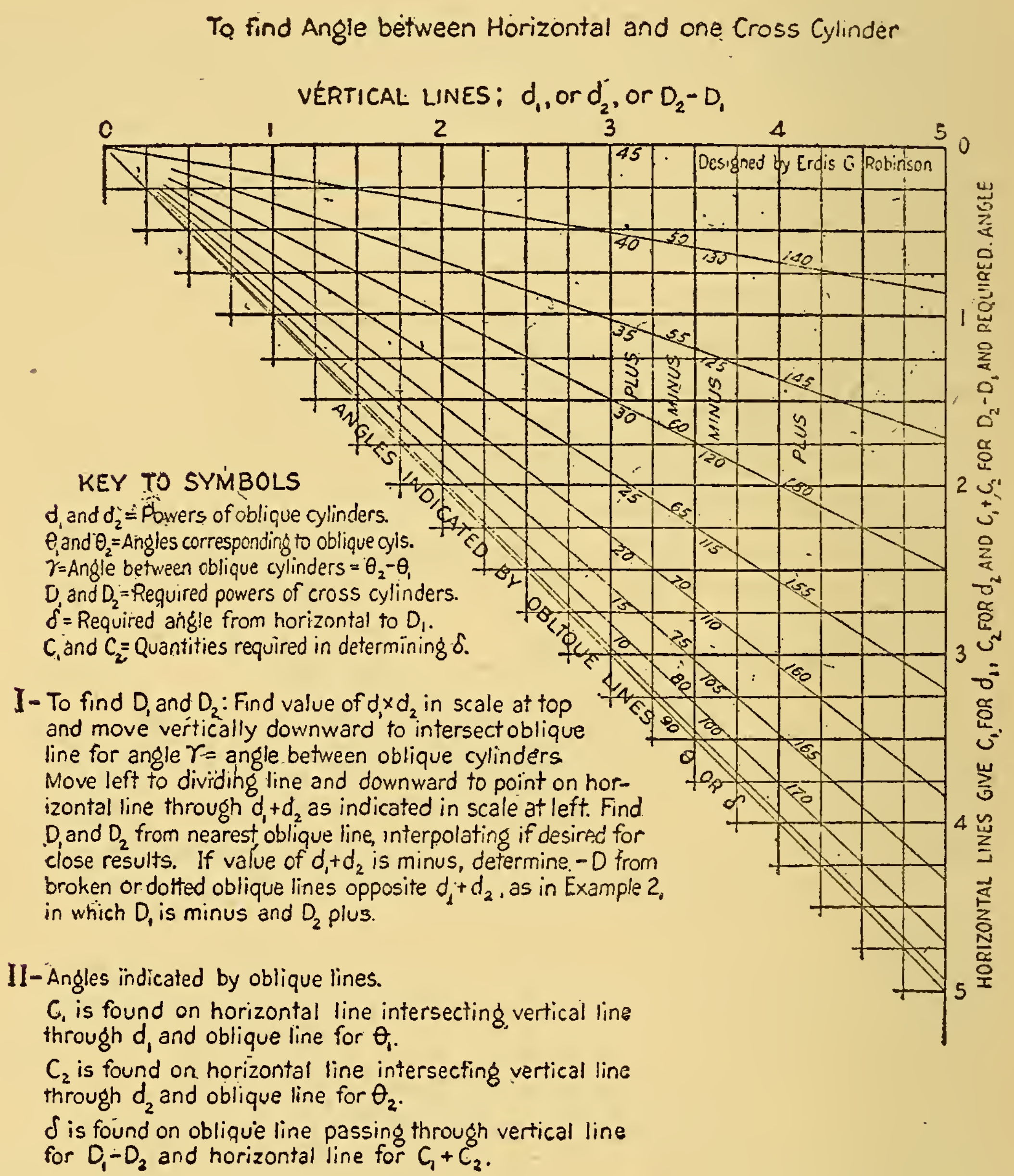
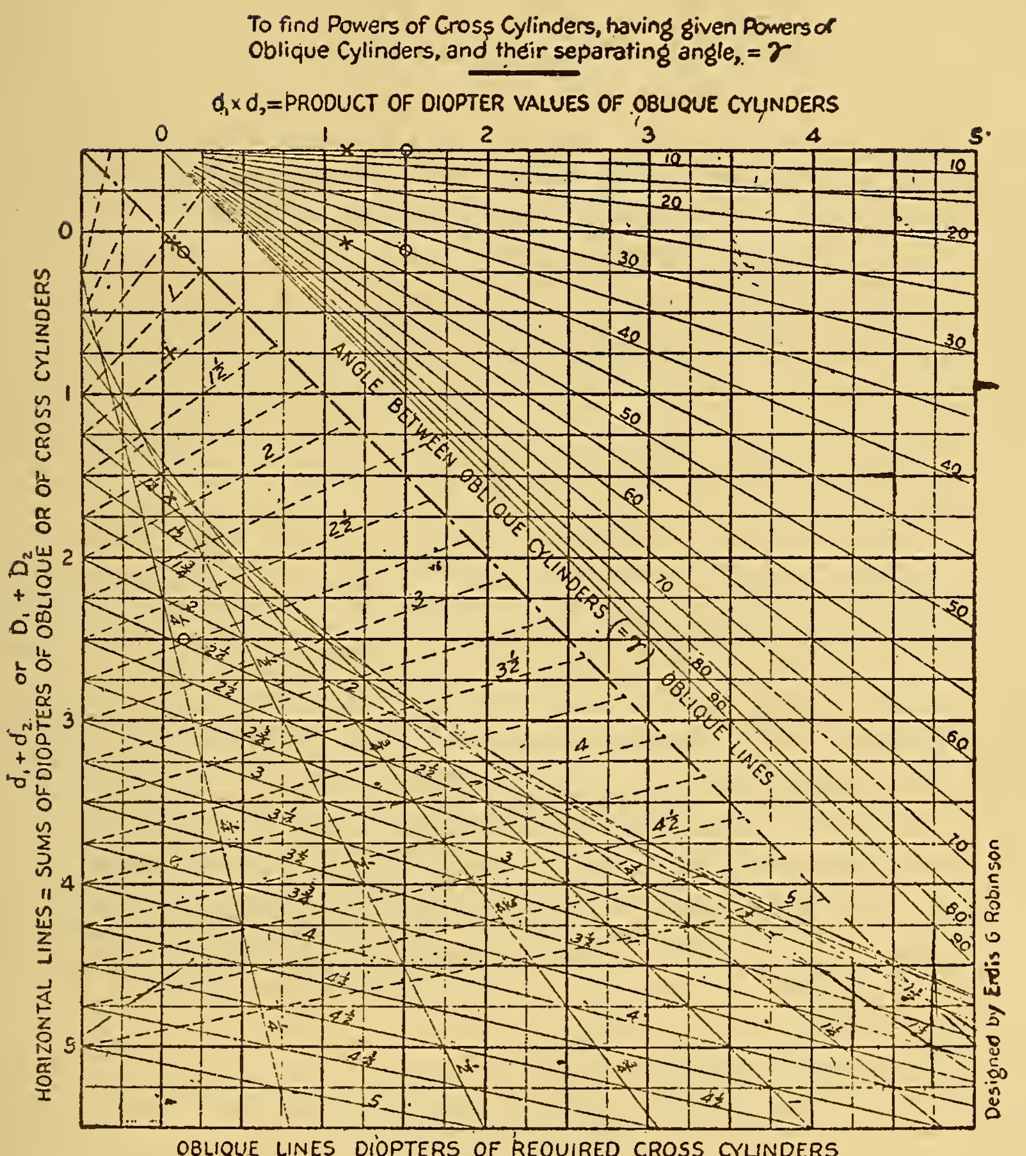


Fig. 73.—(B). Chart for Obtaining the Equivalence of Cylinders Crossed at Oblique Axes.

and 70° respectively. From equations (1-4) the value of $A = + 1.839$ or $- 3.839$ and $B = - 3.839$ or $+ 1.839$, and the value of angle δ from equation (5) is practically 22°. This gives as a cross-cylinder combination, therefore, + 1.839 cyl. ax. 22° ⊂ — 3.839 cyl. ax. 112° or — 3.839 D. S. ⊂ + 5.678 cyl. ax. 22. From the data recorded in the files in connection with the case being described we find that — 3.75



EXAMPLE 1 – Transpose into equivalent cross cylinders + 1.50 cyl. axis 120° $\subset$ 1.00 cyl. axis 80°. Solution on charts indicated by small circles, ⊙. We have $d_1 = 1.50$, $d_2 = 1.00$, $d_1 \times d_2 = 1.50$, $d_1 + d_2 = 2.50$, $\gamma = 40°$. Enter CHART I at top for $d_1 d_2 = 1.50$ and move vertically downward to oblique line for 40°; then to left to Dividing Line (—·—) and down to point horizontally to right of $d_1 + d_2 = 2.50$. Read from nearest oblique lines the values of $D_1 = 0.30$ D, and $D_2 = 2.20$ D. Enter CHART II at top for $d_1 = 1.50$, and move downward to intersect oblique line for θ_1 = "120° minus" and from scale at right read $C_1 = -0.75$. Likewise find $C_2 = -0.95$, and $C_1 + C_2 = -1.70$. From values found by CHART I, $D_2 - D_1 = 1.90$. Enter CHART II again and find δ, the required angle = 15° on oblique line intersecting vertical through $D_2 - D_1 = 1.90$ on top scale, and horizontal through $C_1 + C_2 = 1.70$ in scale at right. Hence required solution 0.30 cyl. axis 15° $\subset$ 2.20 cyl. axis 105°.

EXAMPLE 2 – Solution indicated by small cross X. Transpose +0.75 cyl. axis 135° $\subset$ −1.50 cyl axis 180°. $d_1 = 0.75$; $d_2 = -1.50$, $d_1 d_2 = -1.12$, $d_1 + d_2 = -.75$; $\gamma = 45°$. CHART I gives $D_1 = -1.20$, $D_2 = 0.45$. CHART II gives $C_1 = 0$, $C_2 = 1.50$, $C_1 + C_2 = 1.50$; $\delta = 13°$. Solution −1.20 cyl. axis 13° $\subset$ 0.45 cyl. axis 103°

Fig. 73.—(C). Chart for Obtaining the Equivalence of Cylinders Crossed at Oblique Axes.

D. S. $\bigcirc$ + 5.75 cyl. was introduced into the trial frame and the axis of the cylinder subjectively found to be best at 175°. The final lens values determined upon differed but slightly from the calculated values; the monocular ''best-acuity'' finding was — 3.50 D. S. $\bigcirc$ + 6.00 cyl. ax. 175°. The question of the disagreement of axes of cylinders as subjectively determined and mathematically calculated is discussed in a paper by the writer in the *Ophthalmic Record,* Vol. XXV, pages 558-567, 1916.

Some of the important points to which attention may be directed in the handling of cases involving bi-cylindrical corrections are the following:—(1) The use of the narrow stenopaic slit for the determination as accurately as possible of the best and poorest visual meridians and the attainment of the highest visual acuity in each meridian by the use of spheres according to the customary methods. (2) The preliminary mathematical determination of the values of the two cross-cylinders, which should be converted into equivalent sphero-cylinders and substituted in the trial frame; determine subjectively the best position of the cylinder and finish the test. (3) This method of procedure precludes the possibility of a prescription involving a sphere in combination with two oblique cylinders. (4) The advantage to the practitioner of having the final form of sphero-cylindrical correction before his patient's eye and the nicety of adjustment of the cylinder thus permitted. (5) The fact that equivalent sphero-cylindrical corrections are not equally acceptable warrants the use, in turn, of each form of correction in order to select that which is most satisfactory all points being considered.

Mr. Erdis G. Robinson, C. E., of Columbus, Ohio, has given graphical methods of determining solutions of equations (1-5). These are inserted as Figs. 73 (B) and 73 (C) together with his explanatory notes and directions.

116. *Relations between corneal and total astigmatism.* Quite a difference is often found between the ophthalmometric and subjective measurements. This was first pointed out by Donders and Knapp who attributed this difference to an astigmatism of the crystalline which would act in a contrary direction to that of the cornea. Javal, Pflüger, Tscherning and others have investigated the relations between corneal and total subjective astigmatism. The curves in Fig. 74 are due to Javal and Pflüger; the ordinates represent the amount of astigmatism in diopters. Javal employed concave and Pflüger convex cylinders exclusively. Hence the differences in their cylindrical lens corrections for various ophthalmometric determinations is due in part to the effectivity of plus and minus lens corrections situ-

ated at a certain distance from the eye; the reader is referred to the discussion on vertex refraction and to the effectivity of lenses in the correction of myopia and hyperopia given in previous paragraphs and to treatises on ophthalmic lenses (see particularly Landolt's *The Accommodation and Refraction of the Eye*). Quite apart from physiological reasons, then, apparent changes in the astigmatism of the eye as a whole may arise from optical sources. There is a change due to effectivity as the cylinder is placed in advance of the cornea and as it is convex or concave. This causes the subjective determination of astigmia to differ from the ophthalmometric so that, for example, 4 diopters of corneal astigmatism is corrected by a — 4.25 D. cyl. or a + 3.75 D. cyl. when placed 15 to 17 mms. from the cornea.

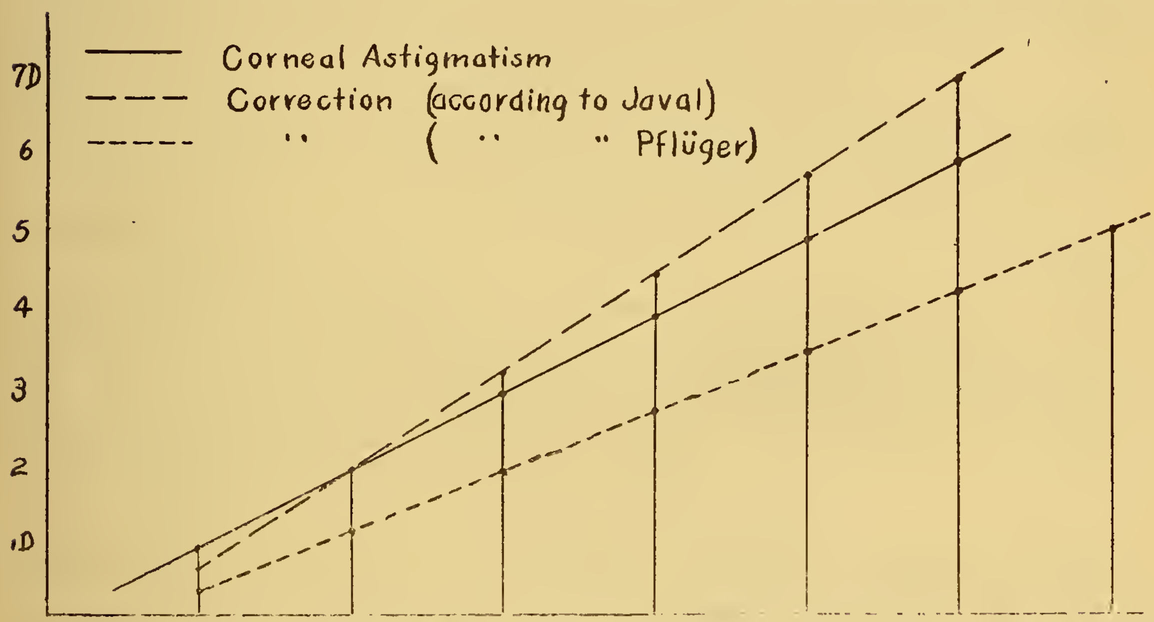


Fig. 74.—Corneal Astigmatism and Lens Connections According to Javal and Pflüger.

There is likewise a change of the astigmatic value of any cylinder as this is combined with a sphere. For this reason a degree of astigmia measured in the eye or at the cornea by keratometric methods is different than the power of the cylinder which will correct it when the latter is combined with a spherical. Suppose, for instance, that there is found 4 D. of actual corneal astigmatism; there will then be required, if one meridian of the eye is emmetropic, a cylindrical lens of focal power + 3.75 D. or — 4.25 D.: if, however, one of the meridians of an eye is hyperopic 6 D. and the other is hyperopic 10 D., the focal lengths of the lens would need to be (at 15 mms. from the cornea) 166 + 15 = 181 mms. and 100 + 15 = 115 mms. or powers of 5.50 D. and 8.75 D., that is + 5.50 D. S. ⊃ + 3.25 cyl. In this case then 4 D. corneal astigmatism would be corrected by a + 3.25

cylinder when in combination with the specified sphere. These conditions may be transposed, however, and the meridians of an eye corrected in the reverse order, i. e., as though the correction were fundamentally 10 D. ⊃ — 4 D. The focal lengths of the lens would then be 100 + 15 = 115 mms. and — 250 + 15 = 235 mms. or powers of + 8.75 D. and — 4.25 D., that is to say, + 8.75 D. S. ⊃ 4.25 cyl. ax. ($x + 90°$) if the plus ⊃ plus combination as just calculated has its cylindrical element at axis "x."

Furthermore, there are the reports of John Rowan (*British Medical Journal*, 1912) in which the astigmatism in one thousand eyes was measured, first by the ophthalmometer of Javal and Schioetz and then by retinoscopy, with atropin or homatropin as cycloplegic. The results of his measurements show that, out of the one thousand eyes examined, *the total astigmatism and the corneal astigmatism were the same* in 475 cases, or 47.5 per cent. This is certainly an interesting conclusion and comes as a rather brisk rejoinder to those who minimize the value of the ophthalmometer in ocular refraction.

117. The rules of Javal, commonly accepted by most investigators, are:—

(1) If there is no ophthalmometric astigmatism, we generally find a slight subjective astigmatism against the rule.

(2) If the ophthalmometric astigmatism is against the rule, the subjective astigmatism is usually against the rule and of greater amount.

(3) If the ophthalmometric astigmatism is with the rule and of a value intermediate between 1 and 3 D., the subjective astigmatism generally differs only slightly from it.

(4) If the ophthalmometer gives an astigmatism with the rule and greater than 3 D., the subjective astigmatism is also with the rule and frequently greater.

Javal expressed the difference between the subjective astigmatism (As_s) and the ophthalmometric astigmatism (As_c) by the empirical rule that

$$As_s = k + p \cdot As_c$$

in which k and p are two constants, $k = 0.5$ D. against the rule and $p = 1.25$. The formula gives the following relations:—

	Against the rule			With the rule					
Ophthalmometric astigmatism..	2	1	0	1	2	3	4	5	6
Subjective astigmatism........	3	1.75	0.5	0.75	2	3.25	4.5	5.75	7

118. Among the factors which may be mentioned which may account for these differences are:—

(a) *Deformity of the internal surfaces.* The vertical meridian of the posterior surface of the cornea presents a more pronounced curvature than the horizontal meridian, whether the condition in toto is one of astigmatism with or against the rule. It is doubtless for this reason that eyes which have no ophthalmometric astigmatism generally have subjective astigmatism against the rule. The term k of Javal's formula must be influenced in part by the posterior surface of the cornea.

(b) *The obliquity of the crystalline lens.* This produces astigmatism against the rule which may be compensated in large measure by the special structure of the crystalline lens (Hermann). This compensation due to crystalline structure, as hypothecated by Hermann and some other investigators, is open to serious criticism however.

(c) *The influence of the distance of the correcting lens from the eye.* In consequence of this the concave correcting cylinder is stronger and the convex weaker than the true astigmatism.

(d) *Astigmatic accommodation of the lens.* This would have the effect, if such an action is possible, of correcting in part or in whole—or even over-correcting—the corneal deformity.

(e) *The astigmatism in the different zones of the cornea.* The peripheral zones frequently possess a value and sometimes a direction more or less different from those of the central zone.

These factors are merely pointed out in concluding this discussion of astigmatism; they have been considered at some length in the preceding paragraphs.

XI. ENTOPTIC PHENOMENA

119. If one approaches a luminous point the circle of diffusion to which it gives rise increases in size; when the luminous point is at the anterior focus of the eyes the rays are parallel after refraction and the circle of diffusion is equal to the size of the pupil. Entoptic phenomena are then observed; that is to say, shadows which the "corpuscles," as Tscherning calls them, or particles situated in the various refracting media of the eye, project upon the retina can be rendered visible to the eye itself. Thus opacities and normal striæ in the crystalline lens and those due to cataract can be seen by looking at a surface, such as a white cloud, through a pin-hole disc placed at the anterior focus of the eye. Another method of observing entoptic phenomena is to look at a luminous point, placed at a far distance from the eye, through a convex lens of high dioptric value. Among some of the simple entoptic phenomena which may be cited are the following:—

(1) On winking the eyes transverse striæ are produced due, prob-

ably, to wrinkles of the epithelial layer. If this is continued for some length of time, as may be the case after reading for a long time in a horizontal position, striæ which last for several hours are produced and give rise to a marked diplopia of horizontal lines. This condition has been called *tarsal asthenopia.* The striæ produced by winking (after the drawing by George Bull) are shown in Fig. 75.

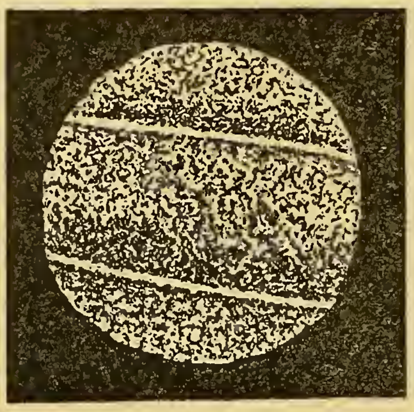


Fig. 75.—Entoptic Striæ Produced by Winking the Eyelids. (After George Bull.)

(2) The luminous spot is always limited by the shadow of the border of the iris: the irregularities of the latter can, therefore, be studied. The pupillary contraction is very readily seen on opening or closing the other eye.

(3) Small circles with bright centers are frequently seen and these

Fig. 76.—Speckled Appearance of the Entoptic Field Produced by Rubbing the Cornea. (After George Bull.)

have an apparent motion after an excursion of the eyelid; they are due to small specks on the anterior surface of the cornea and actually move in a direction contrary to their apparent motion.

(4) On closing and then opening the eyelids, after looking at a distant luminous point, long striæ running vertically are often seen. They are produced by the layer of tears in the conjunctival sac and

which assume near the borders of the eyelids a prismatic form with an outer concave and inner convex surface. Because of this shape of the tear-prism we have a prism of varying power and hence striæ rather than doubled images are produced.

(5) A rubbing of an eye causes the luminous spot to become

Fig. 77.—Star Figure of the Crystalline Lens. (After Helmholtz.)

speckled or mottled, due to the slight irregularities of the cornea or the irregular laying down of tear fluid. This soon disappears. Fig. 76 (after G. Bull) shows the speckled appearance of the entoptic field produced by rubbing the cornea.

(6) The star figure of the crystalline lens can frequently be seen.

Fig. 78.—Incipient Cataract Seen Entoptically. (After Darier.)

A drawing by Helmholtz is shown in Fig. 77. This star figure is sometimes bright and sometimes dark with more luminous borders. Crystalline opacities are outlined with great distinctness. Hence many persons having such conditions can diagram and follow the development of the cataract step by step. Fig. 78 (after Darier) shows an incipient cataract as seen entoptically.

(7) Objects situated in the vitreous are easily seen: they become

partly visible by simply looking at the sky or when reading or working over a smooth white surface. This is particularly true when these objects are near the retina. The name *"muscæ volitantes"* has been given to this phenomenon. If the particle is in motion, its direction can be determined by looking at the sky through a window upon

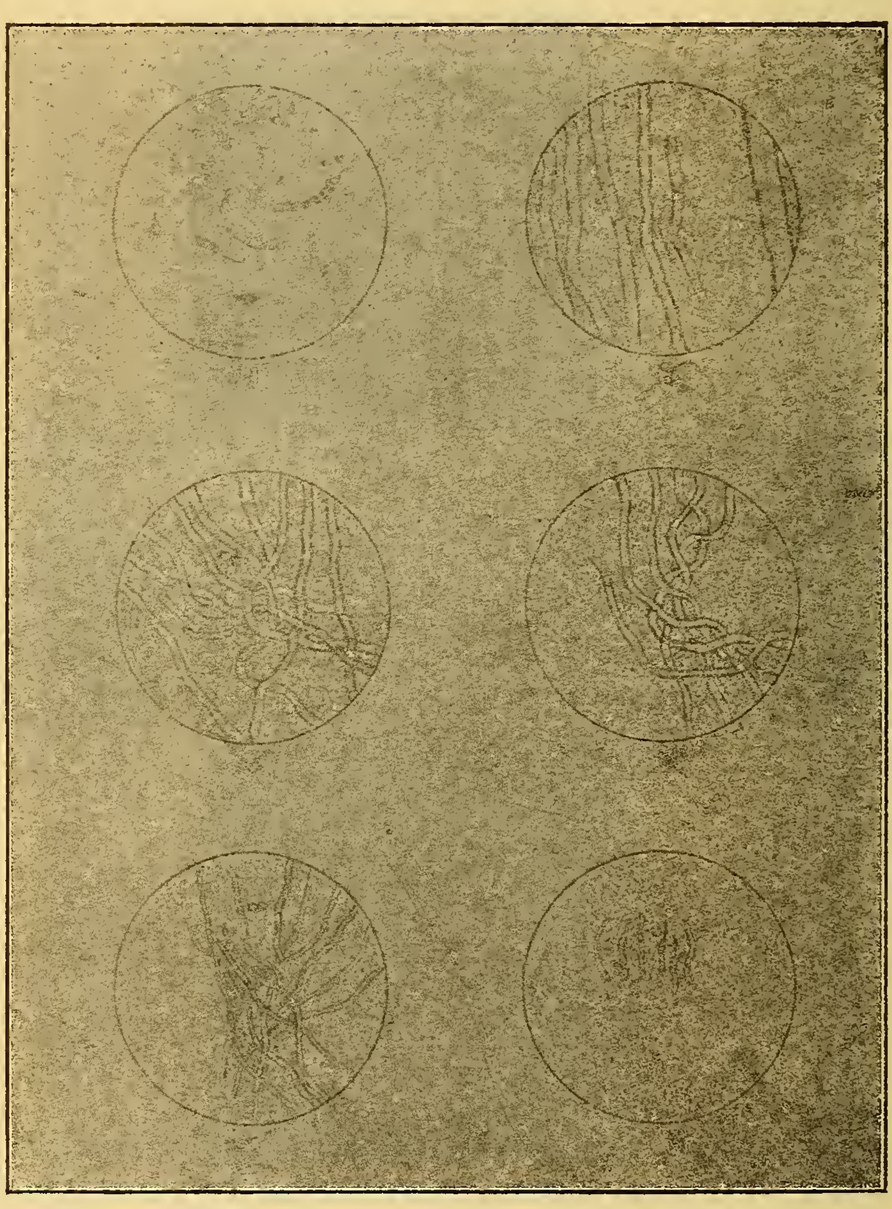


Fig. 78.—(A) Entoptic Figures. (After Bourdon-Cooper.)

which a point is taken to assure fixation and noticing whether the particle ascends or descends; the actual motion will be contrary to the apparent by virtue of the laws of projection of retinal images in space. J. Bourdon-Cooper (*Ophthalmic Review,* December, 1908) found that muscæ volitantes can be satisfactorily studied in the field obtained by using a low power objective in a microscope and placing a high power objective upside down on the top of the eyepiece. With the usual condenser and diaphragm the field illumination can be regulated to give the greatest distinctness of the shadows. A simpler but very serviceable device can be made by fusing the end of a capillary glass tube until it forms a small glass sphere of which all but the sphere can be blackened to avoid light reflections. Studying entoptic phenomena by this means it is found that the string of beads commonly regarded as fixed in the vitreous have a movement which Bourdon-Cooper regards as an argument for a lymph space existing between the retina and the hyaloid membrane.

(8) Looking toward the sky bright points are frequently seen which move rapidly and then disappear giving rise, in turn, to others (Purkinje). Cobalt glass often aids in their observation. This phenomenon is explained as due to the pressure which is exerted on the sensitive layer by a globule of blood which is stopped in a narrow capillary.

(9) By compression of the eyeball for some time we can see the retinal vessels and notice the blood globules magnified about 50 times. The retinal vessels appear bluish. Before perceiving them, however, those of the chorio-capillary membrane, red on a black background, will be observed (Vierordt, Laiblin).

(10) On making, in a darkened room, rapid movements with the eyes we observe two luminous circles corresponding to the places of entrance of the optic nerves. These are due to the traction produced by the nerves during the movement.

(11) When making an effort of accommodation in a darkened room, ofttimes a very large luminous circle is seen. This is attributed to the traction which the ciliary muscle exerts on the interior membranes of the eye during the act of accommodation (Czermak).

(12) By looking towards the sky through a Nicol prism we see the brushes of Haidinger. This is in the form of an indistinct cross. One of the arms is yellow and the other blue. The phenomenon rotates in unison with a turning of the Nicol.

(13) If pressure is applied to a small portion of the sclera there is produced a phosphene corresponding to the inflection of the retina. This experiment, when performed in the dark, exhibits a feebly luminous disc surrounded by a bright border. Young was able to produce a phosphene corresponding to the macula in eyes which were rather prominent. External objects in the position of the phosphene were still visible but with pronounced deformities. By exerting a sufficiently strong and uniform pressure on the eyeball, the entire field will be darkened in consequence of the anemia of the retina. Ohle-

mann (*Ann. of Oph.*, 1909) gives a case of exophthalmic goitre in which the patient perceived a circular phosphene, like a radiating corona, ascribed to mechanical irritation of the retina. It was, without doubt, a pressure effect for it was seen at night at the height of the disease when the tension was the greatest.

(14) Looking at the sky through a narrow slit, the granulated ground and fine vessels which surround the macula will be distinctly seen, but the stenopaic opening must be kept in continuous motion for otherwise the phenomenon disappears. No results will be obtained by looking at the sky without the slit because the shadow of the vessel is too short to reach the sensitive layer. Similar phenomena are frequently observed when working with a microscope; if the field is illuminated with daylight the vessels can be seen by placing the eye at the ocular and giving it a to-and-fro movement.

(15) By observing a bright streak or line through a prism Maxwell observed a dark spot, corresponding to the fovea, which rose and fell with the direction of observation when this was confined to the blue portion of the spectrum but which disappeared immediately when the visual regard wandered from the blue. This is known as the *spot of Maxwell.* One can see this dark spot after having fixed the attention upon a yellow colored paper for a little time and then turning to a blue paper, or by observing the sky through a blue colored glass. The greenish-blue wave lengths (of the order of 5100 to 5200 Angstroms) are in the region for which the sensibility of the fovea is very inferior to what it is for the remainder of the spectrum. It might be possible to explain this phenomenon on the basis of the absorption of these radiations by the yellow pigment of the macula if the existence of this pigment in the living eye were definitely proven.

(16) Entoptic phenomena give a means of studying very *slight displacements of the eye* as a whole. For such experimentation Tscherning invented a small instrument known as the *entoptoscope.* It is shown in Fig. 79. It consists of a plate of wood (*a*) which is held between the teeth; on the vertical rod (*b*) is a spherical cup (*c*) pierced in the center by a small (1/10 mm.) opening which is to be on a level with the eye. At (*d*) are stretched two threads, one vertically and the other horizontally. When the instrument is adjusted and the observer looks toward the sky he sees the entoptic field occupied by the cross which is greatly enlarged. A point in the cross is selected for fixation. The position of the cross is thus dependent on that of the head. If then a displacement of the cross in the entoptic field is produced it is because the eye suffers displacement. It can be shown by this method that the eye is slightly displaced upward when we

wink the eyelids and a little downward when we open the eye very widely. When the head is tilted to one side the eye undergoes a slight displacement in the direction of the weight. These phenomena are more pronounced with the eye under eserine since the field is then very much smaller.

120. *Analysis of entoptic phenomena.* 1. *Observation of their parallax.* Upon fixing various points in the entoptic field or observing some of the phenomena which have been rehearsed above, the

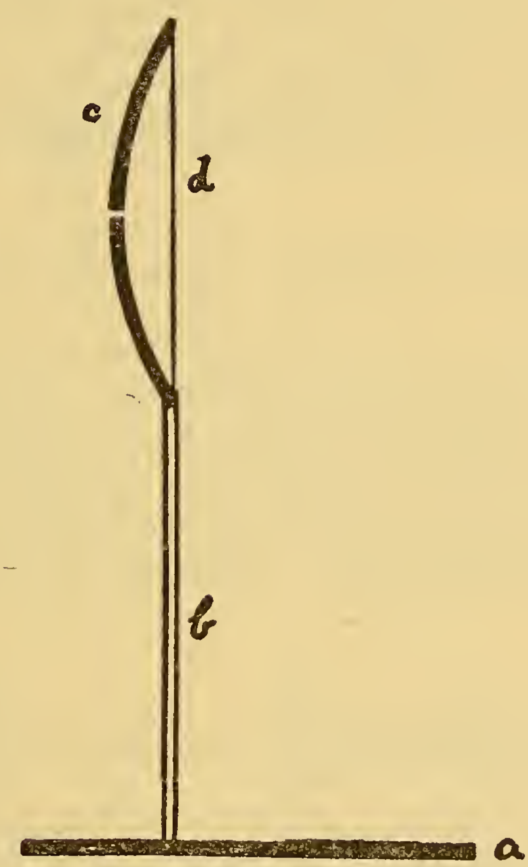


Fig. 79.—Tscherning's Entoptoscope.

entoptic phenomena are displaced in the field. If the particle which gives rise to the shadow is behind the pupillary plane, the shadow will move in the same direction as the visual line. This is shown in Fig. 80 in which *a* represents the pupil, *b* the particle and *V. L.* the visual line. In Fig. 80 (A) is represented a certain condition of visual line, particle and shadow. Suppose now that the eye is directed upwards. The visual line is directed upwards and the shadow, Fig. 80 (B) has descended to the lower portions of the retina. By the law of projection, however, the shadow will appear to have descended or the shadow moves in the same direction as the visual line. The contrary parallax occurs if the object is in front of the pupillary plane and disappears if the object is in this plane.

2. *Measurement of the distances of the corpuscles from the retina.* Brewster proposed the use of two luminous points, as *A* and *B* in

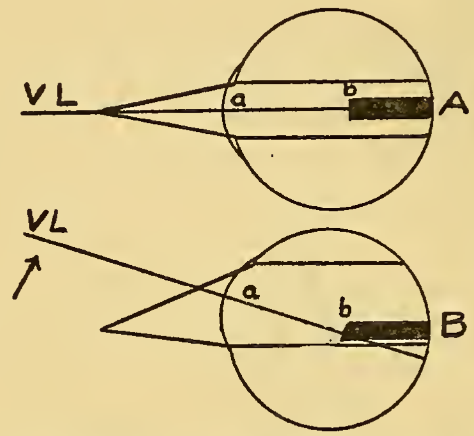
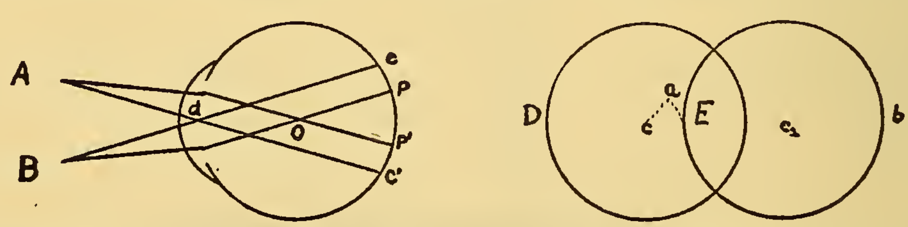


Fig. 80.—Parallax of the Entoptic Phenomena.

Fig. 81, of 0.1 mm. diameter and 2.5 to 3 mms. from each other. These points must be in the anterior focal plane of the eye in order to give parallel light within the eye. Let *d* be the middle of the pupil and *O* the object (or particle). Then *p* and p_1 will be the shadows cast by *O* and *c* and c_1 the centers of the circles of diffusion since *d* represents the central point of the pupil. Also from the diagram, *dc* and *op* are



Figs. 81-82.—Determination of the Position of an Entoptic Object. (After Brewster.)

parallel; likewise dc_1 and op_1 are parallel. Hence from the triangles dcc_1 and opp_1 we have the ratio

$$\frac{pp_1}{cc_1} = \frac{op}{dc}$$

Two circles of diffusion are seen which partly overlap. We measure the distance pp_1 between the two shadows of the same object and the diameter, *DE,* of the free part of one of the diffusion circles as in Fig. 82. The ratio between these measurements is equal to the ratio between the distances of the object from the retina and that of the retina from the pupil. Fig. 82 shows that $cc_1 = DE = R + a$, if R is the radius of the circle of diffusion. One has, therefore, only to project the mutual distance of the two centers, i. e., the breadth of the uncovered part of the circles and that of the double shadows of any object in order to obtain the experimental data necessary to solve the equality of ratios given above and determine *op*, the distance of the particle from the retina. For more complete details and description of Brewster's method the reader is referred to the volume on the *Accommodation and Refraction of the Eye*, by Donders. The measurement of the projected images is most readily effected by the method *à double vue* due to Doncan; looking through the two small openings downwards on a mirror reflecting the light we can with the other

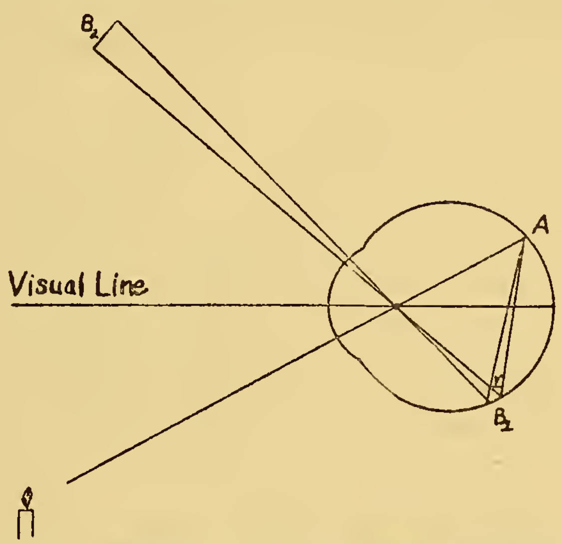


Fig. 83.—Entoptic Observation of the Vessels. (After H. Müller.)

eye project and measure the forms on an adjoining sheet of white paper. Taking into account the distance at which we project, the magnitudes of the retinal shadows are readily obtained.

121. *Entoptic observation of the vessels of the retina.* The retinal vessels, greatly magnified, may be seen projected into the dark portions of a room if, in a dark room, a candle is held at some distance from the eye and the gaze is directed straightforward. The retinal vessels appear of a dark bluish color on a semi-luminous orange background. If we move the candle toward or away from the visual line the vessels are apparently displaced in the same direction. The fovea is without vessels; in some eyes it has a star-like appearance and in others it appears as a luminous disc.

Henry Müller explained these phenomena. In Fig. 83 there is formed at *A* a retinal image of the candle. This portion of the retina thus illuminated sends diffuse light in all directions. A retinal vessel at r, for example, intercepts the rays Ar so as to form a shadow B_1 on the sensitive layer of the retina. This is the shadow which is ultimately seen; it will be appreciated that the shadow B_1 and the vessel r are actually very near together. The shadow at B_1 is seen projected in space as B_2. It can also be seen from the figure that a movement of the light source toward the visual axis will cause a movement of the retinal vessels in the same direction. Direct illumination also produces these images of the vessels on the sensitive part situated behind it, but the shadow is rarely perceived under these conditions probably because this shadow is always formed at the same place in direct fixation and the retinal layer has thus become accustomed to this as a normal procedure.

Light may be concentrated on the sclera by means of a convex lens, as shown in Fig. 84, as near the sclero-corneal border as possible. The

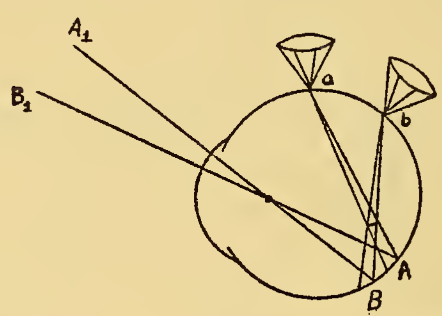


Fig. 84.—Entoptic Observation of the Vessels by Illumination of the Sclera.

dark vessels on an orange background can be seen. The vessels move in the same direction as the luminous focus as shown by their projections in Fig. 84. The explanation is the same as given in connection with Fig. 83; the light of the image of the flame formed on the sclera passes through this membrane and the choroid and causes shadows of retinal vessels. H. Müller measured the distance ab of the displacement of the focus of light on the sclera and the displacement AB of the shadow of a vessel corresponding to this displacement of the light source. Necessarily the distance between the projected images A_1 and B_1 and the distance from the point A_1 or B_1 to the nodal point of the eye must be known in order to calculate the size of AB. Müller calculated that the vessel should be 0.17 mm. to 0.33 mm. in front of the sensitive layer. This experimentation seems to prove that it is the layer of rods and cones that is the sensitive layer, for the distance of the small vessels near the macula from the layer with the cones is about 0.2 to 0.3 mm.

Letters are sometimes seen vividly colored in red when reading in strong sunlight. This may be explained as due to the component of the sunlight, namely the red, which is transmitted through the membranes of the eye. This would be added to the light coming through the pupil. The red would be too feeble to affect the white tint of the paper but would tinge with red the black letters which reflect to the eye practically no light.

If one eye is illuminated while the other is in the shade it will be found after a little that, on alternately closing the eyes, a white object appears greenish to the illuminated eye while it appears reddish to the other eye. The explanation lies in the fact that the light which passes through the membranes of the outer eye is colored red by the vessels of the choroid coat. This red light fatigues the retina of the illuminated eye and this has the effect of making a white object appear greenish in color; the other eye sees it red by contrast.

Both these phenomena may be readily observed in the refracting room and may indeed be sources of annoyance as well as of error unless the presence of brilliant luminous sources, such as incandescent lamps, near the patient's head or in his direct field of view is guarded against. A uniform illumination, such as is obtainable by the present indirect or semi-indirect lighting systems, will afford relief from some of these entoptic phenomena which may arise from no other cause than the presence of side-lights or the promiscuous distribution of luminous sources in refracting rooms and which may have thereby aroused a suspicion of pathologic conditions (or changes which may be associated with such disturbances in color-vision) as would occur to a practitioner when a patient says: "The letters are equally readable with each eye, but they appear sort of reddish with my left (or right) but all right with my other eye." Cases of this kind have arisen in practice which have been found to be uniformly correct and normal in each eye (i. e., letters black and backgrounds white) when the room was flooded with subdued sunlight.

122. *Diffraction in the eye.* If an observer looks at a very brilliant light source he will observe the phenomena of diffraction due to the non-homogeneity of the ocular media. These phenomena are very striking and are designated by the name of "ciliary coronæ." This ciliary corona is composed of an infinity of very fine, many-colored radiations which cross through the whole of the luminous area. Its extent depends upon the intensity of the light source. With an arc lamp or the image of the sun reflected from a convex mirror there will be obtained a diameter of the corona which may reach 8 degrees or more. In addition to the ciliary corona most people see around

the entire luminous source a vivid diffraction ring, which we shall specify as ring *A*, presenting the colors in the order red to blue from the outside toward the center. This ring is separated or distant from the light by a diameter of about 3 degrees for the blue. If the luminous source is not very bright this ring forms the limit of the corona, but if the intensity is high the diameter of the corona may attain a value double that of the ring. The phenomenon appears to be a universal one. Anyone walking along a street lighted by gas or electric lamps may readily observe the phenomena and will be interested to see the manner in which the appearance of the corona and diffraction ring depend upon the proximity to the light source. Viewing a luminous source from some distance when passing along a darkened street one will often see two diffraction rings both of which are fairly faint. Upon closer approach the inner ring becomes more luminous while the outer one disappears. These phenomena will be referred to in the succeeding paragraphs; suffice it to say that the pupil is dilated when in the dark and contracts as one approaches the luminous source.

123. Druault (*Compte rendu du Congrés d'Ophthalmologie d'Utrecht*, 1899) describes the second diffraction ring, designated as *B*, which is seen when the eye is dilated with cocaine, in addition to ring *A*. It presents the colors in the same order as ring *A* but is more irregular and composed of radial striæ. The blue border of the outer ring seems to be superposed on the red portion of the inner ring. An examination of these phenomena using monochromatic light causes some minor changes in their character. For under these latter conditions the ciliary corona presents the form of a luminous dust within which one sees some radial striæ. Quite near the luminous source are one or two very fine black rings due to the diffraction by the border of the pupil. That portion of the luminous "dust" or haze close to the luminous source appears to have a constant motion of the nature of contractions and dilatations which probably correspond to changes of the pupil. The ring *A* exhibits itself as a concentration, regular and circular in form, of the luminous dust. Using yellow light the diameter of this ring is about 4.5 degrees. If one covers a portion of the pupil, all portions of ring *A* disappear at once. The ring *B*, of about 7 degrees diameter, presents the same irregular and striated appearance with yellow light as with white. If one covers a part of the pupillary area the corresponding portion of the ring *B* disappears and the other half becomes much more regular. It is probable that the ring *A* is due to the epithelial cells of the cornea and is of the same nature as those which can be seen by looking

through a plate glass covered with lycopodium. We can calculate the diameter D of the particles causing the phenomenon from the formula

$$D = \frac{2k\lambda}{\sin a}$$

in which λ signifies the wave length and a indicates the angle of deviation or one-half of the angular diameter of the ring and k a constant which, for the first ring, has a value of 0.819. When $a = 2° 12'$ and $\lambda = 5900$ Angstroms, D has a value of 25 μ. Schioetz measured the dimensions of the superficial cells of the epithelium of the cornea and found sizes varying from 25 μ to 40 μ. He likewise showed that on exposing the cornea to the action of distilled water for some time one observes a system of rings of which the first corresponds practically to the ring A; it is a little smaller however. Druault, on the other hand, on looking through a dead cornea found a ring the dimensions of which differed but little from those of ring *A* and which was undoubtedly due to the endothelium of Descemet's membrane, for he could remove the entire epithelium of the anterior corneal surface without producing any effect, but the ring disappeared as soon as the endothelial layer was touched. The ring *B,* on the other hand, is doubtless due to the crystalline fibers which are arranged in the form of a network or grating. The size of the openings or slits between these fibers can be calculated from the formula

$$a = \frac{\lambda}{\sin b}$$

which gives for $\lambda = 5900$ Angstroms the value $a = \frac{0.59\ \mu}{\sin 3° 33'} =$ 9.5 μ. This corresponds closely to the size of the crystalline fibers (10 to 12 μ). The ciliary corona is probably, like the ring *B,* a phenomenon due to meshes or, in other words, is to be attributed to the structure of the crystalline. With feeble luminous sources and dead crystalline lenses suspended in air one can observe phenomena closely resembling the corona.

124. Glaucomatous patients usually see rings which resemble those just described but they are generally larger and of about 10 to 12 degrees angular diameter. The size of the rings increases as the distance between the corpuscles or cells producing them decreases.

Schioetz attributes the origin of the glaucomatous rings to the deepest layer of the corneal epithelium, the cells of which are much smaller than the superficial ones. Experiments on pigs' corneæ, so arranged that salt water could be forced into them, showed that a large circle of diffraction was produced at the time that the liquid penetrated between the deeper epithelial cells. Rings are also often seen by persons suffering with conjunctivitis and are analogous to those which can be produced by the introduction of a drop of blood in the conjunctival sac. A prominent ring of diameter 7.5 degrees for yellow is found, surrounded by a second paler ring. The space between these rings is not black, however, as in the preceding cases but is yellowish to maroon in color.

XII. THE MECHANISM OF ACCOMMODATION

125. In emmetropia the macula lutea coincides with the posterior focal plane of the eye and such an eye, if otherwise normal, will receive a clear image of a distant object. An image of a near object would be formed at its conjugate focus but behind the posterior focal plane. The light would thus be intercepted by the retina before reaching this conjugate focal plane, forming upon the retina diffusion circles. The image of the object, being an aggregation of such diffusion circles, would be blurred. In order to prevent this indistinctness of vision the power of accommodation must be exercised. This is a dynamic function and is called into play by an eye whenever it fixes an object within its far point. The various refractive conditions of the eye and their near and far points, together with their amplitudes of accommodation have been discussed at some length under the portion of this monograph devoted to *Refractive anomalies.*

Five theories have been advanced to account for the manner in which the eye accommodates: (a) increase of curvature of the cornea, (b) increase of curvature of the crystalline lens, (c) elongation of the globe, (d) advance of the crystalline lens and (e) contraction of the pupil. These last two can be readily disposed of, for it can be easily seen that, if the crystalline lens could advance so as to touch the cornea, this would not be sufficient to account for any considerable amplitude of accommodation. The accommodative contraction of the pupil, discovered by Scheiner, is not sufficient to explain accommodation. The theory of the change of curvature of the cornea was supported by the measurements of Home and Ramsden made toward the end of the eighteenth century. This hypothesis and the discussion which arose in connection with it resulted in the very valuable researches of Sturm and of Arlt. The investigations of Sturm on the form of the astigmatic pencil were undertaken to show that accommodation did not exist but that near and distant points were seen with the anterior and posterior parts respectively of the focal interval. When Arlt discovered that myopia depended in general upon an elongation of the eyeball, he labored under erroneous ideas as to accommodation for he thought that an elongation of the globe was brought about by the action of the external muscle when a near object was viewed. By making autopsies on some excessively myopic eyes he was able to prove a lengthening of the globe and believed that he had confirmed his hypothesis. The hypothesis was, in the end, of no value but his experimental demonstrations of the connection between elongated globes and myopia were of the greatest value.

126. *Researches of Young.* The first definite proof that accommodation is due to an increase of convexity of the crystalline lens was given by Young. This great savant wrote his treatise on the *Mechanism of the Eye* (*Philosophical Transactions*) in 1801. He eliminated possible corneal curvature changes during accommodation by observing that during this act there was no change in corneal images but that an easily visible change could be produced by exerting a pressure on a peripheral part of the cornea; this change of curvature was considerably less than that which would be necessary to explain accommodation. But Young added further to the evidence on this subject by his classic experiment of "putting the eye under water." This he did by taking the objective of a microscope which had as nearly as possible the same refractive power as the cornea, filling the tube with water and placing it in front of his eye also submerged in water. The cornea was thus surrounded on both sides by the same liquid practically and was thus eliminated and replaced by that of the unchangeable objective. In this experiment the amplitude of accommodation (about 10 diopters for Young) was preserved, proving that it was not a function of the cornea. To show that accommodation is not produced by an elongation of the globe Young devised the scheme of turning his eye inward as far as possible and applying to its anterior surface a strong metal ring. He then applied the ring of a small key to the exterior side between the eye and the bone until the phosphene, produced by pressure, reached the fovea. The rings were maintained fixedly in place; the eye thus pinned down could not lengthen. He found that accommodation was not abolished and that the phosphene, which would have extended over a great area due to pressure if elongation should have resulted, did not change its size. By using his optometer Young proved that persons operated on for cataract lost their accommodative powers. He thus firmly established

his belief that this power resides in the crystalline lens and hence that it could only result from an increase in the curvature. Young knew nothing about the nature of the ciliary muscle and its contractility, however, and was unable to formulate a theory to explain the mechanism of accommodation.

127. *Researches of Langenbeck, Cramer and of Helmholtz.* The fact that accommodation is accomplished by an increase of curvature of the crystalline lens was first objectively demonstrated by Langenbeck

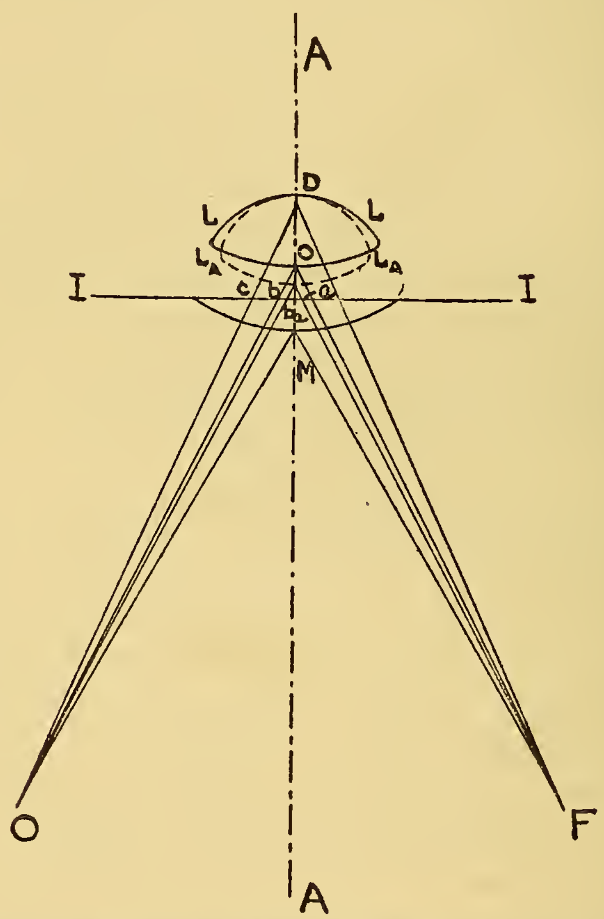


Fig. 85.—Diagram Showing How Reflected Images are Formed. Illustrative of the classic experiments of Cramer and Helmholtz.

in 1849. He observed the changes in the images formed by reflection at the anterior surface of the lens. The images of Purkinje, discussed in the paragraphs devoted to the *Catoptrics of the eye,* were examined and an increase of curvature of the anterior surface of the crystalline lens observed. In 1851 Cramer constructed a magnifying instrument, which he called an *ophthalmoscope,* with which he was able to demonstrate clearly the movement of the images which occurred during accommodation, proving that the anterior crystalline surface made a quite extended centripetal movement. This centripetal movement has been erroneously interpreted at different times by various experimenters to indicate a see-saw movement of the crystalline lens. It is probable, however, that there is a slight trembling or shaking of the lens in the act of accommodation. Fig. 85 attempts to diagram schematically the positions of the three Purkinje images seen during repose and during accommodation when the eye is illuminated by a light source at *F* and observations of the phenomena are made with the observer at *O,* the line of the subject's vision being *AA.* It is to be noted that refractive changes influencing the directions of rays are neglected in Fig. 85; this is to simplify and to present the essential points involved in the classic experiments of Cramer and of Helmholtz. *M* represents the anterior corneal surface, *II* represents the plane upon which are seen the reflected images, *LL* and $L_A L_A$ the crystalline positions of repose and of accommodation respectively. The ray *FM,* by reflection to *O,* gives the anterior corneal image at *a.* The ray *FD,* neglecting refraction as previously explained, after reflection at the posterior surface of the lens, gives the image due to reflection from the concave posterior lenticular surface at *c.* With the lens in a position of repose the anterior lenticular image is at *b,* but upon accommodating the image is found at b_a as can be seen by following the course of the dotted lines in Fig. 85. This indicates a centripetal movement and shows that the accommodative change takes place in the anterior crystalline surface. Cramer attributed this change, however, to the contraction of the iris: he believed that the iris was, in a state of repose, in a swollen condition and that it flattened during accommodation, thus exerting a pressure on the peripheral parts of the crystalline lens and that the ciliary, contracting simultaneously, exerted a traction on the choroid pushing the vitreous forward, thus subjecting the crystalline to a pressure except on the pupillary part. These ideas had to be abandoned when v. Graefe published his case of complete aniridia in which the amplitude of accommodation was intact, thus limiting the active agent in accommodation to the ciliary.

128. At the same time, and independently of Langenbeck and of Cramer, Helmholtz found the same results but added that the posterior surface of the crystalline is also modified though but slightly. He finally invented his ophthalmometer by means of which observations of the forms and variations in forms of the various surfaces of the eye have been rendered so precise. Helmholtz used two sources of light, a lamp and its image by reflection from a mirror. His results are pictured in Fig. 86. In these diagrams *A* represents the condition of affairs with the eye in a state of repose, and *B* that of accommodation; *a,* the corneal reflexes; *b,* reflexes from the anterior surface of

the crystalline, smaller and consequently nearer each other during accommodation (*B*) and nearer to those from the cornea; *c*, reflexes from the posterior surface of the crystalline lens, least luminous of all, keeping their positions and becoming slightly smaller during accommodation. It is now universally agreed that the accommodation of the eye is produced by a change in the form of the crystalline lens, by virtue of which the anterior surface of this lens advances and becomes more convex while the posterior surface increases but little in convexity and changes its position but slightly if at all. From his investigations Helmholtz adopted certain values for his schematic eye and these have been quite generally accepted. These numbers, together with his results upon the dead eye, are tabulated below.

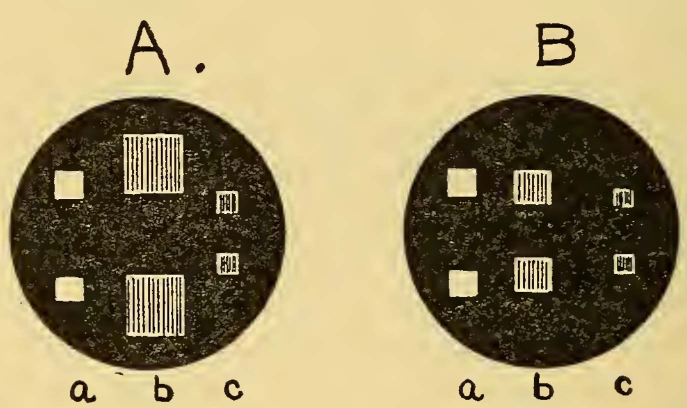


Fig. 86.—Purkinje Images During Repose and Accommodation.

A, state of repose. B, state of accommodation. *a*, corneal reflexes; *b*, reflexes from anterior surface of crystalline; *c*, reflexes from the posterior surface of the crystalline lens.

	Schematic		*Dead Eye*	
	Repose	Acc.	A	B
Radius of anterior surface...	10 mm.	6	10.16	8.87
Radius of posterior surface..	6 mm.	5.5	5.86	5.89
Thickness	3.6 mm.	4	4.2	4.31
Focal distance	43.71	33.79	45.14	47.44
Total index of refraction....	1.4545		1.4519	1.4414

129. The ciliary muscle is the agent by or through which the lens becomes more convex during accommodation. It is still a question whether the zonula is relaxed or is tense during accommodation and there are different views which are held as to the exact form the lens assumes during the act of accommodation. The two chief theories are due to Helmholtz and Tscherning; some of the essential points of difference with supporting proofs will be outlined in succeeding paragraphs. The reader is referred to the masterly writings of these

men for details. The anatomical facts which may with propriety be succinctly stated here relative to the ciliary region are as follows: The lens, enclosed in its capsule, is supported between the aqueous and the vitreous by a delicate ligament, the *zonula of Zinn* (or the suspensory ligament). This ligament is attached to both surfaces of the capsule near the peripheral border of the lens. The ligament thus attached to the lens has its outer border attached to the ciliary processes and the depressions between them. The ciliary processes are a network of blood vessels and pigment which line the inner circumference of the sclero-corneal ring and which, running backward, become united with the choroid. The muscular nature of the ciliary was discovered by Wallace, an American physician, in 1836; the credit is usually given to Bowman and Bruecke (1846).

The ciliary muscle lies beneath the ciliary processes, is composed of non-striated fibers and is made up of two parts. One portion consists of longitudinal or meridianal fibers which are attached anteriorly to the sclera near the canal of Schlemm and passing backward are inserted in the anterior portion of the choroid. The outer surface of the muscle is in contact with the sclera. On the inner side of the meridianal fibers are the transverse or circular fibers ordinarily known as the annular muscle of Mueller. This consists of a circular band of fibers surrounding the margin of the iris. Some of the fibers, after proceeding for a certain distance transversely, penetrate this portion of the muscle and join the meridianal part.

130. *Statement of the theories of Helmholtz and of Tscherning.* Helmholtz first presented a rational explanation of the way in which accommodative changes are accomplished. He believed that in a state of repose the crystalline lens is kept flattened by a traction exerted by the zonula. A contraction of the ciliary muscle, of which the anterior extremity is attached to the firm sclero-corneal border, would then draw forward the anterior portion of the choroid, to which the posterior extremity is attached, during accommodation. As a consequence of this forward motion the ciliary processes and the suspensory ligament of the lens would also be drawn forward and a relaxation of the zonula would occur. The crystalline lens would then swell by its own elasticity, approaching the spherical form.

On the other hand, according to the views which have been recently elaborated by Tscherning and his collaborators at the Sorbonne, it is considered that when the eye is adjusted for distance it is entirely at rest. The ciliary muscle is relaxed, the zonula is also relaxed and the convexity of the lens is just sufficient under conditions which are normal, to produce a clear image upon the retina. When, however, accommodation is effected, the contraction of the ciliary muscle produces a tension of the zonula and this is of such a character as to cause an increase in the convexity of the anterior surface of the lens or, as Tscherning calls it, a temporary *"anterior lenticonus."*

Helmholtz confined his measurements to the portions of the surface near the optic axis. Tscherning has carried on numerous researches involving measurements of curvatures of more peripheral parts of the lens surface. Briefly stated, he has found that the curvature of the anterior surface, which is chiefly instrumental in accommodation, diminishes very rapidly as the distance from the axis increases, and concludes that the anterior surface of the lens assumes in accommodation a form closely approximating an hyperboloid.

131. *Helmholtz theory—Observations and experiments in support thereof.* The first experimental observations undertaken to test the correctness of the assumptions of Helmholtz were by Hensen and Voelkers (*Arch. für Ophthal.,* 1873) performed upon the lower animals. They thrust very fine needles into the eye a little behind the ora serrata; on stimulating the ciliary ganglion they saw the free extremity of the needle describe a movement backward. They were able to demonstrate:—(1) a contraction of the pupil with a forward motion of the pupillary border of the iris and of the anterior surface of the lens with an increase of curvature of this latter surface and (2) contraction of the ciliary muscles with advancement of the ciliary processes and anterior portion of the choroid.

132. Coccius (1867) and Hjort (1876) observed the changes which occur in the living eye during accommodation; the first named experimenter observed eyes upon which peripheral iridectomies had been performed, while the latter made use of a person in whom there existed total aniridia, due to accident, but who possessed an accommodation of 5.8 D. The changes which these investigators observed were analogous to those described by Hensen and Voelkers. They were also able to view the ciliary region directly and to demonstrate that the ciliary processes advance during accommodation. In recent years Hess has conducted experiments in which he concludes:—(1) that the suspensory ligament is in a relaxed condition during accommodation because he succeeded in demonstrating that the observations of Coccius and Hjort are correct, (2) a sinking of the lens from gravity when the eye makes a maximum effort of accommodation and (3) a change of position of the lens during accommodation with change of position of the head; that is, a forward movement when the head is inclined forward and so forth. The experiments of Hess are given in detail in the *Transactions of the A. M. A. Ophthalmological Section,* 1907.

133. A considerable amount of discussion has ensued over the measurements of the thicknesses and curvatures of surfaces of dead and living eyes as made by Helmholtz. Helmholtz measured the thickness of the crystalline lens and found it a little greater during accommodation than in a state of repose; he also measured two dead crystalline lenses and found their thicknesses greater than those of living eyes in a state of repose. This may depend on elongation of the lens through tension of the zonula of Zinn during life as a result of the pressure of the vitreous humor, while after death, when the pressure ceases, the tension may diminish and the lens consequently become thicker. Tscherning doubts whether these autopsies tell in favor of the theory of Helmholtz because (1) measurements of the thickness of a living lens are not within the limits of error experimentally; "the much disputed question of knowing whether the crystalline lens changes its thickness during accommodation can with difficulty be decided by the observation of the crystalline images, for the alleged change (an increase of 0.4 mm.) does not exceed the limit of error" (Tscherning) and (2) dead crystalline lenses do not possess the accommodative form but the radii of curvature are those corresponding to the living eye in a state of repose. Helmholtz's measurements of the radii of curvature of the anterior surfaces of three living eyes in a state of repose were 11.9 mm., 8.8 mm. and 10.4 mm., while for the dead eyes he found 10.16 and 8.87 mms. Stadfeldt, in 1896, measured eleven living human crystalline lenses in repose obtaining with the ophthalmometer an average radius of curvature of 10.6 mms. for the anterior surface of the crystalline. The average with a half-dozen dead crystallines taken from the eyes and measured with a Javal ophthalmometer without any traction being applied was 11.4 mms. Souter, in his textbook on the *Refractive and Motor Mechanism of the Eye* reports some investigations made upon the healthy lens of a man twenty-five years of age immediately after the enucleation of the eye. Upon removal of the lens from the eye he observed that the usual flattened aspect of the anterior surface was absent and resembled very closely the accommodative form described by Tscherning. Traction made at opposite points of the equator of the lens produced a decided flattening of curvature which disappeared on release from traction: "the action of the lens did not in any way justify a belief in Tscherning's theory."

134. If we assume with Helmholtz that there is a relaxation of the suspensory ligament as a consequence of the forward motion of the ciliary processes and the zonula, we need only to glance at Fig. 87 and recollect the nature of the constitution of the crystalline lens in order

to see what will be, in a general way, the effect of such a relaxation upon the shape and position of the lens. The crystalline lens and ciliary region during repose and when in a state of tension are represented by heavy lines; the dotted outline represents the changes which occur during accommodation.

In childhood, when accommodation is most active, the lens consists of a gelatinous mass enclosed in a contractile capsule. Such a mass will assume a shape approximating the spherical form according to the physical law that a fixed volume of liquid presents its smallest area of external surface when in this form and the contractility of

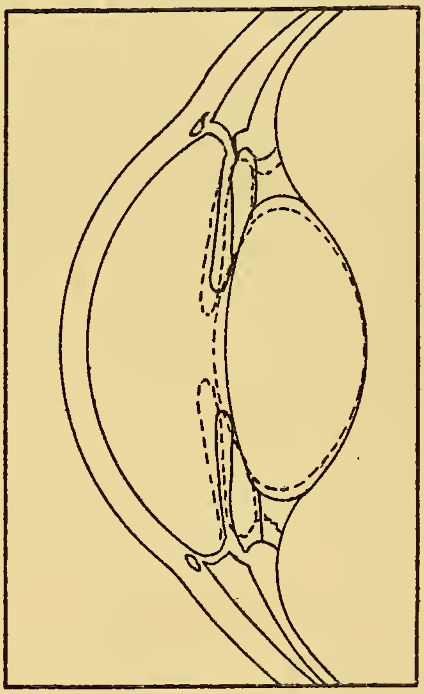


Fig. 87.—The Crystalline Lens and Ciliary Region.

The dotted outline represents the change which occurs in the act of accommodation according to Helmholtz.

the capsule is ever acting to reduce this surface area. A rubber bag filled with water illustrates this condition very well; by pressure or traction on the bag its shape is altered but its original form will be resumed by release from pressure or traction. In the case of the lens there are modifying conditions resulting from its characteristic structure which prevent its assuming a spherical shape even though all external pressure is removed. Again, the anterior portion of the suspensory ligament is shorter than the part attached to the posterior surface of the lens and as a result—presumably—the tension will be greater upon the anterior than upon the posterior surface of the lens; hence the effect of relaxation must be greater upon the anterior crystalline surface, allowing this to advance with a decided increase in curvature. The posterior surface, on the other hand, will be but little affected either in curvature or position.

135. *Experiments of Tscherning and his collaborators. I. The overcorrection of spherical aberration during accommodation and the peripheral and central amplitude of accommodation.* Aberroscopic phenomena show that most persons see the shadow concave toward the periphery, but on effecting accommodation the forms of the shadows change: these shadows turn their concavity toward the middle or the center (see Fig. 57). The central refraction must, therefore, have increased more than the peripheral refraction. Fig. 61 (l. c.) exhibits a condition of overcorrection of the aberration, proving that the pupillary contraction cannot have been responsible for the effects. The optometer of Young affords a means of measuring directly the difference between the central and peripheral amplitudes of accommodation. The central accommodation can be measured with the two narrow and closely situated slits placed near the pupillary center and the peripheral accommodation with the triangular plate lowered just enough to permit of seeing the two lines. Some measurements recorded by Tscherning (*Encyclopédie française d'Ophthalmologie*, Vol. III) are as follows:—

Subject	Central Amplitude (Interval = 0.75 mm.)	Peripheral Amplitude (Interval = 5 mms.)
Young	9.8 D.	4.2 D.
Koster	8.0 D.	3.3 D.
Demicheri	7.5 D.	3.7 D.
Mme. T.	6.7 D.	3.8 D.
Tscherning	3.0 D.	1.25 D.

136. The methods of skiascopy with a luminous point furnish an objective method of studying accommodative changes. A fixation stand is placed before the party under observation at a point very close to his punctum proximum; the observer throws the light from a well-screened lamp into the observed eye using a concave mirror which forms an image of the luminous point at about the same position as the point of fixation. To make the observation it is best to select, when possible, a person whose pupil is well dilated with cocain (since the accommodative power is not to be allayed) who is also emmetropic and who does not have too much aberration when the eye is in a state of repose. Skiascopically, then, when the person under observation

accommodates, the borders of the pupil will be illuminated and separated from the small bright luminous area in the center by a dark zone as shown in Fig. 88 (b). As long as the person does not accommodate the whole pupil will be entirely illuminated. Fig. 88 (a) shows by contrast with diagram (*b*) the appearance in a non-accommodated eye made myopic by a convex lens. The degree of aberration can be determined from skiascopic observations by measuring the distance from observer to observed. For example, if the fixation point is placed at 10 cms. (10 D.) from the observed eye and if the operator when at 50 cms. (2 D.) sees the ring there is an aberration of 8 diopters. By approaching closer and closer to the point of fixation the ring becomes thinner and thinner but it is rare that it disappears entirely before the accommodation attains a high degree.

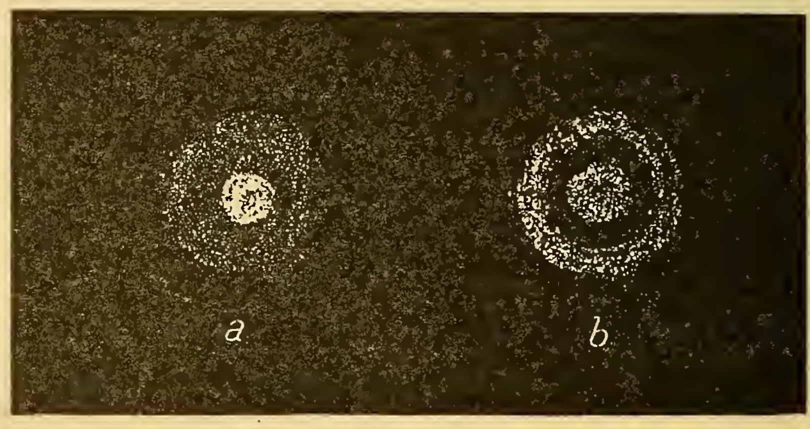


Fig. 88.—Skiascopic Examination of Accommodation.

a, appearance of the emmetropic eye made myopic with a lens of + 5 D. *b*, appearance of the same eye, accommodating 5 D., without lens. (After Tscherning.)

From these experiments Tscherning concludes that the amplitude of accommodation diminishes toward the periphery of the pupil.

137. *(II). Changes in the form of the crystalline surfaces during accommodation.* Tscherning and his pupils conducted a series of experiments with the ophthalmophakometer to determine the curvatures and thicknesses of the crystalline during repose and accommodation. They differ little in many essentials from the results of Helmholtz but do appear to indicate that the increased thickness of the crystalline when in the accommodated state is due to a recoil or retrogression of the posterior surface. It also appears that the crystalline undergoes slight displacements, forward or backward, at the same time it changes form, but the determinations of the positions of the surfaces cannot be made with sufficient accuracy to warrant any

definite conclusions. The dioptric power of each surface can be calculated from the relation

$$D = \frac{n - 1}{R} = \frac{1.074 - 1}{R}$$

The following table gives a representative set of data on an eye, (A) from the experimentation of Helmholtz and (B) from Tscherning. Table (C) gives the dioptric values of the changes during accommodation as taken from Tscherning's data.

Table A (Helmholtz)

	Repose	Accommodation
Radius of anterior surface	11.9 mms.	8.6 mms.
Radius of posterior surface	5.8	...
Depth of chamber (anterior)	4.0	3.7
Thickness of lens	3.2	3.5
Position of posterior surface	7.2	7.2

Table B (Tscherning)

	Repose	Accommodation
Radius of anterior surface	9.7 mms.	5.4 mms.
Radius of posterior surface	5.7	5.3
Depth of anterior chamber	3.6	3.5
Thickness of lens	4.0	4.3
Position of posterior surface	7.5	7.8

Table C

	Repose	Acc.	Diff.	Repose	Acc.	Diff.
Anterior surface	7.6D	13.7D	6.1D	6.5D	11.3D	4.8D
Posterior surface	13.0	14.0	1.0	13.2	16.5	3.3
Total diopters	20.6	27.1	7.1	19.7	27.8	8.1

These and similar results indicate that the rôle of the posterior surface of the crystalline is not negligible in accounting for accommodation.

138. Young believed that it was impossible to explain the negative spherical aberration of an eye other than by a flattening of the peripheral parts of the crystalline. Tscherning succeeded in giving a tangible form to this conception by using the arc of the ophthalmophakometer in a horizontal position carrying three lamps so situated as to permit of all three images formed by the anterior surface of the crystalline being visible in the pupil. The gaze of the observed party is so directed that the three images are situated near the upper border. In a state of repose these images are all in a straight line or else slightly concave toward the center; during accommodation they form a curve convex towards the middle, the curvature of which is greater in proportion as the accommodation is increased. Fig. 89 shows these effects and is taken from the work of Crzellitzer: diagram (1) is for an accommodation of one diopter, (2) for an accommodation of 5

diopters, (3) for an accommodation of 6.7 diopters and (4) shows the positions of the three images under an accommodation of 9.1 diopters. These phenomena indicate a greater curvature at the middle than at the periphery. This can be seen more readily by employing three other lamps on a similar cursor to that used in the above experimenta-

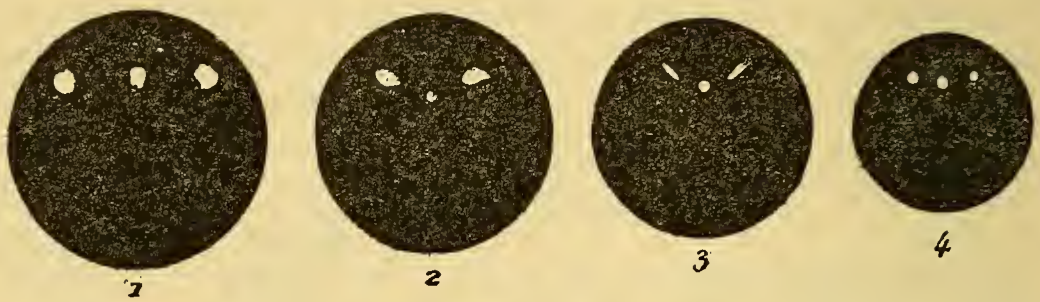


Fig. 89.—Images of the Anterior Surface of the Crystalline Lens. (After Crzellitzer.)

1, Accommodation of 1 D.; 2, Accommodation of 2 D.; 3, Accommodation of 6.7 D. and 4, Accommodation of 9.1 D.

tion and placed vertically above or below the first carrier. Let us assume the conditions represented in Fig. 90 and consider as objects the distances between the two lamps situated on the same vertical line. During repose there would be three images all of the same size represented by a_1, a_2 and a_3 in Fig. 90, showing that the curvature is every-

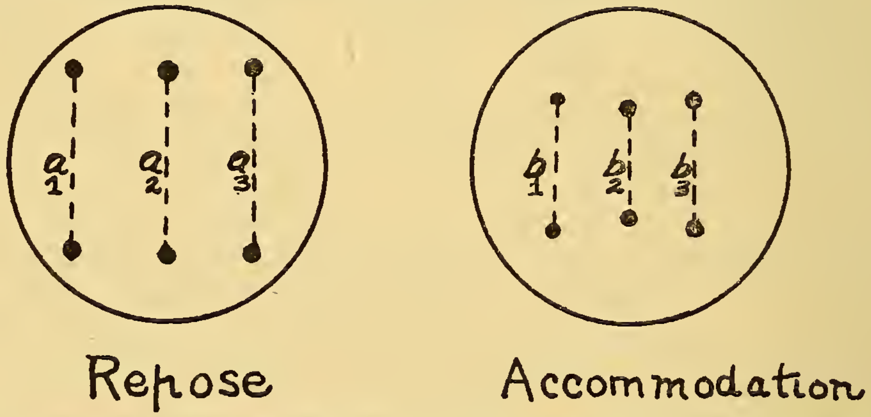


Fig. 90.—Diagrams Showing that the Curvature is Everywhere the Same During Repose, but that the Curvature Increases at the Center During Accommodation.

where practically the same. But during accommodation the middle image, b_2 of Fig. 90, is considerably smaller than the other two images, b_1 and b_3, showing that the curvature is greater.

139. The peripheral parts undergo a real flattening which causes, however, an increase of refraction; it might appear from the above considerations as though the curvature of the peripheral parts in-

creased during accommodation but not as rapidly as at the center. It is to be remembered that, except at the axis, it is the normal and not the radius of curvature which plays the part of the radius of the refracting sphere, assuming that the luminous point is on the axis. Tscherning gives a clear and mathematically elegant treatment of this subject and the method of applying his ophthalmophakometric observations in the *Encyclopédie française d'Ophthalmologie,* Volume III, pages 268-272. We shall quote his results later, but will include at this point a simple example illustrating the calculation of the radius of curvature, ρ, from the formula

$$\rho = \frac{N^3}{\rho_0^2}.$$

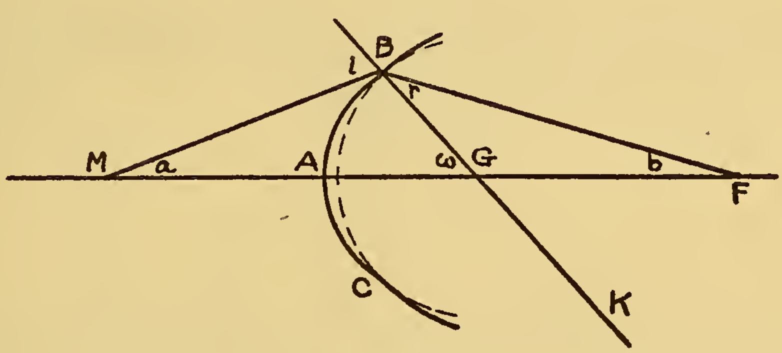


Fig. 91.—Refraction by a Parabolic Surface.

In Fig. 91 let *BAC* represent a curve of the second degree, *MF* its axis and *BK* the radius of curvature at the point *B, BG* the normal and the dotted curve, a circle, drawn with *BG* as radius. The luminous ray *MB* is refracted in the direction *BF* exactly as if the surface were replaced by a circle of radius *BG.* Assuming that the accommodation is accomplished solely by the anterior surface and taking the data previously given with regard to the accommodation of Demicheri, who had centrally an accommodation of 7.5 D. and at 2.5 mms. from the axis an accommodation of 3.7 D., let us suppose that 10 mms. represents the radius of the anterior surface in a state of repose and that 1.06 is the index of the crystalline relative to the aqueous humor. Then the refractive power centrally of the anterior surface is

$$D = \frac{n - 1}{r} = \frac{0.06}{0.010 \text{ meter}} = 6 \text{ Diopters}$$

During accommodation the central refraction increased 7.5 diopters, giving a total refracting power of 13.5 diopters. The radius of curvature at the centre, ρ_c, can therefore be gotten from

$$\frac{n-1}{\rho_c} = \frac{0.06}{\rho_c} = 13.5\text{ D.}$$

which gives $\rho_c = 4.44$ mms. At 2.5 mms. from the axis the accommodation was 3.7 D. and therefore the refraction at this point of the anterior surface in a state of repose was 9.7 D., and the normal, N, is given by

$$\frac{n-1}{N} = 9.7 = \frac{0.06}{N}.$$

This equation gives $N = 6.1$ mms. The radius of curvature, ρ, at this point can be found from the expression which holds good for all surfaces of the second degree, namely

$$\rho = \frac{N^3}{\rho_c^2}.$$

This gives $\rho = 12$ mms., while for the same surface in repose $\rho = 6.1$ mms. and the central radius of curvature, ρ_c, $= 4.44$ mms. These results show that the surface is flattened during accommodation as one proceeds toward the periphery. The surface will, therefore, have the form of a flattened hyperboloid.

140. The curves given in Fig. 92 are taken from Tscherning's and Besio's recent work on the subject: the curves show the lengths of the radii of curvature of the anterior surface of the crystalline at different distances (y) from the axis. The ordinates give the radii in millimeters. The lower curves correspond to a condition of repose, the upper curves to a state of accommodation in three different cases. The figure shows that the curvature diminishes toward the periphery during a state of repose but in a degree differing for different eyes; during accommodation the peripheral flattening is accentuated; the curvatures of the central portions increase while the peripheral curvatures diminish.

141. The flattening toward the periphery of the anterior lenticular surface explains in part the difference which is observed, by skiascopy or otherwise, between the central and peripheral accommodation. Calculations from the data of Tscherning and Besio, according to the

formula $D = \frac{74}{N}$, give as the average difference between the central and peripheral parts about 3 D., while skiascopy and the subjective methods give a difference of 5 to 6 diopters. A part of the accommodative overcorrection must then be attributed to the posterior surface. No direct measurements are at hand establishing the peripheral flattening of the posterior surface, but Grossmann, who studied the

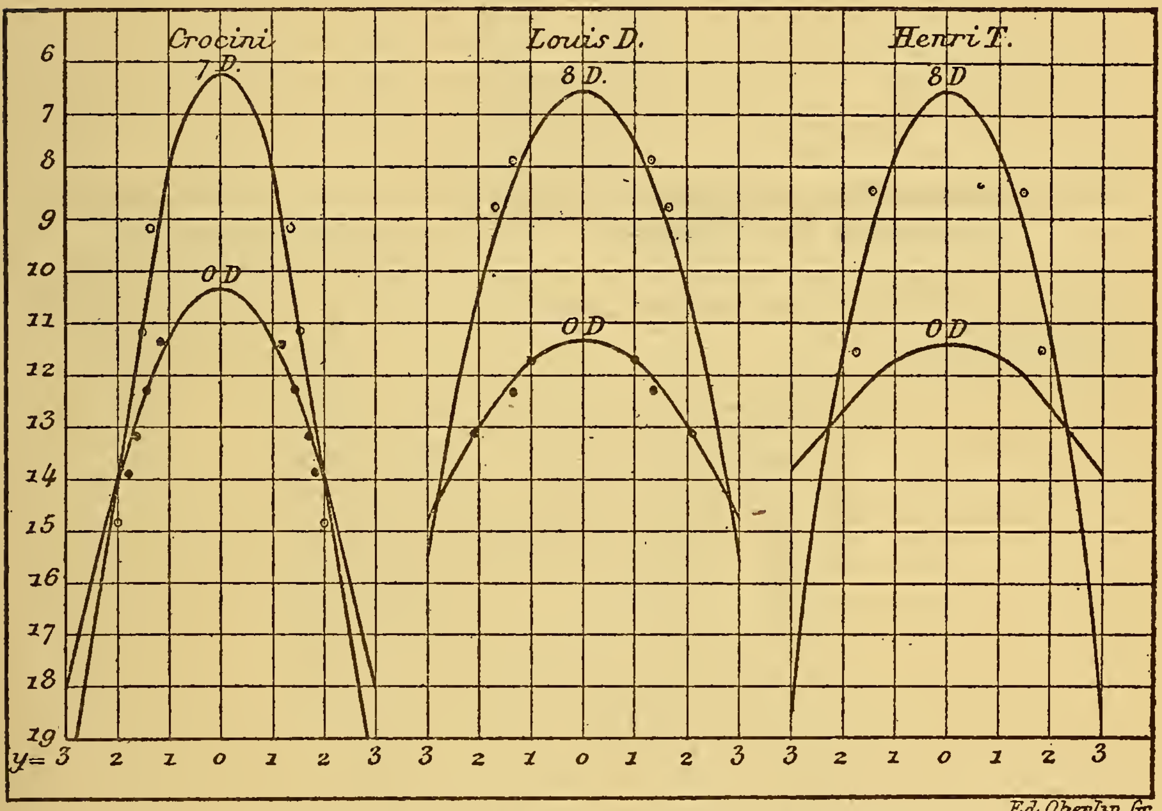


Fig. 92.—Curves Showing the Lengths of the Radii of Curvature of the Anterior Surface of the Crystalline at Different Distances from the Axis. (After Besio.)

accommodative phenomena in a subject having congenital aniridia, has drawn the forms of the images due to this surface and they are analogous to those represented in Fig. 89. In fact the figures of Grossmann indicate that the flattening is much more pronounced at the posterior lenticular surface. Furthermore, the observations of Grossmann show that there is a diminution of the diameter of the crystalline during accommodation. Likewise, according to the investigations of Besio, the increase in the thickness of the lens during accommodation is to be attributed to the flattening of the surfaces near the borders.

Fig. 93 is drawn from the measurements of a living eye as given by Tscherning and show in heavy lines the form of the lens during repose

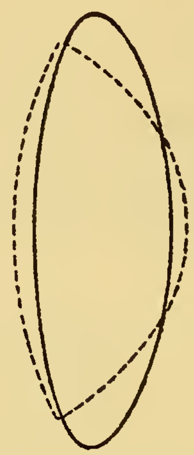


Fig. 93.—Form of the Crystalline During Repose (————) and During an Accommodation of 8 Diopters (. . . . . .). (After Tscherning.)

and by dotted lines the shape of the lens during an accommodation of 8 diopters.

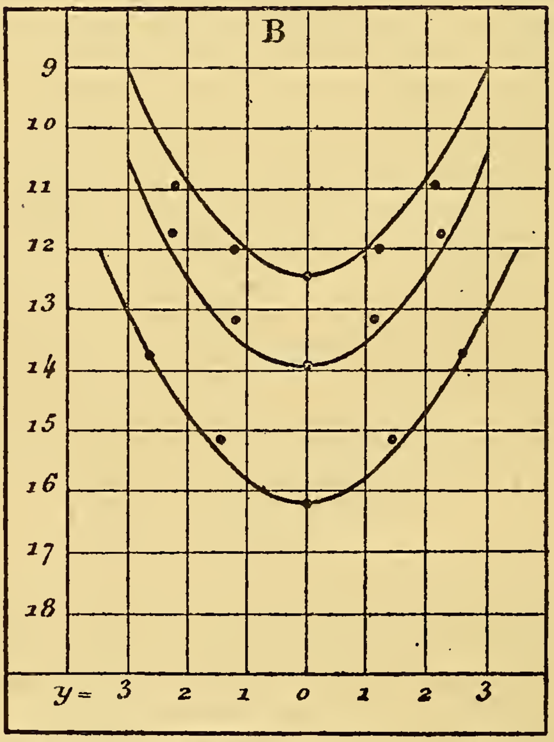


Fig. 94.—Curves Indicating the Lengths of the Radii of Curvature of the Anterior Surface of the Dead Crystalline Lens at Different Distances from the Axis. (After Tscherning.)

142. In the light of the foregoing results the hypothesis of von Helmholtz appears rather improbable, for it is difficult to understand how a relaxation of the zonula can effect a bulging of certain parts of

the surfaces while it causes a flattening of other portions. From a study of the radii of curvature of the anterior surfaces of dead crystalline lenses, Tscherning has shown that there is a flattening at the center and an increase of curvature toward the borders, which is just the reverse of the observations recorded in Fig. 92 for the crystalline under accommodative action. The graphical results of measurements on the radii of curvature of the anterior surfaces of morbid crystallines at different distances from the axis are shown in Fig. 94. A comparison of Figs. 92 and 94 is very instructive. If the living

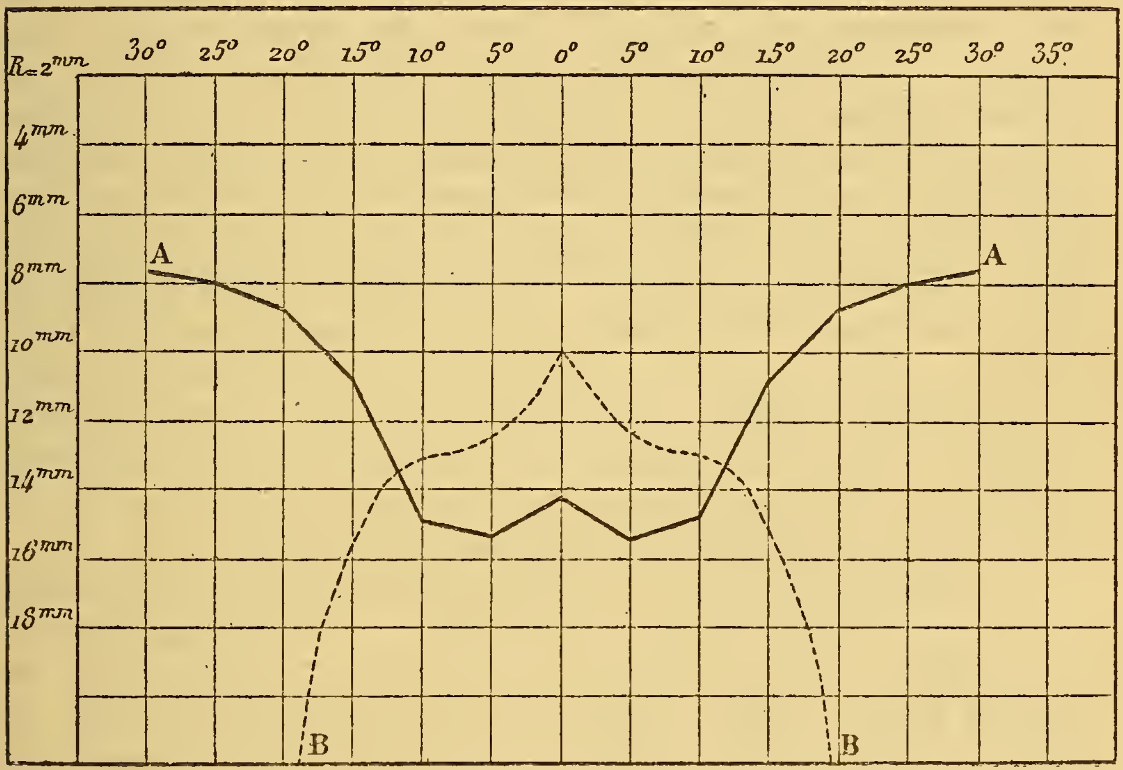


Fig. 95.—Radii of Curvature of the Anterior Surface of the Crystalline Lens of Dead Ox Eye. (After Crzellitzer.)

Curve A corresponded to state of equilibrium and curve B was obtained by traction upon the zonula.

crystalline should assume during accommodation the form of the morbid crystallines there would result a very rapid diminution of the refraction and an accentuation of the spherical aberration. The contrary is, however, experienced.

143. Crzellitzer constructed an instrument by means of which he could exert a traction upon the zonula in all directions at the same time. In his experiments with an ox eye he found that the radius of curvature diminished at the center of the surface, following traction, from 14 millimeters to 10 millimeters, while at 17 degrees from the axis it increased from 10 millimeters to 18 millimeters. The curve *A*

in Fig. 95 indicates the radii of curvature of the surface in repose or static equilibrium conditions; the curve *B* gives the radii when the crystalline is subjected to a pulling force along the zonula. The curve *A* is analogous to the dead human crystalline (see Fig. 94) and the curve *B* to that of the living crystalline in a state of accommodation (see Fig. 92).

144. The result of these experiments, "which," as Tscherning says, "at first sight may appear paradoxical," is a simple consequence of the structure of the crystalline lens. The nucleus has a much more pronounced curvature than the surfaces of the crystalline lens and its shape is not changed except with great difficulty. The superficial layer can be readily changed in shape and curvature and has been referred to as "the accommodative layer." This diminishes with age and the amplitude of accommodation consequently decreases. By the exertion of a traction on the zonula the peripheral parts must flatten, while at the middle the curvature increases on account of the greater resistance and curvature of the nucleus. The same result will follow, according to Tscherning, if there is no nucleus, as in the case of children, if the curvature and resistance increase toward the center of the lens. The evidence is fairly satisfactory that there is increased resistance as the center is approached since the index of refraction increases in the same direction; the increase of curvature of the central layers is visible in any preparation of the lens.

145. Tscherning has given us an account of the order in which the accommodative procedures and phases occur. By placing a cursor of the ophthalmophakometer above the telescope and requesting the observed person to look at the latter, the following phenomena are observed. There are four apparent phases which occur during an act of accommodation followed by a relaxation. (I) The image of the anterior surfaces descends quickly towards the corneal image and is ultimately hidden behind this. The pupillary contraction begins toward the end of this phase. (II) The small image due to the posterior crystalline surface descends in turn by an abrupt movement. The displacement is less than for the anterior surface image but moves in a curve with its concavity turned toward the middle. The pupillary contraction is greatest at this period. (III) When relaxation of accommodation occurs the posterior surface image ascends to its original position with a rapid movement. (IV) The large anterior surface image re-ascends by a slow, hesitating movement. These four phases are diagrammed in Fig. 96. The corneal image is represented by a large blackened circle (●), the anterior crystalline surface image by an open circle (○) and the posterior surface image by a small full

circle (●). Diagrams I and II indicate the positions of these images during accommodation, III and IV during relaxation. The arrows indicate the directions of movement.

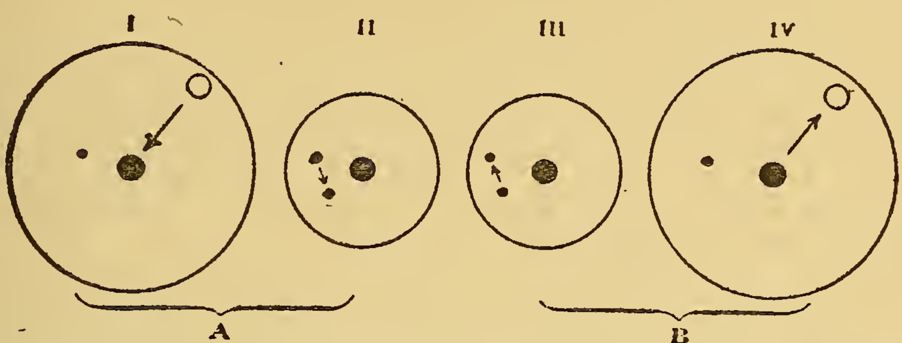


Fig. 96.—The Four Apparent Phases of Accommodation. (After Tscherning.)

Fig. 97 shows the displacements of the image due to the posterior crystalline surface during the maximum effort of accommodation; *C* represents the state of affairs when looking straightforward; *H*, looking upward; *B*, looking downward; *D*, to the right and *G* to the left. The corneal and posterior lenticular images only are shown; the arrows indicate the direction of the displacement during the act of accommodation.

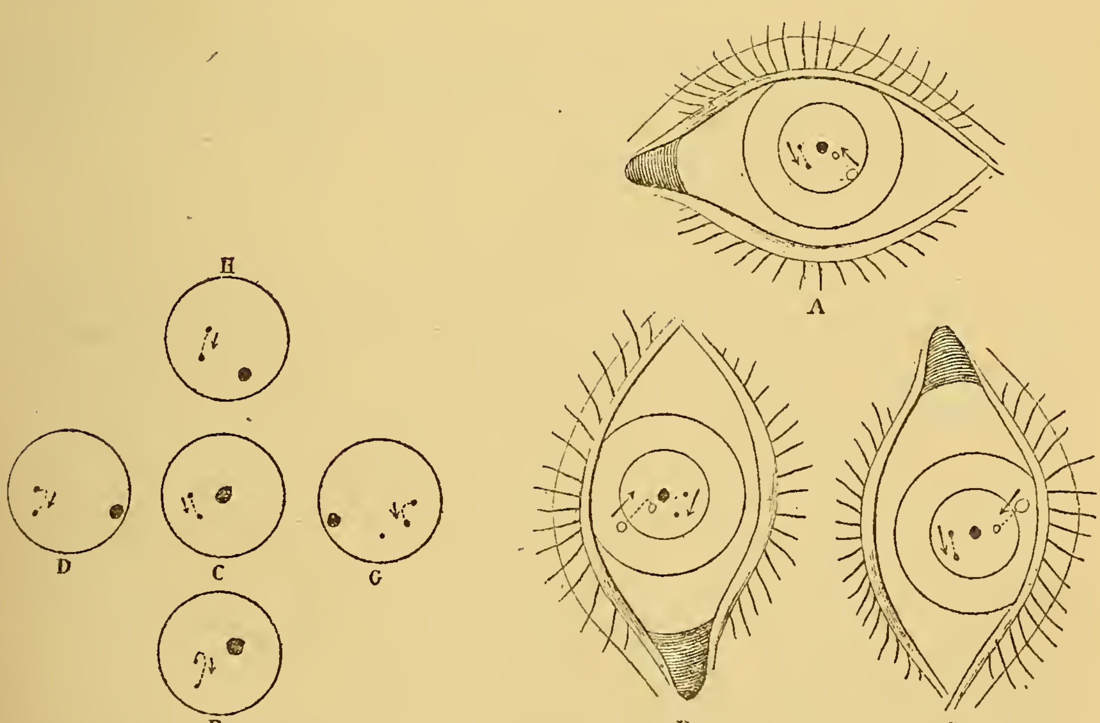


Fig. 97.—Displacements of the Image of the Posterior Surface During Accommodation, Observed with the Ophthalmophakometer. (After Tscherning.)

146. The conclusions which Tscherning has drawn from his experimentation have been included in the rehearsal of his work on accommodation. We shall gather together at this juncture some of the essential points in support of his theory and state them in succinct form.

"(1) The increase of refraction of the lens in accommodation takes place only near the apex of the lens. This is established by a study of the spherical aberration of the eye. Aberration, which is positive when the eye is at rest, diminishes or even becomes negative during maximum accommodation."

"(2) Measurements with the ophthalmophakometer show that the increase of curvature of the anterior surface of the lens is confined to the portion near the summit of the lens. Accommodation is effected by the temporary formation of an anterior lenticonus."

"(3) Experiments made upon the eyes of animals show that traction upon the ligament of the lens produces an increase of curvature near the summits of the surfaces and relaxation produces diminution of curvature."

Hence Tscherning and his collaborators have been led from their researches to the conclusion that the zonula is relaxed when the eye is in a state of repose and that it is in a taut condition or under traction during accommodation.

XIII. THE PUPIL

147. The pupil contracts and dilates under different influences: these movements are rather complex and still without complete or adequate elucidation. In 1812 Maunoir revealed the existence of circular fibers in the iris of the bird; the microscope has since demonstrated the existence of the sphincter muscle of the iris in man. It is known to be a circular muscle-band about a millimeter wide and situated at the pupillary margin of the iris and nearer its posterior surface. The existence of radiating muscle fibers, or the pupil dilator, has been the subject of much discussion. Grunhagen, Collins and others have taught that there is elastic tissue in the iris that effects the dilatation of the pupil when the action of the sphincter is inhibited. In 1894 Juler presented evidence before the International Congress of Ophthalmology to show the existence of a radiating muscular structure of the iris; he demonstrated that these fibers have their origin at the attached margin of the iris and pass to the pupillary margin where they become blended with the sphincter muscle. The movements of the pupil are under the control of the motor oculi and the great sympathetic. Cutting the motor oculi produces a dilatation

of the pupil which is less, however, than that produced by atropine. The pupillary contractions which accompany accommodation and the incidence of light cease together. Hence the contraction which accompanies the incidence of light is produced by a reflex action between the retina and the optic nerve on the one hand and the third nerve on the other. The third nerve fibers and those of the cervical sympathetic causing contraction and dilatation pass first to the ciliary ganglion where they are joined by fibers from the fifth nerve. The several short ciliary nerves, each containing fibers from the three different sources, pass out from this ganglion and proceed to the posterior part of the eye and are finally distributed to the ciliary body and iris. Brown-Sequard produced a contraction of the pupil by concentrating light on an enucleated rabbit's eye. The writer of this monograph has recently had a similar experience with enucleated dog eyes in which it was found that, in an occasional case, the pupil of such an eye responded to light stimulation. This would indicate that light has a direct influence on the muscles of the iris. An irritation of the oculo-motor produces a contraction of the pupil, while an irritation of the great sympathetic at the neck produces a marked dilatation.

The action of mydriatics and myotics is a topic beyond the purview of this work. Briefly, however, it may be stated that atropine produces a marked dilatation of the pupil, paralyzing its movements and those of the accommodation as well. It probably also irritates the terminal fibers of the great sympathetic, thus causing a greater dilatation than would be produced by cutting the motor oculi. Cocaine (5 per cent.) dilates the pupil but does not act upon the accommodation. Scopolamine ($\frac{1}{5}$ per cent.) produces paralysis of accommodation with marked dilatation of the pupil. Eserine (0.5 per cent.) causes a very large contraction of the pupil; under this treatment an eye reaches its maximum accommodation since the decrease in size of the pupillary area produces a reduction of the diffusion circles with a corresponding improvement in visual acuity.

148. *The movements of the pupil.* I. *The pupil contracts under the influence of light.* If it does not do so, but does when light impinges upon the retina of its mate, a complete amaurosis of the eye in question can be inferred. In normal conditions both irides respond, giving usually equality of sizes of pupillary areas, when one eye is illuminated and the other less strongly lighted or even screened. This is due to the consensual action. In darkness the pupil reaches its maximum dilatation so that the iris is often practically invisible; if the iris is not visible at all it can be shown that it is an apparent phenomenon due to refraction through the cornea. The very evident purpose of

the contraction and dilatation of the pupil is to regulate the quantity of light which enters the eye; it acts, therefore, as a *photostat.*

II. *The pupil contracts during accommodation.* The normal iris should not only respond directly and consensually to light stimuli (reflex by the optic nerve) but also when an effort of accommodation is made. Accommodative contraction may exist without the reaction to light and vice versa. Hueck says that the most peripheral portions of the iris show a centripetal movement during accommodation and that this is generally not the case in a reaction to light.

III. *The pupil contracts when the aqueous humor escapes.* The observations of Arlt showed that there was still pupillary contraction after paracentesis had been performed on a dead eye. Some experiments made by Tscherning show that, by inserting the point of a syringe into the anterior chamber and, depressing and withdrawing the piston in turn, the pupil can be made to dilate or contract at will. In fact, by a removal of all the contents of the anterior chamber the diameter of the pupil could be reduced to 1 or 2 mms., and by forcing the injection the iris could be made to disappear. These effects do not appear to be due to pressure, for the eye can be compressed without any change in the diameter of the pupil resulting and it cannot be effected by injecting into or removing liquid from the vitreous.

IV. The pupil is *contracted during sleep* even in amaurotic persons; during narcosis; generally when a person is suffering; at the moment of death the pupil is generally dilated, soon followed, however, by a contraction. The pupillary contraction present during sleep does not inhibit the reaction to light.

V. Under a magnifying glass *rhythmic contractions* are observed corresponding in part to the systole. This is greater when the systole is coincident with respiration.

VI. A *dilatation* occurs during fright; it also occurs during vigorous muscular action or a sharp irritation of any sensitive nerve.

149. *Advantage of the situation of the pupil near the nodal point.* In physical optics, a lens or lens system is often demanded which is *rectilinear.* By this we mean that the image produced shall be free from distortion and the images of straight lines, for example, placed peripherally in the field shall be straight and not curved. This condition of rectilinearity or orthoscopy may be approximated by employing a diaphragm so placed before a single lens as to be at the point through which all rays from the object must pass in order that the subsequent refraction by the lens may produce an image proportionate in all respects to the object. The image will be free from distortion when the so-called tangent condition is fulfilled.

The question may, then, be very appropriately raised: Is there any advantage in the situation of the lens near the nodal point? Distortion in a two-element system is eliminated by a stop placed between the components in such a position as to cause the distorting effect of the front lens to be neutralized by that of the back lens. Now, the refraction by the cornea produces a constriction of the image near the border. The anterior surface of the crystalline lens, however, will have little compensating effect since the pupil is practically in contact with it. The posterior lenticular surface compensates somewhat for the corneal error but not sufficiently to rectify the system. Hence retinal images will be deformed with barrel or negative distortion. Yet without doubt and in spite of the foregoing statements, the position of the pupil near the nodal point of the eye (assuming one nodal point only, without affecting the argument, since the two nodal points

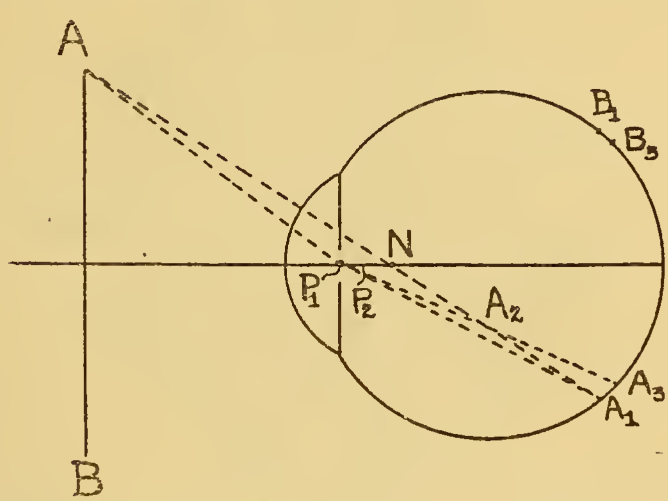


Fig. 98.—Advantage of the Position of the Pupil Near the Nodal Point.

of the eye are so near together) probably plays some part in the correct vision of objects seen indirectly.

150. Young remarked that if the pupil of an eye were to be situated nearer the cornea than it is the apparent size of objects would change whenever an effort of accommodation is made. The image of a point for which the eye is not accommodated forms a circle of diffusion the center of which, corresponding to the middle of the pupil, is frequently brighter on account of spherical aberration. If the pupil is not too large this central portion may be considered an indistinct or vague image of the point. Let us assume that, in a state of repose, the eye is focussed for an object AB as in Fig. 98. The image of the point A is found at A_1 on the line AN passing through the nodal point N. The image is, however, moved forward to A_2 during the act of accommodation. Let P_1 represent the center of the pupil of

entrance: therefore, to find the place where the diffuse image is formed on the retina, the ray AP_1 is drawn passing through the point P_1. This ray, after refraction and after passing through the middle point P_2 (P_1 and P_2 are practically coincident, however, being in reality but 0.7 mm. apart) of the pupil of exit and through A_2, the position of the accommodated image, will form a diffuse image upon the retina at A_3 and the image of the entire object A_3B_3 is smaller than the distinct image A_1B_1. "In the human eye we may observe a slight effect of this kind using our accommodation while observing distant objects; it is more pronounced when the pupillary action is replaced by a stenopaic opening at some distance from the eye" (Tscherning).

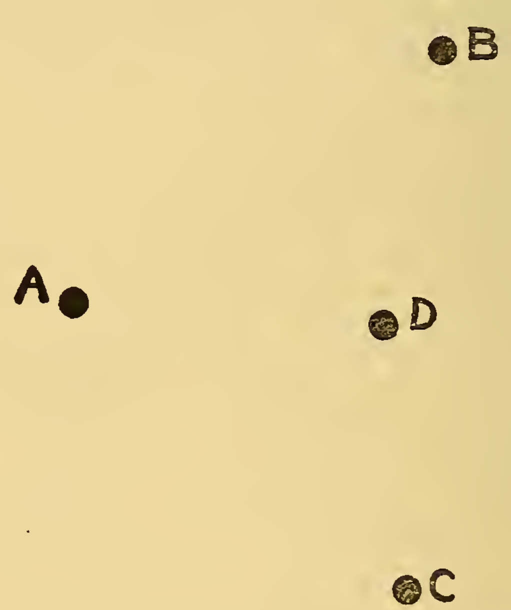


Fig. 99 A.—Test for Showing that the Eye is not Rectilinear.

151. That the eye is not rectilinear may be proven by some simple experiments due to Helmholtz.

I. Place upon a table a small piece of paper, *A*, Fig. 99 A, which shall serve as a fixation point and then put two other small objects, *B* and *C*, as far as possible from *A* so that they may be seen distinctly in indirect vision. While fixing *A*, the experimenter endeavors to put a fourth piece, *D*, in a line joining *B* and *C*. Generally *D* will be too far inwards toward *A* thus giving the barrel-shaped distortion.

II. If a strip of paper with parallel borders about 8 to 10 centimeters in width is taken and the center visually fixed, the borders appear concave towards the point of fixation, hence the strip appears larger at the middle than at the ends.

III. An experiment of a similar kind consists in placing a circular piece of cardboard in the periphery of the visual field; it is seen elongated in the horizontal direction above or below, while when laterally viewed it appears elongated in a vertical direction.

IV. Helmholtz constructed from theoretical considerations his celebrated hyperbolic chess-board of which Fig. 99 B is a representation reduced in the ratio of 1 to 8. When this was viewed at a large distance the lines appeared to have the curvatures which they were given; on moving it nearer and nearer he found that the curvatures diminished and that at 20 centimeters distance from the eye the curvatures disappeared, agreeing with his calculations. On closer approach the lines assumed the reverse curvatures. Tscherning repeated this experiment with an artificial eye, practically duplicating the dioptrics of the eye, together with a hollow hemisphere of ground glass of practically the same curvature as that of the retina of the normal eye. He found that as long as the object was remote the image was like it; very near to the drawing the lines of the image became concave on the inside. The point at which the image of the figure appeared most rectilinear was at 20 centimeters from the artificial eye. These results are interpreted to indicate that all these deformities are dependent primarily upon the form of the retina.

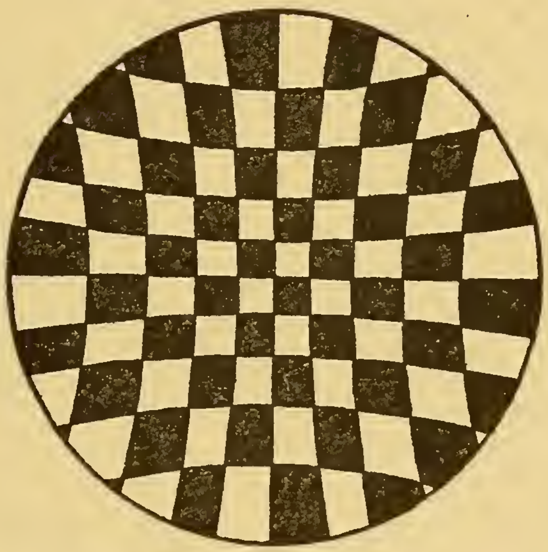


Fig. 99 B.—Hyperbolic Chess-board of Helmholtz.

The query has been raised by us as to whether or not there is any advantage in the position occupied by the pupil or, in other words, could it not have been more advantageously situated elsewhere? Considering the hemispherical shape of the retina, we can proceed to project upon a screen visual lines, both direct and indirect, drawn from *retinal points equally spaced as to arc measurements* in three apparent ways. These are shown in Fig. 100. In (I) of this figure the various points are projected straightforward by lines perpendicular to *EF;* we see that the space image is extremely constricted in size and narrowed at the periphery. In (II) projection is carried out through the center *C;* the image expands rapidly toward the periphery. In (III) we have stereographic representation, in which projections of rays are carried out through a point *P* on the corneal surface. This gives us the nearest approach to rectilinearity: that is to say, if the pupil were reduced to a point and situated at the anterior pole of the retinal hemisphere, the image would be as rectilinear as possible. The pupil is, however, situated behind the anterior corneal

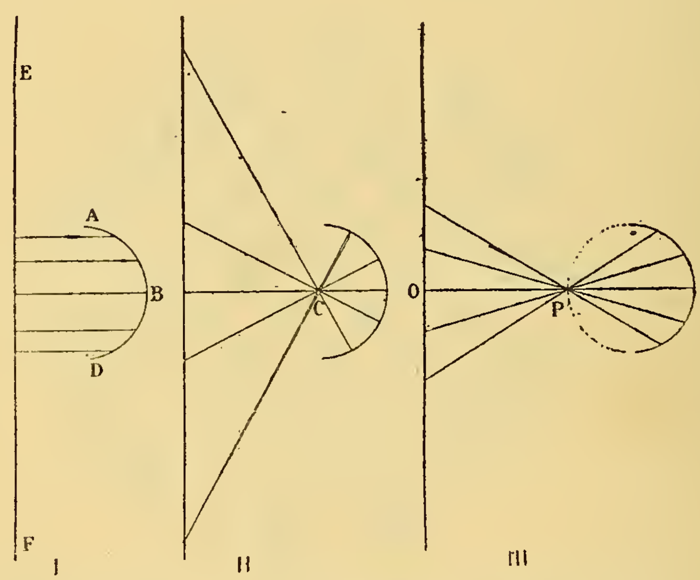


Fig. 100.—Deformity of Image Due to the Shape of the Retina.

pole and the incident rays suffer spherical aberration in the peripheral regions, hence the impossibility of rectilinearity when a distant field is under observation with fixation at a definite point. The aberroscope shows the presence of this distortion. But on accommodation it is found that the aberroscope generally shows no distortion and even an over-correction giving evidence of the formation of an anterior lenticonus. At the same time the iris together with the anterior lenticular surface is displaced forward toward the corneal pole. It therefor seems possible to explain the changes observed by Helmholtz in using the chess-board as due to (1) elimination of spherical aberration by contraction of the pupil, (2) the closer approach of the pupillary area to the corneal apex and (3) the change in the obliquity of the rays incident upon and emergent from the crystalline lens due to changes in curvature of the lenticular surfaces. All of these effects are probably intimately tied up with retinal curvature effects.

152. *The relation between pupillary area and asthenopia.* A subtle and synchronous balance between retinal perception, uveal stimulus and iridic response must exist if the iris is to perform its functions as a photostat and diaphragm. It is probable that a disturbed equilibrium of these functions is the cause of asthenopia in low degrees of ametropia in which small corrections are advantageous. In cases where quarter diopters appear to relieve asthenopia it will often be found that the pupils are comparatively large; hence asthenopia may be experienced as much on account of the size of the pupil as on account of errors of refraction. In a case of hyperopia of low degree, for example, with extremely large pupil, we have a comparatively small central area of diffusion due to the refractive area covered by a much larger area of diffusion and illumination. A slight error of accommodation would either sustain or increase the discrepancy. If, then, the aberration is to be abolished, the iris must receive an increased stimulus to bring about a contraction of the pupil in excess of that which is associated with accommodation. A correction of the slight error eliminates the diffuse central image but the area of total illumination and peripheral aberration is increased; the fact, however, that the lens improves vision and relieves asthenopia is satisfactory proof that the aberration is dispelled, indicating an increased iridic contraction. And again, in normal pupils with retinal perceptive powers excellent, and errors of refraction slight, retinal stimulus will prompt contraction of the pupil sufficient to exclude aberration. In cases with large pupils it is probable that prolonged efforts of iridic contraction will cause a fatigue of the iris and the resultant asthenopia. (See the section on Lenses and Prisms in *The American Encyclopedia of Ophthalmology,* Volume X, pages 7349-7358.)

XIV. APPLICATIONS OF THE LAWS OF CONJUGATE FOCI TO RETINOSCOPY AND OPHTHALMOSCOPY

153. *Retinoscopy.* Retinoscopy may be defined as the measurement of ocular refraction by means of the real or apparent movements of the fundus reflex. The laws governing conjugate foci are applicable to these objective methods since the eye, although a complicated system, is equivalent to a single convex refracting surface with a posterior focal length of about 20 mms. Object and image positions are always reversible in a lens system; subjectively, light from an object twenty or more feet away will form upon the retina of the human eye of proper axial depth a distinct image without any effort of accommodation. The incident light is, therefore, parallel, being brought to a

focus upon the retina; infinity and the retina are, then, conjugate points. This retinal image may, in turn, be adopted as the luminous object and the refraction may be considered as taking place from the denser (water equivalent eye) to the rarer external medium, with the emergent light parallel. Or again, assuming a myopic eye, fixing at infinity, we know that the image due to the refractive apparatus will be formed in the vitreous; this will, however, produce a diffuse, enlarged image upon the retina. This retinal image, whatever its size or character, may then be considered as the new object and its conjugate found at a finite position in space. The fundamental law of refraction at curved surfaces, which has been previously expressed in the section on *Ocular dioptrics* as

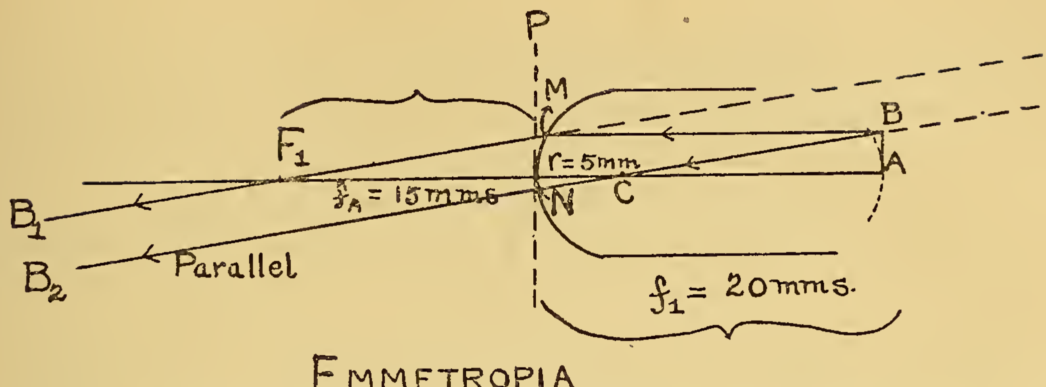
$$\frac{n_1}{f_1} + \frac{n_2}{f_2} = \frac{n_2 - n_1}{r}.$$

holds equally well in ocular calculations when the illuminated area upon the retina is taken as the object and its conjugate in space is desired, provided due regard be had to the algebraic sign of the radius of curvature, r. As an illustrative case, let us assume the axial depth of an eye to be 23 mms. and that it is desired to find the conjugate to a point upon the retina situated at this distance from the cornea. Assume the constants of the reduced eye; our data are, then, $f_1 = 23$ mms., f_2 is to be calculated, $r = -5$ mms. Since the light is now considered as passing from the more dense to the less dense medium, $n_1 = 4/3$ (water) and $n_2 = 1$. By substitution of these values in the above equation one obtains

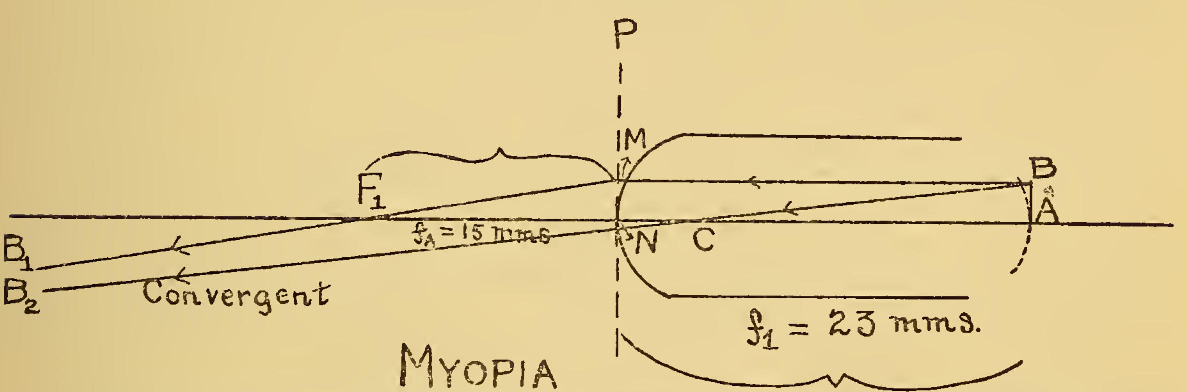
$$\frac{4/3}{23} + \frac{1}{f_2} = \frac{1 - 4/3}{-5}$$

which gives, upon solution, $f_2 = 115$ mms. $= 11.5$ cms. as the conjugate focal length in air measured from the cornea in a direction contrary to that in which the posterior focal length, f_1, is measured. This value of f_2 corresponds to the far point in a myopia of 8.7 diopters. Light diverging from a point on the retina would then be received as light converging toward a point situated 11.8 cms. from the cornea. In a case of hyperopia in which, for example, the depth of the eye, f_1, may be taken as 18 mms., calculations will show that

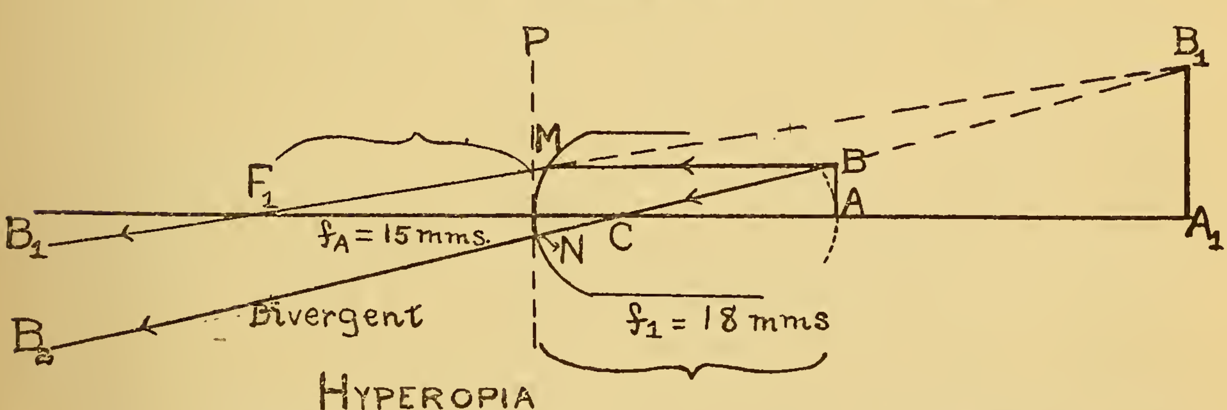
$$\frac{4/3}{18} + \frac{1}{f_2} = \frac{1 - 4/3}{-5}$$



(A).—Illustrative of the Conjugacy of Foci and Nature of the Emergent Beams in Plane Mirror Retinoscopy when Emmetropia Obtains.



(B).—Illustrative of the Conjugacy of Foci and Nature of the Emergent Beams in Plane Mirror Retinoscopy when Myopia Obtains.



(C).—Illustrative of the Conjugacy of Foci and Nature of the Emergent Beams in Plane Mirror Retinoscopy when Hyperopia Obtains.

Fig. 101.—Illustrative of the Conjugacy of Foci and Nature of Emergent Beams in Plane Mirror Retinoscopy when (A) Emmetropia, (B) Myopia and (C) Hyperopia Obtains.

gives $f_2 = -135$ mms. $= -13.5$ cms. This distance f_2 is then to be measured from the cornea toward the retina, which shows that the point conjugate to the retina does not exist in space but lies behind the retina. That is to say, we have a virtual object which means that the rays emerging from the eye under these conditions will be divergent but that, by projection backwards, they can be made to meet at a point back of the retina. These various conditions, using an illuminated retinal area as an object, are diagrammed in Figs. 101 (A), 101 (B) and 101 (C), representing respectively emmetropia, myopia and hyperopia. The center of curvature is represented as at *C* and occupies the same position relative to the principal refracting plane, assumed tangent to the corneal surface, in all cases since all emmetropic and ametropic conditions of the eye are diagrammed and considered as dependent upon axial depth and not upon curvature or indicial variations. The anterior focal point is represented as situated at F_1 such that the distance f_A, representing the anterior focal length, has a value of 15 mms. The radius of curvature of the refracting surface is 5 mms.; the posterior focal length $f_1 = f_P$ of a normal eye is taken as 20 mms. The single nodal point is coincident with the center of curvature. For the sake of clarity the diagrams do not show the paths of the rays incident upon the eye and their positions in the eye after refraction; an illuminated retinal area *AB* is exhibited in each case, however, and this plays the part of a luminous object in our subsequent discussion. The paths of the emergent rays and their conditions of parallelism, divergence or convergence may be readily established by the application of a few fundamental principles of geometric optics. The ray *BM*, being parallel to the principal axis will, after refraction by the curved surface, pass through the anterior focal point F_1. A ray, *BC*, passing through the center of curvature or nodal point *C* will continue without subsequent deviation or refraction. The point at which two emergent rays, such as B_1M and B_2N, meet determines the position of the image of *B*. From the figures we see that the light emergent from the illuminated retina of an emmetropic eye will be parallel and hence meet at infinity; in myopia a real, inverted, aerial image will be formed by the convergent light somewhere between infinity and the eye; in hyperopia the emergent rays are divergent and would, by projection, intersect at a point behind the retina.

154. There is, therefore, every reason to believe that light from an illuminated retina, which acts as a luminous source, emerges from the eye and can be intercepted by another eye if a proper device is at hand to permit of its reception. Ordinarily the pupil of an eye is seen black because the observer is never in a position to receive the light from the retinal image of a source sufficiently bright to enable the fundal light to be received. An extraneous source of light, such

as a lamp, cannot be used in the direct line of sight for the obvious reason that the observer's own head would come between the observed eye and the light used, thereby cutting off the source of the retinal illumination necessary to the production of the returning rays. To overcome this difficulty the observer's eye must be in a line with the direction of the entering ray. For this purpose a mirror having a central aperture and known commonly by the name of *retinoscope* or *skiascope* is employed. This method in essentials was discussed by Cuignet in 1873 and was employed by him in ocular refraction but he attributed the phenomena to the cornea instead of the fundus oculi. It remained for Parent in 1880 to specially develop the method and give it its correct explanation. Briefly, light from some external

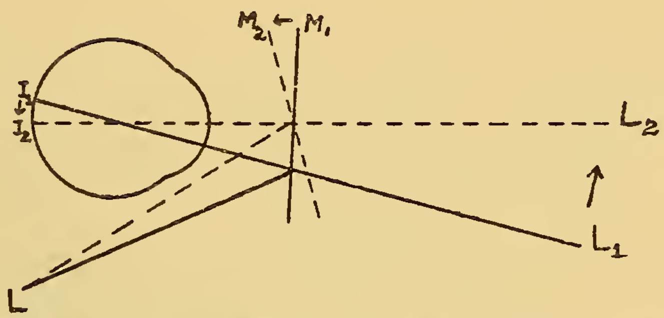


Fig. 102.—Illustrating Movements of Mirror and Reflex in Skiascopy with the Plane Mirror.

L, lamp; M_1, first position of mirror; L_1, image which it forms of the lamp; I_1, retinal image.
M_2, second position of the mirror; L_2, image of the lamp; I_2, retinal image.

source is received by the mirror and reflected into the observed eye where it comes to a more or less well defined focus on the retina. The returning light is, however, diffused by the choroid and not the retina, which is transparent in health, and gives to the reflex its characteristic reddish color. In retinoscopy and ophthalmoscopy the sole function of the mirror is to supply the illumination; the intensity of the beam and the brightness of the fundus reflex are dependent, however, upon the nature, the focal length and the aperture of the mirror.

155. It is common practice at present for the observer to take his position about one meter from the patient whose eye he illuminates with a *plane* mirror. By rotating the mirror about a vertical axis he sees the luminous spot thrown by the mirror flit to and fro horizontally across the face and vice versa, the spot traveling in the same direction as the mirror is rotated. By throwing the light upon the pupil, the fundus is illuminated and it can be shown that the direction of motion of the *retinal* image is always in the same direction as that in

which the mirror is rotated provided a *plane* mirror is employed; the contrary condition or direction of retinal image holds when a concave mirror is employed. An examination of Fig. 102 will show this clearly: L represents the lamp, M_1 the first mirror position, M_2 the second position, L_1 and L_2 the corresponding positions of the images of the lamp L formed by the mirror and I_1 and I_2 the retinal images. The arrows show the directions of motion. *The image upon the retina always moves in the same direction as the plane mirror is rotated, irrespective of the ametropic condition present; the direction of the movement of the reflex and its accompanying shadow is another matter entirely and dependent upon other phenomena for its existence.*

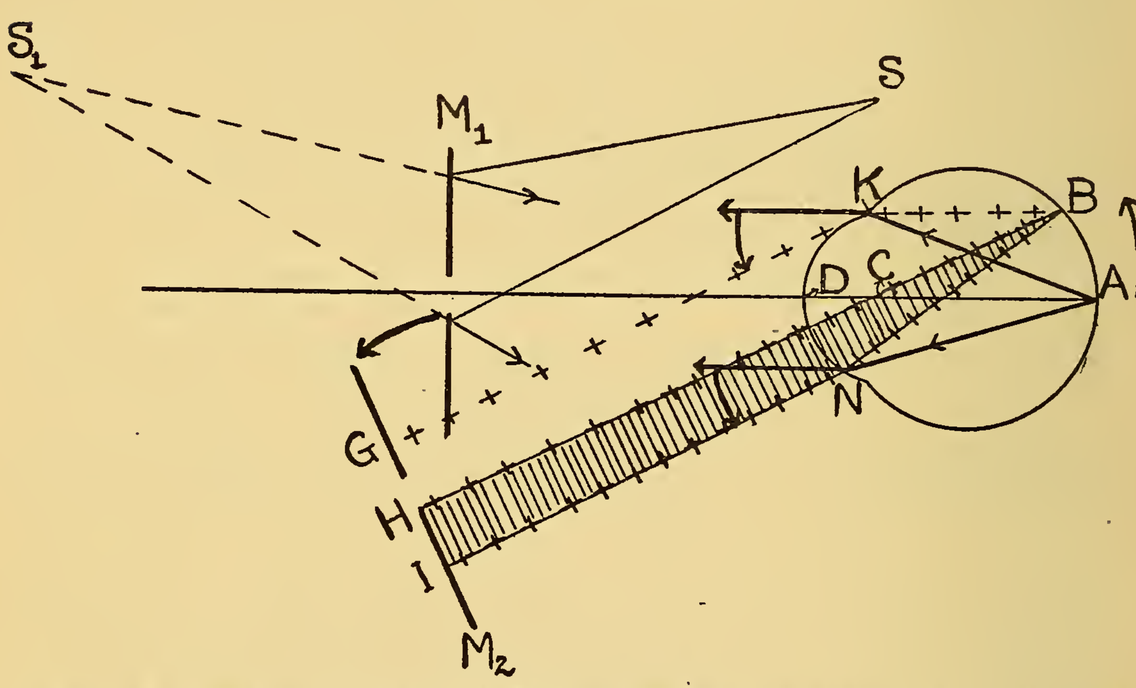


Fig. 103.—Detailed Diagram of Mirror, Retinal Image and Reflex (Shadow) Movement in Emmetropia.

156. Having, then, illuminated the retina and obtained the red reflex, the mirror is rotated and the observer watches the reflex as it moves across the pupil. As a matter of fact it is the boundary between the illuminated and non-illuminated portions of the fundus reflex which is noticed or what is technically known as the *shadow*. The direction of the reflex or shadow movement, whether "with" or "against" the rotation of the mirror, depends upon the refraction of the eye, the nature of the mirror and the position of the observer. Figs. 101 *A, B* and *C* show the nature of the emergent rays in various refractive conditions (simple ametropias) of the eye. Figs. 103, 104 and 105 attempt to show:—(a) the plane mirror in two positions, M_1

and M_2, (b) the cone of light, *AKN*, emerging from the retinal point *A* (drawn as heavy lines), (c) the cone of light, *KBN*, emergent from the luminous point *B* after rotation of the mirror from position M_1 to M_2 (indicated by dotted lines), (d) the nature of the emergent rays, whether parallel, convergent or divergent and (e) the portion of the pupil which appears luminous. The diagrams are of necessity exaggerated and the mirror in its second position not only rotated but slightly displaced in order to emphasize certain points. In Figs. 103 and 104, representing emmetropia and hyperopia respectively, a portion of the emergent cone of light, *AKN*, is intercepted by the observer's eye situated back of the aperture in the mirror in position M_1; this being the primary position the pupil of the eye under observa-

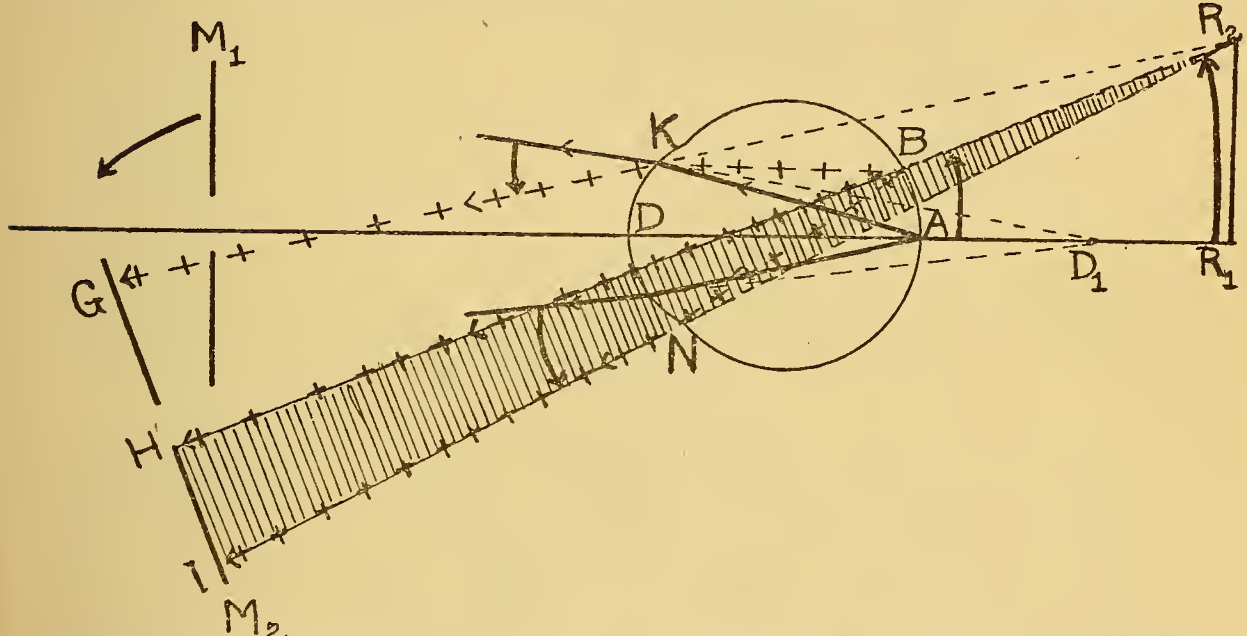


Fig. 104.—Detailed Diagram of Mirror, Retinal Image and Reflex (Shadow) Movements in Hyperopia.

tion is seen filled with a reddish glow: upon rotation of the mirror into a secondary position, as M_2, a portion of the emergent cone *BKN*, passing into space between the limits of *KG* and *NI* as diagrammed, is intercepted and received by the observer. The eye sees as luminous that portion of the pupil which sends rays to it, i. e., the portion *KD* in Figs. 103 and 104, while *DN* is dark because the rays which come from this point are not received by the observer or are, as another mode of expression, intercepted by the iris of the observer. We see, therefore, that in emmetropia and hyperopia, using a plane mirror, the reflex will travel in the same direction as that in which the mirror is moved and will be followed by a shadow due to the fact that the light from a portion of the pupil does not reach the observer's eye.

The light apparently moves on the retina in the same direction as it in reality does. If, on the contrary, a sufficient amount of myopia exists in the observed eye such that the conjugate to the retinal point A is at O_1 between the observed and observing parties, it can be seen from Fig. 105 that the light or reflex moves in the contrary directions to

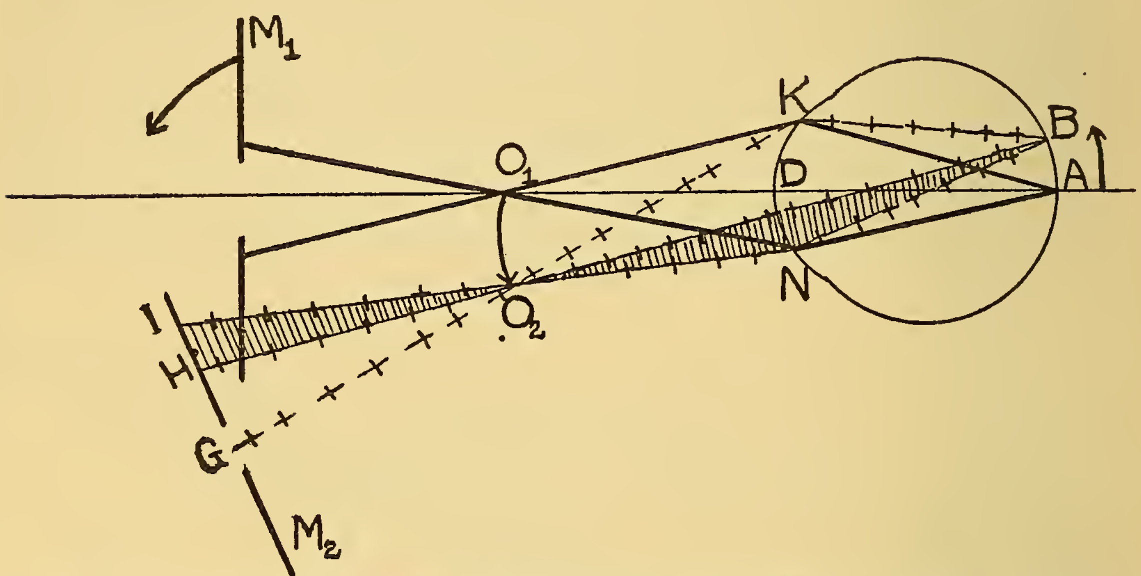


Fig. 105.—Detailed Diagram of Mirror, Retinal Image and Reflex (Shadow) Movements in Myopia.

the mirror rotation, because the light comes to the observer from an inverted aerial image which the operator observes. These two images are represented as being at O_1 and O_2 and the arrows show that their motions are contrary to those of the retinal images A and B. Likewise, a casual survey of Figs. 104 and 105 will show that the relative

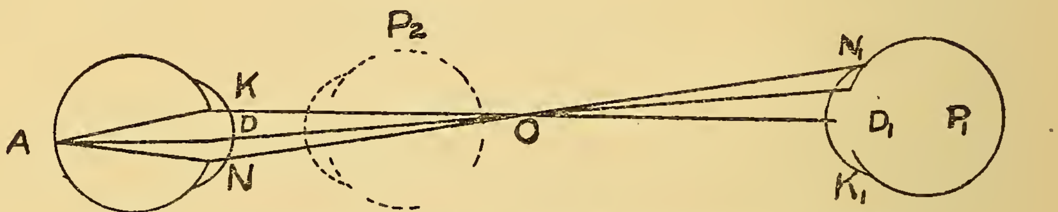


Fig. 106.—Diagram Illustrating Leroy's Theory of Skiascopy.

orders of the rays KG, DH and NI are reversed in the two cases; it can also be seen that further rotation of the mirror as shown in Fig. 105 will cause the cone of light O_2HI to enter the mirror and hence cause the observing eye to see the luminosity travel from K to N. This condition is illustrated in a further and possibly simpler manner in Fig. 106. Let A be an illuminated point of the retina of the observed eye, supposedly myopic, and O its aerial image. The luminous cone *AKN* leaves the observed eye, giving the aerial light cone *KON* of which, however, only the portion OD_1K_1 enters the observer's eye. Hence to the observer the portion of the pupil *KD* will appear luminous while *DN* is dark because the rays which come from this part are intercepted by the iris of the observer.

157. The movements of the reflexes and shadows as seen in the pupil have been discussed with some degree of fullness in connection with the preceding diagrams in order to correlate the laws of conjugacy of foci with the fundamentals of skiascopy. But nothing has as yet been said about the relative positions of the observed and of the observer. This is in practice one of the most important considerations. Referring again to Fig. 106 it will be seen that if the observer should move his eye from position P_1 to that indicated at P_2, the patient still fixing infinity and with accommodation properly relaxed or under a complete cycloplegic, there would be a reversal of the direction of movement of the shadows in the two positions. That is to say, in position P_1, under the conditions diagrammed, the movement would be against that of the mirror and therefore to be interpreted as *myopia;* in position P_2, however, the motion will be with the mirror indicating a *hyperopic condition with respect to the far point O of the observed eye.* The point or position *O* is, as has just been stated, the far point of the patient's eye; it is, then, the meeting point of all rays conjugate to the retina; it is the *neutral* or *reversal point* skiametrically. If, therefore, the image of the luminous retinal point formed in space at *O* is at the nodal point of the operator no motion of reflexes or shadows will exist but the pupil of the patient will appear uniformly illuminated. This is due to the fact that the aerial image *O*, falling at the nodal point of the observer's eye, becomes in and of itself an illuminated object which sends out to the retina in all directions rays which are not acted upon by any optical apparatus to produce an image anywhere. So long as there is any single point of the luminous retina which has its image at the nodal point of the observer's eye the fundus will be fully illuminated. If, therefore, the nodal point of the observer's eye is between the observed eye and its far point, whether this be the natural or artificially produced point, the shadow movements will be contrary to those experienced when the observer's nodal point is outside or more remote from the patient's eye than its punctum remotum. The natural far points may, however, range theoretically between minus infinity and extremely small finite quantities dependent upon the degree of hyperopia or myopia present. For emmetropia the punctum remotum is at infinity; assuming a point twenty feet away from such an eye to be sufficiently remote as to be

considered at infinity it can readily be seen that retinoscopy could not be successfully practised under such conditions. As a result an arbitrary or artificial far point is produced by the addition of such a lens quantity to the normal eye as would bring the skiametric reversal point to any point desired. Since light emerges as parallel rays from a normal eye fixing at infinity, the addition of a + .1 D. S. before this eye will cause these parallel emergent rays to focus at 40 inches or 1 meter. The observer stationed at this point would, therefore, be at the reversal point of such an eye thus optically modified and would view a fully illuminated pupil free from shadow movements. When the observer, working at forty inches, with a + 1 diopter lens before the eye, the other being occluded by a cover, sees a "with" motion when using a plane mirror he is justified in concluding that hyperopia exists and continues to add plus lenses until the reversal is reached. In case the motion is "with" when no lens is present to compensate for the "working distance" but is "against" when such a lens is inserted, the observer knows that a myopia of less than the dioptric value of the working distance is indicated. When an "against" motion exists before any working distance lens is inserted the operator realizes that he has a case of myopia of greater amount than that represented by the dioptric equivalent of his working distance. The one essential, therefore, in skiascopy is that the position of the observer's nodal point shall be at the conjugate focus of the retina of the observed eye. The operator, being thus desirous of bringing this conjugate focus to his own nodal point, must always add algebraically to the total lens quantity in the trial frame the negative dioptric value of the distance at which he has worked. For example, the artificially fixed far point is to be 1 meter; let us assume that a total lens quantity of + 4 D. S. before an eye barely produces reversal at this distance. The difference between the actual far point of this eye and the one diopter of artificial myopia for the working distance of a meter, or + 3 D. S. in this case, expresses the strength of the auxiliary lens which must be placed before this eye to make infinity and the retina conjugate foci. If, again, with observation at 26 inches, a — 2.50 D. S. neutralizes the shadow movement or just causes reversal, the total correction indicated is a — 4.00 D. S. The following is a digest of fundamental rules in practical skiascopy using a plane mirror; if the mirror is a short focus concave the movement is in every case the opposite of that specified but the calculations are identical. (1) The shadow movement is *with* in all cases of refraction in which the punctum remotum is negative or, if positive, behind the observer's nodal point. If the punctum remotum is between the observed and observing eyes the movement will be *against.* The shadow is neutralized when the far point coincides with the observer's nodal point. (2) The neutralizing lenses are those which overcorrect hyperopia and undercorrect myopia to a degree equal to the dioptric distance at which the observer operates.

158. The factors upon which the *rapidity of movement* of the light on the retina and the form of the light area depend are worthy of brief consideration. The speed with which the light and shadow appear to travel across a pupil depends upon the rapidity of the real movement of the light area upon the retina and upon the magnification of the retina. The rapidity of the real movement on the retina is determined by (a) the rate of movement of the mirror by the observer, (b) the distance of the mirror from the observed eye, (c) the distance of the original source of light from the mirror and (d) upon the distance of the retina from the nodal point of the observed eye. The rapidity of the *apparent* movement of light in the pupil is much more dependent upon the extent to which the retina and the real movement of light upon it are magnified, than upon the actual rate of the real motion. It is found that the closer the observer's eye is to the point of reversal the more is the real movement of light upon the retina magnified and hence the swifter it appears. The farther the observer's eye from the point of reversal the less is the real movement of light on the retina magnified, since the observer receives rays from an increasing area of the retina and more and more of the retinal image occupies the same space in the pupil.

159. The *form of the light area* upon the retina will be circular except under certain conditions in astigmatic eyes. If the light is perfectly focussed upon the retina the light area will be circular because that is the form of light source employed in practice. If the light is imperfectly focussed, however, the circular pupil gives its form to the resulting area of diffusion. In eyes free from regular and irregular astigmatism and aberration effects the form of the light area varies with the distance of the observer's eye from the point of reversal. If the magnification of the retina is of such a slight degree as to permit of the whole of it being visible in the pupil at one time, the light area will appear circular. When, however, the point of reversal is approached, in which case the magnification of the retina prevents all of the retinal light area being seen at one and the same time, it will be found that only a portion of its outline is visible as an arc of a greatly enlarged circle. The nearer the observer comes to the point of reversal the closer will the boundary between light and shade approach a straight line; in fact, when the eye is almost corrected, one

sees the glow very bright and its border very nearly straight. This straight ocular border is, however, a portion of the boundary of a circle and hence will show itself in whatever direction or meridian the mirror may be rotated; this is in contradistinction to the "band-like" appearance in astigmatism in which the direction of the border always conforms to one or the other of the principal meridians. That the form of the light area is due to the superposition of circles of diffusion of the same form as the pupil of the observed eye can be readily demonstrated by using as a luminous source a very long, bright ribbon of light. In this case the border of the ocular glow remains straight even in the event of strong ametropia because the superposition of the circles of diffusion cannot produce a circular form. Furthermore, if the pupil is given a triangular shape by using a stenopaic opening of this form before the eye under observation, the shadow will be found to retain its rectilinear border, for triangular diffusion spots cannot give a round form to the diffusion area.

160. Our discussion has thus far dealt with the skiascopy of uniform axial ametropic conditions or the equivalent thereof. The determination of *astigmia by skiascopy* rests upon the fundamental principles previously rehearsed, but two meridians of unequal power must be considered in astigmatism. The movement of the retinal image being of necessity in the same plane as the mirror movement, which is at right angles to its axis of rotation, it is the refraction in a single meridian only corresponding to that plane which is determined in retinoscopy. The axis of mirror rotation can, however, be made to correspond to any meridian of the eye which it is desired to examine, hence it is possible to determine separately the refraction in any two meridians at right angles to each other such as exist in regular astigmatism. The points of reversal for the meridians of greatest and least refraction being known, the value of the interval of Sturm, which represents the astigmatic error, is known.

Various methods are used in the practice of skiascopy in the determination of the astigmatic error: many operators neutralize each meridian separately with spheres, while others first neutralize one meridian with spheres and then, without removing any lens quantity present, proceed by the use of cylinders to bring the second refracting meridian to the same point of reversal. The objection to this latter method is, of course, the liability to error in placing the cylinder axially correct. It must be remembered in the calculation of the astigmatic error by the first of these methods that the optical deficiency of the eye is considered as made up essentially of two cylinders at right angles to each other, although the value of one of these cylinders may be zero as in simple astigmatism, and that the maximum power of a cylinder is always at right angles to its axis. If, then, the mirror is turned about a horizontal axis so as to sweep vertically up and down over the eye, the observer neutralizes by means of spheres the ametropic error in the vertical meridian; i. e., he brings the far point of the observed eye in a vertical meridian to his own nodal point. For example, if the meridian with its axis at 90° requires + 3 diopters to bring about a reversal at one meter and the meridian with its axis at 180° requires + 4 diopters, each meridian being operated upon separately, then the total lens quantity needed to cause reversal simultaneously at one meter is + 3 cyl. ax. 90° ⊃ + 4 cyl. ax. 180°. This is equivalent to + 3 D. S. ⊃ + 1.00 cyl. ax. 180° and the correction for this eye, allowing one diopter for the artificial myopia at the working distance, is + 2 D. S. ⊃ + 1.00 cyl. ax. 180.

161. It is as true of the astigmatic as of the non-astigmatic eye that the image of the pupil becomes magnified as the point of reversal is approached. Hence, when the observer's eye is nearer to the point of reversal for one meridian than it is for the second meridian, the retinal image is more magnified in the direction of the principal meridian to which the nearer point of reversal belongs. When the observer's nodal point is at the point of reversal for one of the observed meridians, the retinal image becomes indefinitely magnified in the direction of this meridian while it is magnified comparatively little in the direction at right angles to it. Every retinal point appears in the pupil, therefore, as a line running in the direction of the principal meridian and the retinal light area assumes the form of an elongated *band of light* running in the direction of the meridian which has its reversal point at the observer's eye. In order to bring out this band-like appearance it is necessary to secure as perfect focusing as possible in the principal meridian at right angles to the one in which the band is sought. Hence the principle, developed by Jackson, that "the band-like appearance is most perfectly developed when the observer's eye is at the point of reversal for one principal meridian and the immediate source of light at the point of reversal for the other principal meridian."

162. *Aberration* and *irregular astigmatism* add to the difficulties of making an exact determination of the refractive condition of an eye. Spherical aberration appears under the forms known as positive and negative, and is the condition in which, during the process of neutralization, two areas arise, one of which is central and the other peripheral, in which the refraction is not the same. The peripheral refraction is stronger than the central in positive aberration and weaker in

negative aberration. In the positive form, when the point of reversal for the center of the pupil is close to one meter for example, the peripheral illumination grows broader and will often crowd in upon the smaller central illumination, causing the observer to believe neutralization has been accomplished or even an over-correction given because of the reversal in the peripheral regions. The operator must therefore regard the motions in the central portions of the pupil. The area in the center of the pupil of comparatively uniform refraction is the *visual zone* and is the portion which is of practical importance for purposes of distinct vision. The cause of the paracentral shadow is explicable by reference to Fig. 107. Assume the patient's eye emmetropic but possessed of a strong positive spherical aberration. The rays coming from some luminous point of the retina will then have the position indicated in Fig. 107. When the observer's nodal point

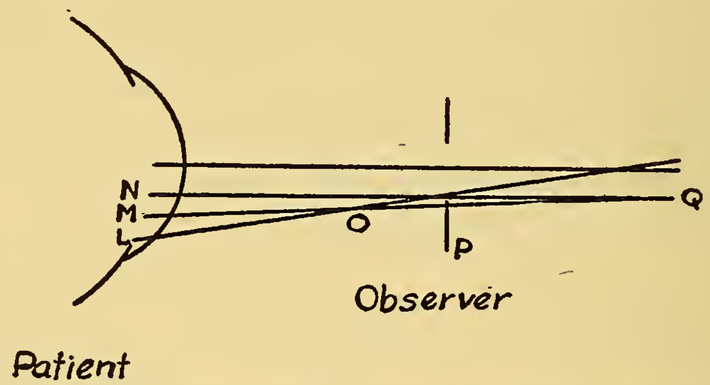


Fig. 107.—Illustrative of the Paracentral Shadow.

is at *P* he will receive the rays *NP* and *LP* and will see as luminous the portions of the observed pupil corresponding thereto, while in the pupillary region corresponding to *M* there will be no luminosity since the ray *MOQ* does not enter the observer's pupil. The observing eye would, therefore, see a bright center separated from a luminous peripheral area by a dark ring. If *P* be displaced a little downwards, so that the pupil can receive all the rays drawn in the figure, the whole will then appear luminous, but the rays coming from the upper portion of the observed eye (not drawn in the figure) will be then affected in that some of them will be occluded from the observer's eye giving rise to the paracentral shadow. A comprehensive and clear account of the appearances of positive and negative aberration effects as the observer moves with his mirror away from the point of reversal for the most myopic part of the eye can be found in Jackson's *Skiascopy*. Spherical aberration is ordinarily not seen in undilated pupils (whether this dilation be natural or by drugs is immaterial);

while the positive form is common to the great majority of eyes it is usually too slight to cause any inconvenience in retinoscopy when the pupil is of normal size.

163. *Irregular astigmatism* exhibits itself under the retinoscope as a more or less broken reflex giving no definite shadow when the neutralization has been carried to the approximate reversal point. If this irregularity, which may have its seat either in the cornea or the lens, is accompanied by a high degree of ametropia the pupillary shadows may be fairly distinct but as higher dioptric powers are inserted before the eye the shadow loses its definite form, thus permitting of an approximate correction only. Irregular astigmatism of the lens due to spicules pointing in from the periphery is often exhibited in retinoscopic examinations and is rarely seen with the ophthalmoscope in its incipient stages.

164. *Conical cornea* is a variety of irregular astigmatism in which the cornea bulges forward in the shape of a cone which may or may not have its apex at the anterior pole of the eye. The result is a highly myopic condition in the apicial region with a diminishing refraction toward the periphery. Under the retinoscope, opposite movements for the center and periphery may occur; the outer portions of the shadow move comparatively rapidly while the central portions are sluggish and may appear stationary. This gives rise to the peculiar appearance of the reflex which has a ''swirling'' motion around the apex of the cone.

165. *Scissor movement* is a term applied to the retinoscopic appearance of the pupil in which two areas of light are seen and which, as the mirror is rotated, advance from opposite sides of the pupil and merge into each other. The cause of the phenomena is probably *coma* due to the obliquity of one or more dioptric media of the eye. The medium involved is without doubt, in the majority of cases, the crystalline lens. Monographs by Jackson, Thorington and Sheard discuss practical methods of handling such cases.

166. *Ophthalmoscopy.* We now pass to the applications of the principles of conjugacy of foci to ophthalmoscopy.

The ophthalmoscope is an instrument employed for examining in detail the fundus of the eye through the pupillary opening. A concave mirror similar to that used in retinoscopy may be used as an ophthalmoscope, but when such a mirror is used at the ordinary retinoscopic distance the red fundus reflex only, with no details, is seen. This is due to the fact that the emergent rays seldom have that divergence for which the observer's eye is at that moment adapted, and also to the fact that a very small portion of the retina under observation is visible at ordinary retinoscopic working distances. In order then to be able to obtain a detailed view of the fundus it is necessary to do one of two things, either (1) approach the eye very close to that of the patient in order to enlarge the field of view, i. e., to carry the fundal image to the observer's distance of distinct vision, or (2) by means of an auxiliary convex lens (in practice a + 13 to + 16 diopter sphere) form a real image of the fundus in the air which can be produced within the observer's range of distinct vision. The first of these affords us the *direct* method of ophthalmoscopy since the image which is viewed is a virtual one, while the second is known as the *indirect* method because the fundus is seen by virtue of the aerial image formed by the auxiliary condenser.

167. The *optical principles* involved in the *direct method* are sketched in Fig. 108. The concave mirror has a focal length of about 3 inches. This is held in such a position with respect to the observed

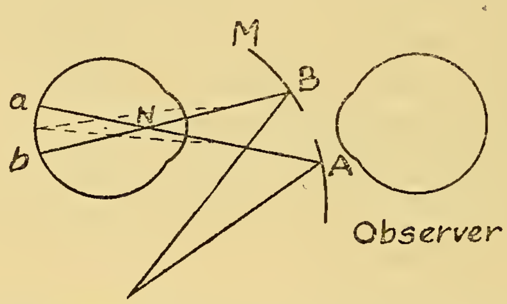


Fig. 108.—Optical Principles Involved in the Direct Method of Ophthalmoscopy.

eye that the light from the source *S* will be converged to the nodal point *N*, from which point it will diverge and illuminate an area, *ab*, of the fundus. The retina then becomes a source of illumination and diffuses the light which emerges from the eye in the manner discussed under *Retinoscopy* as parallel, divergent or convergent beams, depending upon the refractive condition of the observed eye. Hence the distinctness of the image seen by the observer depends upon his own refraction and that of the patient. The observer should be emmetropic, or rendered so by means of his correction, and should likewise be able to suppress all accommodative action at will if he is desirous of using ophthalmoscopic methods to obtain an estimate of the refractive condition of the eye under observation. Suppose, then, that the observer is emmetropic; he can then see the fundus of another emmetrope without any further aid optically, since the rays emerging from the observed eye are parallel and the observer's eye is adjusted for such rays. If the eye being observed is not emmetropic the light emerging from it will be either divergent or convergent; such rays cannot be brought to a focus on the emmetropic non-accommodated eye of the operator. In the examination of ametropic conditions, therefore, lenses must be revolved into the sight-hole of the mirror—convex in hyperopia and concave in myopia—in order that the emergent light may be rendered parallel and therefore adjusted to the emmetropic observing eye. One, therefore, looks for the strongest convex glass or the weakest concave lens with which he can see the details of the fundus most distinctly; the lens thus turned up before the mirror aperture gives a measure of the refraction of the observed eye. A difference between subjective and ophthalmoscopic refraction generally occurs and this latter method is but little employed in refractive work today.

The chief reasons for these discrepancies are: (1) lack of ability to perfectly relax the accommodation on the part of the observer; (2) distances of the lenses from the observed eye vary considerably between subjective and ophthalmoscopic examinations, hence introduce vital lens differences; (3) the papilla may have a different refraction from that at the macula. The direct method of ophthalmoscopy does, however, furnish a means of judging of the depth of a papillary excavation by a measurement of the difference of refraction between the edge and the bottom of the cup, bearing in mind that a difference of one diopter corresponds to practically one-third of a millimeter. By the same process one can measure the tumefaction of the disc in cases of optic neuritis or estimate the distance from the retina of an opacity in the vitreous body.

168. The *indirect method* in ophthalmoscopy does not give as magnified a view of the fundus as is afforded by the direct method and it is of practically no service in estimating the refraction. The optical principles involved in the indirect method are diagrammed in Fig. 109. The mirror M should have a focal length of about 10 inches. This is used at from 20 to 30 inches from the observed eye close to which a 13 diopter convex lens is held. In the diagram, the reflecting mirror M sends a converging beam of light on L, the condenser, which in turn further converges the light entering the eye. A portion of the fundus, ab, is illuminated. Light returning from any point D emerges and is brought by the lens L to a focus at B from which point it diverges to be focussed on the observer's retina R by the aid of his accommodation or an auxiliary convex lens turned up before the aperture of the mirror. The observer sees, therefore, at B a real inverted image of the fundus magnified about five times.

169. *Size of the ophthalmoscopic images.* (A) *Direct method.* Let us assume for simplicity's sake that the observed and observing eyes are emmetropic and at a distance apart equal to the sum of their

anterior focal lengths so that their focal points coincide at F as shown in Fig. 110. Let OE be a portion of the illuminated retina of the observed eye. Then any ray from E, parallel to the axis, will after refraction pass through F and this point being, in turn, the anterior focus of the observer's eye, this ray will then be refracted parallel to the axis of the observing eye and will reach the retina at a point E_1 such that O_1E_1 is the image of OE. Since the static refractions of the

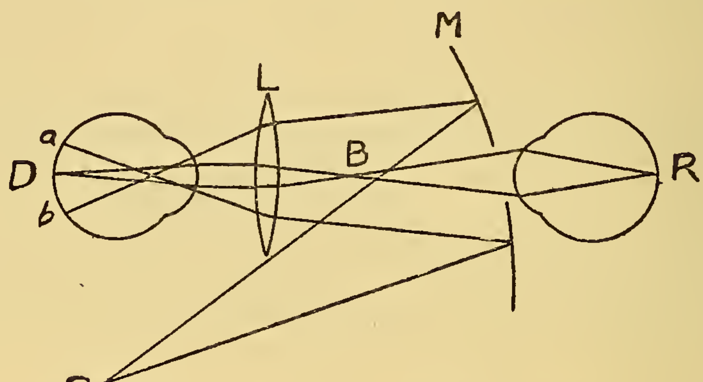


Fig. 109.—Optical Principles Involved in the Indirect Method of Ophthalmoscopy.

eyes involved are equal, then the image (I) received by the observer will be identical with that of the object O. Since the rays issuing from the observed eye are parallel and the observing eye is in a condition to focus upon the retina incident parallel rays, both eyes being assumed emmetropic, the distance between patient and operator is immaterial, hence the size of the image (I) does not alter as the observer approaches or recedes; the only result is a greater or lesser

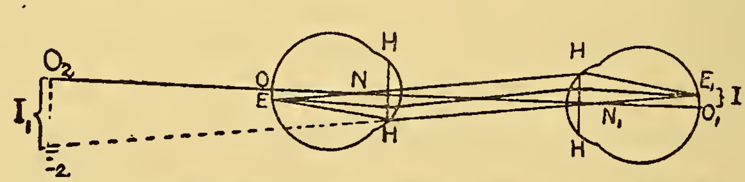


Fig. 110.—Magnification Under Direct Ophthalmoscopy.

field of view. But the observer's retinal image (I) is projected to his distance of most distinct vision and assumes the size of the virtual image diagrammed in the figure as O_2E_2 or, for brevity, indicated as I_1. The distance of distinct vision may be taken as 250 mms. The question as to the ophthalmoscopic magnification becomes, therefore, the following: What is the relation of the visual angle under which the virtual image of the illuminated retinal area (O_2E_2) appears to that visual angle under which the disc itself appears at the distance of distinct vision? The answer is readily obtainable from an inspection of Fig. 110; for the size of the retinal image I is, under the assumed conditions of emmetropia, equal at all distances of the two eyes to the size of the object OE. The object being viewed through the dioptric apparatus of the eye under examination, the visual angle is equal to $\frac{O_1E_1}{O_1N_1}$. From the similarity of the triangles $O_2N_1E_2$ and $O_1N_1E_1$ it will be seen that the apparent magnification is the ratio of O_2E_2 to O_1E_1 which is equivalent to the ratio of N_1O_2 to N_1O_1. We have, therefore,

$$\text{Magnification} = \frac{I_1}{I} = \frac{N_1O_2}{N_1O_1} = \frac{250}{15} = 17 \text{ (appr.)}$$

It will also be observed that the object O, the retinal image of the

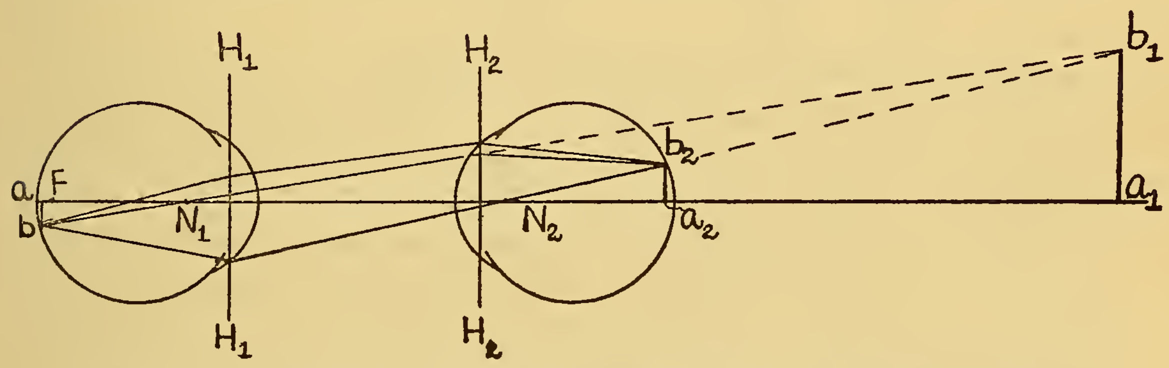


Fig. 111.—Magnification of the Upright Image in an Emmetropic Eye.

observer I and the final projected image I_1 are all formed under the same angle.

As a second case, let the patient's eye be axially myopic by an amount of 5 D. Then the disc lies 1.6 mms. behind the posterior focal point F of the normal eye as indicated in Fig. 111 and the far point a_1 is 200 mms. in front of the principal plane H_1H_1. Let the observer be 40 mms. away from the patient. Under these conditions the hyperopic observer, having a virtual far-point at a_1, sees the object ab at a visual angle given by

$$\frac{a_2b_2}{N_2a_2} = \frac{a_1b_1}{N_2a_1}.$$

The denominator N_2a_1, with the above assumptions, is equal to 200 — 40 — 5 = 155 mms. if we take the distance of the nodal point N_2

to be 5 mms. from the principal plane H_2H_2. The numerator can be determined (by a pair of equal triangles as shown in the figure) to be $\frac{1.5}{16.6} \times 205$ mms., where 1.5 mms. is the size of the object *ab* and 16.6 mms. represents the depth of the myopic eye from the nodal point N_1 to the retina. The equation then reads $\frac{1.5}{16.6} \times \frac{205}{155} = \frac{1}{8.3}$.

Since *ab*, at a distance of distinct vision of 250 mms. appears to the observer, without the magnification of the patient's eye, at a visual angle of $\frac{1.5}{255} = \frac{1}{170}$, and since $\frac{1}{8.3}$ is 20.5 times greater than $\frac{1}{170}$, we find the desired magnification to be 20.5.

By a corresponding process it can be shown that the magnification for a case where a patient has a hyperopia of 5 diopters and the eye is 40 mms. distant from the patient is fifteenfold.

If the correcting lens in the ophthalmoscope be situated at F_A, the anterior focal point of the eye under observation, the angle subtended by the image will be the same in hyperopia and myopia as in emmetropia and therefore the magnification will be unchanged; the image is simply sharpened by the correcting lens. Generally, however, the ophthalmoscopic lens is situated beyond the patient's anterior focal point and in that case the angle subtended by the image will be altered in size, becoming larger in myopia and smaller in hyperopia.

170. There are four conditions which influence the size of the ophthalmoscopic field. (a) The size of the observer's pupil must be considered since the greater the pupillary size the greater will be the image on the patient's fundus. As a matter of fact, however, this plays no particularly important part since the aperture in the ophthalmoscope plays the part of the observer's pupil: hence the statement might be made that the size of the ophthalmoscopic aperture must be considered. (b) The distance of the observer from the patient is of great significance because, as the observed eye is approached, the mirror aperture is brought closer and allows a bigger image to be thrown on the patient's fundus. Hence the rule of practical importance that the observer, in the direct method, should approach the patient as close as possible. (c) The size of the patient's pupil is also of great signifi-

cance. It should be dilated as much as possible by shutting off unnecessary light, excluding light from the most sensitive part of the retina or by the use of mydriatics. (d) The position of the point for which the patient's eye is accommodated has a direct bearing upon the size of the field. In the examination of the upright image, the ophthalmoscopic field increases with increasing hyperopia, since the point of accommodation of such an eye is beyond infinity or negative, and in turn decreases with increasing myopia.

171. *(B). Indirect method.* We have already stated that in the in-

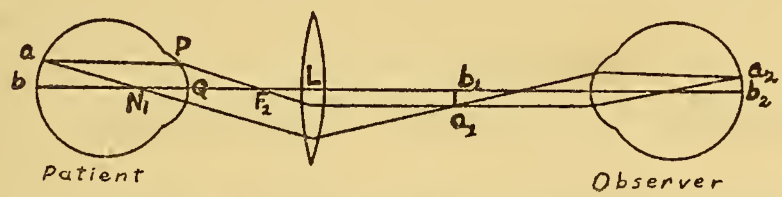


Fig. 112.—Magnification in the Indirect Method of Ophthalmoscopy.

direct method the light emerging from the observed eye is brought to an aerial image by an auxiliary condensing lens, from which image it then diverges to the observer. The observer must, therefore, accommodate for some point between himself and the lens and if his accommodation is insufficient for this purpose he must throw a convex lens into the ophthalmoscopic aperture. In Fig. 112 let *ab* be a portion of the illuminated fundus, N_1 the nodal point, F_1 the anterior focus of the observed eye, and L the condensing lens. Suppose L to be at a distance from F_1. Then light diverging from a will emerge parallel

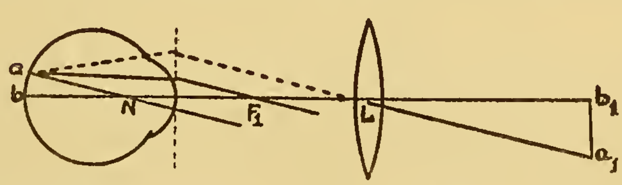


Fig. 113.—Magnification of the Inverted Image in an Emmetropic Eye.

from an emmetropic eye to focus at a_1 in the focal plane of the lens L; b_1a_1 is then the inverted aerial image of the object ab and a_2b_2 is the erect retinal image of the observer secured by accommodating for the plane b_1a_1. The fundus under observation, ab, is therefore seen inverted as b_1a_1. The confines of space in Fig. 112 show a_1b_1 and a_2b_2 smaller or equal to ab; as a matter of fact, each is in practice larger.

Let us determine the size of the indirect image obtained when an emmetropic eye is under examination. Since the light from the observed eye emerges parallel, the ray, such as La_1, Fig. 113, which can pass through the optic center of the lens L must always make the same angle, b_1La_1, with the axis. Also, the aerial image a_1b_1 must always be formed at the focal distance Lb_1, no matter what the distance of the lens L from F_1. Therefore, as the lens is withdrawn from the emmetropic eye the image must remain the same size. Since the angle aNb equals the angle a_1Lb_1, we have the magnification,

$$M = \frac{b_1a_1}{ba} = \frac{Lb_1}{bN}.$$

But $Lb_1 = 77$ mms., the power of the lens L being generally about $+13$ diopters and $bN = 15$ mms. Hence the magnification, $M = \frac{77}{15} = 5$ (approximately). In emmetropia the optic disc is seen as about 10 mms. in diameter situated at the distance of distinct vision of the observer. The actual size of the observer's retinal image depends upon the angle which the image subtends at his nodal point, so that a person with a close near point, or who uses a convex lens in the ophthalmoscope, secures a relatively larger magnification than one who does not possess these advantages.

172. It can be shown that the aerial image formed by the condensing lens will have the same size in all refractive conditions provided the condenser is at its own focal distance from the anterior focus of the observed eye. If the condenser is withdrawn from the eye variations in size of aerial image will be observed in ametropic conditions. An increase in the size of the image on withdrawal of the condenser denotes myopia and a decrease shows hyperopia.

It is difficult or impossible to see under certain conditions the image of the fundus by the direct method. This difficulty, aside from that arising from pupillary contraction, is usually caused by the excessively high magnification in myopia or to the impossibility of neutralizing the great convergence of the emergent light in high myopia. Similar difficulties do not occur in hyperopia since the high degrees are rare and because the divergent light is easily overcome by a convex lens. Taking, therefore, for illustrative purposes a case of 10 diopters of myopia, we know that its far point is 4 inches away and that the ametropia is correctable by about a -12 D. S. placed at 15 mms. from the cornea. As, however, the ophthalmoscope is rarely held as close as a centimeter or two from the eye, a still more powerful lens would be needed to neutralize the convergency of the emergent light. Thus, if the ophthalmoscope were to be used at 2 inches, it would be necessary to turn up — 20 diopters in order to get a clear view of the fundus. And again, taking this same case in which the ophthalmoscope is held 2 inches from a myopic eye whose far point is 4 inches, it can be proven that the magnification obtained would be approximately 30 as compared to the ordinarily obtained ratio of 16 — 20 to 1. Hence the field of view would be correspondingly reduced, so that only a very small part of the fundus would be observable without motion of the eye. In a case of 20 diopters of myopia it can easily be seen that if the ophthalmoscope is held at 2 inches from the observed eye (this distance being likewise its punctum remotum) the magnification will be infinitely great and the field infinitely small. If the ophthalmoscope be held 1.5 inch from the eye there would be required about — 80 diopters to give a clear image. This would give a magnification of about 50. No such lens is found in the ophthalmoscope and as a result the instrument must be approached still closer to the observed eye until the power of lens required falls within the range of those supplied. In practice the difficulties experienced in high ametropia are eliminated by putting the approximate correction in a frame as close to the eyes as possible and proceeding as in emmetropia or low ametropias. Normal magnification and normal field of view will then obtain.

PART THREE

QUALITATIVE AND QUANTITATIVE DETERMINATIONS OF THE RETINAL FUNCTIONS

173. There are three principal functions of the retina, to wit:—

1. The sensation of light and darkness or the *light sense.*
2. The sensation of *color.*
3. The perception of *form.*

Each of these three senses can be quantitatively investigated. They are not identical at the macular region and in the peripheral regions of the retina, hence a separate determination of the light, color and form senses must be made in the central and peripheral regions. We shall, therefore, in the study of these topics be concerned with the determination of the so-called threshold values, state of adaptation of the eye, visual acuity, tone, saturation and brightness of color, visual fields and allied topics. It seems, then, desirable at the outset to briefly define certain terms which will be of frequent repetition in this portion of this work and which must of necessity be introduced and used without any previous assurance that there may not be some overlapping of subject matter in such a manner as to involve correlated topics which may not have been previously discussed.

174. The *field of vision* is the area over which an eye can see indirectly, the visual axis being directed straightforward. The normal field extends, approximately, upwards 50 degrees, downwards 70 degrees, inwards 60 and outwards 80 degrees respectively.

175. The *field of fixation* is the greatest angular distance over which the visual axis can be moved when the head is held stationary and includes the maximum extent of distinct or acute vision. The field of excursion or fixation is normally up 35°, down 55°, in 45° and out 45°.

176. *Visual acuity* is a measure of the ability possessed of receiving, transmitting and mentally interpreting retinal impressions. The transparency of media, the power of the eye, the luminosity under which objects are viewed, the size of the retinal image, the nervous functions of the optic nerve and retina and the mental faculties (i. e., the interpretations of the brain) are factors which influence visual acuity.

177. The *limit distance of vision* is difficult to determine or express; it depends upon the luminosity or the amount of light the object viewed reflects, the clearness of the atmosphere, the color of the object, contrast between object and background, on the elevation of the object and upon whether the object is in motion or not.

178. The *visual threshold* is the lowest limit of light that can be observed by an eye and varies in different people. It is sometimes represented for the normal eye by a piece of white paper feebly illuminated and placed about 600 feet away on a black background.

179. *Illumination* and *visibility* are intimately connected. The visibility of an object increases with the luminosity up to a certain point beyond which the intensity of the light becomes dazzling and confusion from internal reflections and consequent blurring of the image occurs.

180. The *adaptation* of the eye relates to the usual phenomena observed when the eye has been exposed to or obscured from light, as for example when a person passes from a bright into a dull light nothing is clearly distinguished at first. This is partially due to previous exhaustion of nervous energy due to stimulation and partly to other causes connected with the visual purple which presumably undergoes a photochemical modification under the influence of light. Or we may say that the visual process requires time to become adapted to different mean brightness levels; the retina requires time to become accustomed to such brightness changes depending upon the magnitude

of the difference between the brightness levels involved and upon the period of time to which it was previously exposed to the first condition.

XV. THE LIGHT SENSE

181. It is probable that the sensation of light is due to the stimulus given to the terminals of the rods, and discrimination between details to the action of light upon the cones. It is known that the cones are much more abundant than the rods at the highly sensitive macula and that, at the fovea, the rods are absent. Furthermore, the foveal and macular cones are smaller then those in the peripheral regions. Toward the ora serrata, where there is practically complete absence of perception of details, the cones are comparatively few in number. Therefore the deduction is made from anatomical data that acute form vision depends upon the cones and light perception upon the rods. The fovea is, in turn, not the most sensitive portion of the retina for feeble illuminants; these are seen better by means of the rods than by the cones.

The range of light wave-lengths with which we are concerned lies within a very narrow range of radiations of less than one octave and is comprised between the limits of 3900 and 7600 Angstroms as the extreme end points.

182. The range of intensity of illumination over which an eye can see with practically equal comfort is enormous, for the average intensity of illumination at noon on a bright day is about a million times greater than the illumination given by a full moon and yet the eye can see fairly well in both cases. *This adaptability to the enormous range of intensity of illumination* which is met with is secured:

(1) By the automatic contraction or opening of the pupil by the muscles of the iris. In low intensity of illumination the pupil is wide open and contracts at higher intensities. The eye has thus a protective mechanism against the entrance of excessive light power, for at high intensity of illumination the pupil of the eye contracts and a sudden exposure to excessive radiation causes the eyelids to close. This mechanism is, however, mainly responsive to long waves of radiation, that is, to the red and yellow light, but not to the shorter waves of blue or violet light. Natural sources of excessive radiation, such as the sun, are rich in red and yellow waves and especially so when this light is received by reflection rather than by direct passage into the eye. The absence of this automatic protective action of the iris and eyelids against light deficient in long waves, as in the mercury lamp, for example, is important because it means that exposure to high intensity

of illumination from such sources may be harmful while the same or even greater *power* of radiations in yellow light would be harmless.

(2) By the fatigue of the retinal processes and nerves when exposed to high intensity of illumination the nerves become less sensitive, while at low intensity they rest and thus become more sensitive, and as a result the differences of sensation are made much less than correspond to the actual differences of radiant intensity.

(3) The impression made upon any of our senses, i. e., hearing, vision and so forth, is not proportional to the energy which produces the sensation but is approximately proportional to its logarithm and hence the sensation changes very much less rapidly than the intensity. Thus, a change of intensity from 1 to 10,000 units is 10,000 times as great a change in intensity as is produced in passing from 1 to 2 units but the change in sensation in the first case, since the logarithm of 10,000 is equal to 4, is only about 12 times as great as the change in the latter case where the logarithm $2 = 0.301$. This leads us to a further discussion of the hypothesis or psychophysical law of Fechner.

183. *Law of Fechner.* The perception of a difference between two luminous areas or sources occurs when the stimulus has attained a definite increase independent of the initial value of that stimulus. According to this law the smallest perceptible difference of illumination is a constant fraction, about 1 per cent., of the total illumination. This is often mathematically expressed as

$$\frac{\delta I}{I} = A$$

in which I represents the brightness and δI indicates the increment of brightness just perceptible; this ratio should be a constant, A. The law or hypothesis of Fechner was designated by him as psychophysical, since it appears to be as generally applicable to other senses as to that of sight as, for example, in the estimates of differences in weights or intensities of sound. The deduction of Fechner's principle is in reality dependent upon the experimentation of Bouguer and of Masson. The experiments of Bouguer as applicable to the principle under discussion are essentially as follows:—Two light sources, *A* and *B*, Fig. 114, of equal intensity are employed and an obstacle, *C*, so situated between these lights and the screen that two shadows *a* and *b* are formed on it. The shadow *a* is formed by *A* and consequently receives illumination only from *B;* the shadow *b* receives, in turn, light only from *A*. By moving, for instance, *A* away from the screen, the shadow *a* becomes weaker and when the distance of *A* from the screen is nearly ten times that of *B* it ceases to be visible. We can then replace the lights *A* and *B* by others having only half the intensity and repeat the experiments; it will be found that the shadows cease to be visible at the moment when the distance of *A* from the screen is about ten times that of *B*. The same result will follow, to the first order of approximation, whatever may be the intensity of the luminous sources used.

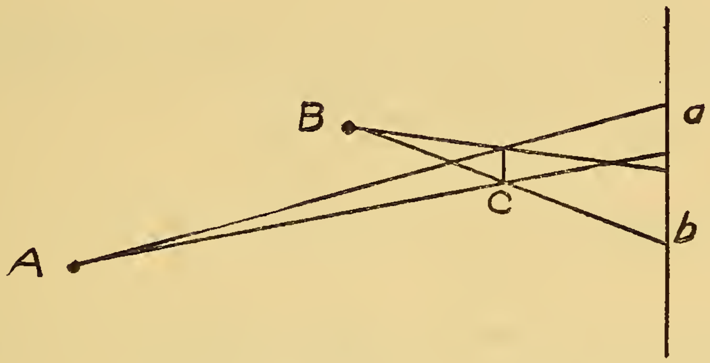


Fig. 114.—Experiment of Bouguer.

From these experiments we may conclude that the differentiable fraction is a constant whatever may be the luminous intensity; that is, $\frac{\delta I}{I} = A$. We know, then, in general that there is a relation between physiological response *S* and intensity of illumination or luminous stimulation *I*; that is to say, the value of the differentiable increment δI is a function of *I*. Hence $S = f(I)$, *f* being the function of I to be determined. The connection between two infinitesimal changes, δS and δI, is expressible as $\delta S = \frac{df}{dI} \cdot \delta I$. Following the hypothesis of Fechner, then, $\delta S = A$ and $\delta I = \psi(I)$ whence $\frac{df}{dI} = \frac{A}{\psi(I)}$ or

$$S = A\int \frac{dI}{\psi(I)} \quad \ldots\ldots\ldots\ldots (1)$$

Bouguer's methods of experimentation established the correctness of the relation $\frac{\delta I}{I} = B$, or $\psi(I) = B \cdot I$. By a substitution of these in equation (1) given above, we conclude that

$$S = \frac{A}{B} \int \frac{dI}{I} = C \log. I + D \ldots\ldots\ldots\ldots (2)$$

If we specify that I_0 shall be the minimum perceptible intensity, or threshold value, then $S = 0$ when $I = I_0$ in the limit, whence $D = - C \log. I_0$, or our complete equation is

$$S = C \log. \frac{I}{I_0} \ldots\ldots\ldots\ldots (3)$$

This equation, then, shows the relation between the physiological effect and the physical stimulus or quantity of light.

As a simple illustration of the verification of the ratio $\frac{\delta I}{I} = A$ by Bouguer's experiments, we will suppose that when the shadow disappears *A*, Fig. 114, is 500 cms. and *B* 50 cms. from the screen. The illumination is, of course, proportional to the intensity of the luminous source and inversely proportional to the squares of the distances. *B* therefore gives to the screen an illumination of $\frac{1}{(50)^2}$, *A* an illumination of $\frac{1}{(500)^2}$, while the shadow *a* receives an illumination of $\frac{1}{(50)^2}$. The difference between the illumination of the screen and that of the shadow is, therefore,

$$\left(\frac{1}{50^2} + \frac{1}{500^2}\right) - \frac{1}{50^2} = \frac{1}{500^2} = \delta I.$$

The ratio $\frac{\delta I}{I}$ then becomes

$$\frac{\frac{1}{500^2}}{\frac{1}{50^2}+\frac{1}{500^2}}=\frac{1}{100+1}=\frac{1}{101}=A$$

As a second case, when the intensity of light sources is only half as great, we find that

$$\frac{\delta I}{I}=\frac{\frac{\frac{1}{2}I}{500^2}}{\frac{\frac{1}{2}I}{50^2}+\frac{\frac{1}{2}I}{500^2}}=\frac{1}{101}=A$$

This law of Fechner explains many of the phenomena of daily observation. One reads nearly as well in the evening under gas or electric light as in daylight, although the illumination in daylight is much greater, because the ratio between the light reflected by the black letters and that reflected by the white paper remains the same.

184. But the *law of Fechner* is true only for *medium degrees of illumination.* The sensibility of the eye to brightness differences is greatest over a wide range of intensities but falls off at very low or very high luminosities. Within the ordinary limits of illumination used in everyday life we may say that the law of Fechner is verified with considerable exactitude. These departures from this law have been investigated by a number of experimenters. The classic experiments are those of Masson, Charpentier and of Koenig and Brodhun. The photoptometer of Charpentier may be used as a differential instrument and is of particular service in very low intensities. The photoptometer, *per se,* consists of a tube about 22 cms. long and 5 cms. wide, the extremities of which are closed by plates of ground glass *A* and *B*. At the middle of the tube are placed two lenses of 11 cms. focal length and between them a diaphragm of changeable aperture. On illuminating *A* the lenses project an image of it on plate *B;* the brightness of this image may be made to change by altering the aperture of the diaphragm. It is the plate *B* which serves for observation; the minimum aperture of the diaphragm which permits the observer to distinguish the plate *B* determines the threshold. A

modified form of the instrument, known as the differential photoptometer and which is particularly serviceable when the intensities are feeble and for the determination of $\frac{\delta I}{I}$, consists of two photoptometers at right angles to each other and each making an angle of 45° with a glass plate placed in the center of the long tube containing the first photoptometer and the sighting tube. The instrument is assembled in a manner analogous to the first ophthalmoscope of Helmholtz.

185. Another method of determining the smallest perceptible difference is due to Masson and depends upon the principle of persistence of vision. The disc of Masson is a white one in which different sectors of varying sizes have been blackened as shown in Fig. 115. By causing a sufficiently rapid rotation of this disc one can see three

Fig. 115.—Disc of Masson.

gray rings separated by white intervals. Supposing the sector *a* is 20° and the sectors *b* and *c* to be 10° and 5° respectively, and further assuming that the black areas do not reflect any light at all, the brightnesses of the three gray rings will be 340, 350 and 355 if we consider the light of the solid white rings as 360. The difference between the extreme gray rings and the white will be 5 and the relation $\frac{\delta I}{I}$ will then be equal to $\frac{5}{360} = \frac{1}{72}$ which represents the value of

Fechner's fraction for the patient under examination if he can distinguish the three images. If he can distinguish only two, Fechner's fraction becomes $\frac{360 - 350}{360} = \frac{1}{36}$. A large number of sectors is used in such experimentation; good illumination must be employed and the patient must not be too remote in order that the influence of diminished visual acuity (because of distance) may not enter into the problem.

186. A simple modification of this rotating disc, made by Masson, consists in drawing upon a white disc a series of equal black and white apertures running in a straight line from the center of the disc to the periphery as shown in Fig. 116. Upon rotation the black lines form gray bands which vary in distinctness as the center is approached.

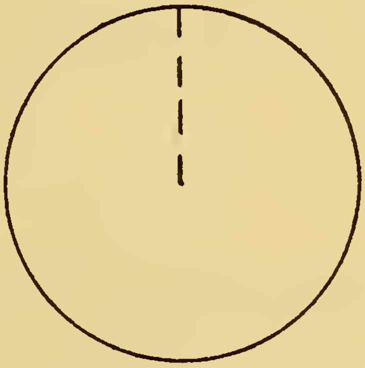


Fig. 116.—Modification of Disc of Masson.

In order to determine the minimum perceptible difference one must count the gray and white rings, produced by the rotation of the disc, proceeding from the center to the circumference. The distance which separates the center of the disc from the black mark which forms upon rotation the last discernible gray circular ring is proportional to the light sense of the subject examined. The value of the minimum differential can thus be obtained as a fractional part of the brightness of the white disc; if d is the width of the black rings, i. e., the size of the black interruptions on the disc as represented in Fig. 116, and r is the radius of the last discernible gray ring, when the disc is rotated, measured from the center of the disc, and if the intensity of the white portions of the disc is taken as unity, then the intensity h of a gray band during rotation is given by the expression

$$h = 1 - \frac{d}{2\pi r}.$$

The minimum perceptible difference, δ, is by definition equal to $1 - h$ and hence we have

$$\delta = 1 - \left(1 - \frac{d}{2\pi r}\right) = \frac{d}{2\pi r}.$$

These and other methods of investigation have shown that Fechner's constant varies when the intensities of illumination are very

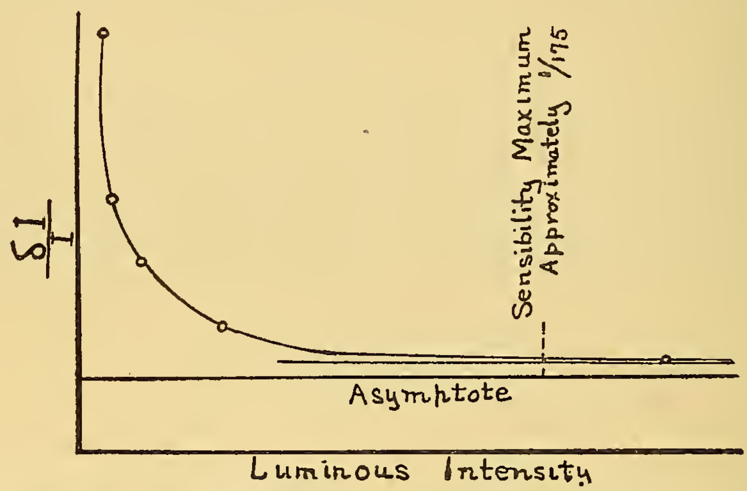


Fig. 117.—Curve Obtained by Broca Illustrating Change of $\delta I/I$ with Luminous Intensities.

high or low. Aubert, following the methods of Bouguer, obtained the following values, the light intensities being expressed in meter-candles:—

Illumination (Meter-Candles)	*Sensibility*
17.7	$\frac{1}{164}$
4.3	$\frac{1}{140}$
2.5	$\frac{1}{123}$
1.1	$\frac{1}{106}$
0.9	$\frac{1}{104}$
0.4	$\frac{1}{94}$
0.2	$\frac{1}{90}$
0.1	$\frac{1}{67}$
0.025	$\frac{1}{35}$

A repetition of similar experiments by Broca using the rotating disc of Masson gave the curve exhibited in Fig. 117, showing that $\frac{\delta I}{I}$ changes rapidly under low intensities.

187. The acuity of the light sense may be very properly expressed as the reciprocal or inverse of Fechner's fraction. If, for example, the latter be $\frac{1}{150}$ we may say that the acuity of the luminous sense is equal to 150 or if, by diminishing the illumination, the fraction increases to $\frac{1}{50}$, we have a luminous acuity of only 50.

We can represent the relation between the light sense and the illumination by a curve which has the form shown in Fig. 118. This has been done by Tscherning and has the following significance. The horizontal line represents the amount of illumination beginning with complete darkness to the left of the point *a* and terminating on the

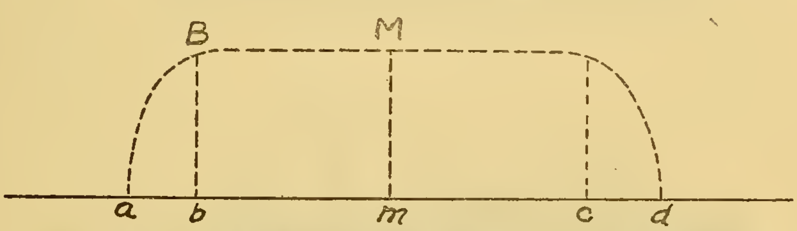


Fig. 118.—Curve Showing the Relation Between the Light Sense and Illumination.

right at the point *d* representing the direct illumination due to the sun with its dazzling effects. The ordinates represent the acuity of the light sense. When the illumination is very weak the eye sees nothing; when it reaches a certain degree, represented in the diagram by *a,* the eye begins to be able to distinguish white objects. The degree of illumination which forms the lowest limit of visibility is called the *threshold.* From this point on the light acuity increases rapidly and when the illumination has reached a certain degree, *b,* the acuity reaches the value which it holds over a considerable range of luminous intensities. *Fechner's law* is true for the range from *b* to *c.* Experimentation has shown that the value of $\frac{\delta I}{I}$ for white light becomes a constant when the luminosity has reached a value of approximately one lumen per square foot.

188. *The threshold.* We have discussed at some length Fechner's law relative to the sensibility of the retina to brightness differences;

there remains still to be considered the determination of threshold values. The threshold may be readily determined by the photoptometer of Foerster shown in Fig. 119, *A* and *B*. This instrument, in essentials, is a box carrying at the rear end a whitened surface, *T*, on which are painted large black stripes. The eyes observe these through two apertures a and a_1 (see Fig. 119, A). The light which penetrates

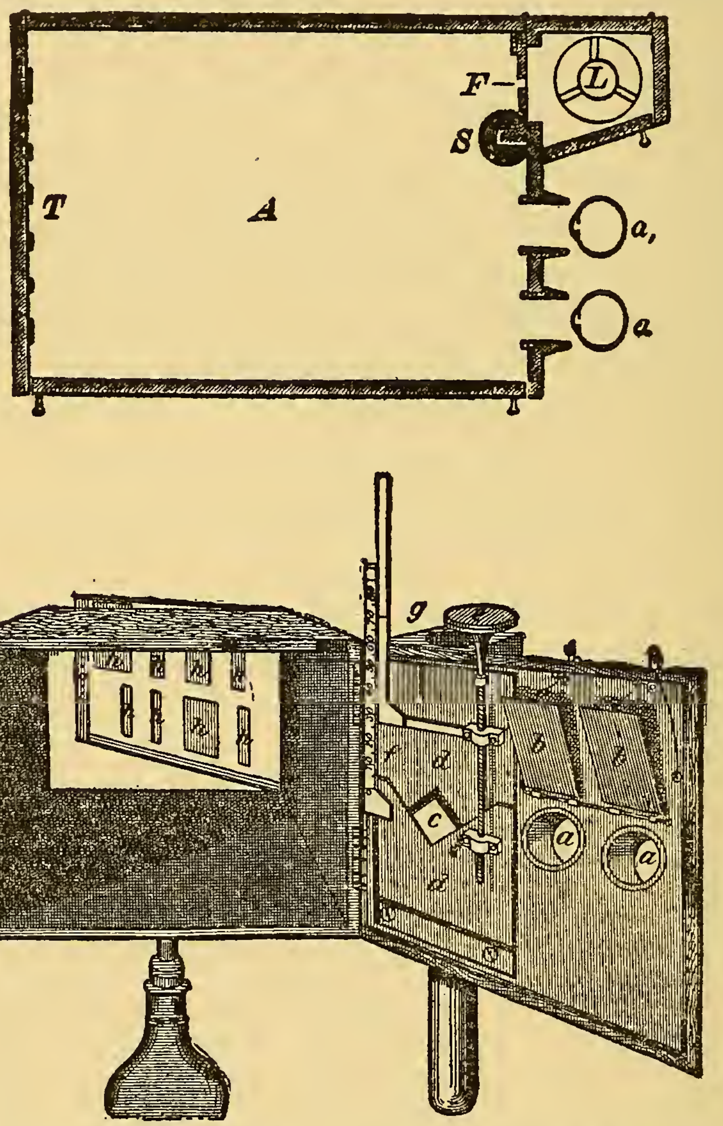


Fig. 119.—Diagrams of the Photoptometer of Foerster.

into the box comes through a window *F*, the aperture of which can be changed. Inside the compartment, screened off by the window *F*, is a standard candle *L*. The minimum aperture of the window *F*, which of course regulates the amount of illumination falling upon *T*, permitting the observer to see the black marks gives the threshold value. The results are subject to the variations which arise from various conditions of retinal adaptation.

189. Aubert determined the threshold of the normal eye; he found that the weakest light that can be distinguished is that of a sheet of white paper illuminated by a candle placed at a distance of from 200 to 250 meters. But the luminous sense varies within wide limits with the state of the retinal adaptation; it is indispensable, therefore, that the state of the adaptation be known in determining the light sense. In fact a determination of the threshold values made after various periods of the exclusion of light from the eye may be said to constitute a means of determining the retinal adaptation. Experiments by Charpentier and others have shown that the minimum perceptible intensity of light diminishes rapidly during the first ten minutes of seclusion in darkness and that after twenty-five minutes this has ordinarily reached a minimum which varies but slightly; the retinal sensibility has, therefore, reached its maximum practically. This means that the subject under examination must be kept in darkness, with eyes bandaged, for at least twenty minutes before determining the threshold values. Charpentier made determinations on the retinal sensibility under different conditions of adaptation and illumination. These latter tests were readily carried out by using as absorbing screens various thicknesses or deeper shades of smoked glasses. Some of the results are tabulated below:—

State of the Eye	*Fraction of light traversing glass plate*	*Sensibility*
Adapted for darkness	0	1000
Adapted for Glass No. 1.......	0.154	40
Adapted for Glass No. 2.......	0.413	17.85
Adapted for Glass No. 4.......	0.617	12.36
Adapted for ordinary daylight..	1.000	4.44

Charpentier observed that the ratio of the sensibilities between an eye adapted for strong light and for darkness was about 1 to 2500.

200. For a weak illumination the light sense of the *macula* is less acute than that of the surrounding parts. By fixing a point a little to one side of the fovea one can distinguish more readily the brightness of objects which differ but little from the background as, for example, when one tries to differentiate very dim stars from their settings. Parinaud and other writers have attributed this phenomenon to the fact that the fovea does not possess the faculty of being able to adapt itself to very weak illuminations as other portions of the retina because the fovea, composed of cones, has no retinal purple. This *retinal purple* has been considered by many as the source of the

adaptation. This hypothesis receives some confirmation from the fact that the time of repose required by an eye to reach complete adaptation is nearly the same as that which is necessary for the reproduction of the visual purple. But the blue rays play a dominant part in vision by weak illumination and it is possible that the inferiority of the macula may be due to its yellow pigmentation which absorbs, in part at least, the shorter wave-length energy incident upon it.

201. In certain pathological conditions the threshold values are affected. In *hemeralopia* the threshold is displaced upwards; it is probable, however, that this is an anomaly dependent rather upon the longer time demanded than in standard or normal conditions for complete adaptation. The existence of hemeralopia may be proven by the photoptometer of Foerster or by an examination of the visual acuity under reduced illumination. Then again there are those who see better when the illumination is low. By comparison of patients having *nyctalopia* with normal persons it is found that the lessening of the illumination causes the acuity of the normal individual to diminish more rapidly than in cases of nyctalopia. The fraction of Fechner is sometimes increased in consequence of which patients do not distinguish gray from white. This may be met with in cases of optic atrophy and in central scotoma. "One of the first cases of this kind was observed at the clinic of Hansen Grut, at Copenhagen, and described by Krenchel. It was a person who presented himself, saying that he did not see well enough to find his way. Examined with the ophthalmoscope, the papillæ were whitish, the visual acuity was normal and the visual field was only slightly contracted. It was puzzling, therefore, to explain the complaints of the patient until the idea of examining him with the disc of Masson presented itself: the fraction of Fechner had increased to 1/10. The patient distinguished perfectly black on white, but was unable to distinguish between gray shades, as they present themselves, for example, in street paving; whence the difficulty which he experienced finding his way." (Tscherning—translation by Weiland.)

202. *The light sense using colors.* The whole of the discussion and data upon the light sense thus far presented have been devoted to brightness differences and threshold values when white light has been employed. The least perceptible brightness increment varies for lights of various colors; we shall treat of these phenomena under the present caption rather than to defer them to the division devoted to the color-sense. In the first place, Purkinje observed that colored papers placed in a chamber in which the illumination falling upon them could be varied were seen first of all without color sensation. Charpen-

tier investigated these phenomena of Purkinje with his photoptometer employing "dark adapted" eyes. He used pure spectral colors and found that the sensation of light only without color sensation was evidenced first and that by increasing the size of the diaphragm and hence increasing the illumination a consciousness of color followed. He differentiated, therefore, between *color minimum* and *light minimum* and specified as the *photochromatic interval* the ratio of the two intensities necessary to produce the sensations of light and color. Charpentier's observations with an eye adapted for darkness showed that the ratio of the apertures permitting light sensation and color sensation varied greatly with the kind of light. His results are as follows:

Color	*Ratio of Apertures*
Red	3.6 to 4
Orange	5.5
Yellow	9.6
Green (average)	196
Blue (French)	625

The methods of procedure in studying the variations of the light-sense using colored stimuli are precisely the same as those used with white light. We are interested, then, in determining the variations in Fechner's law, the threshold values and the effects of the retinal adaptation for various colors and light intensities. The following table gives the data of chief interest to us: for each intensity the unit chosen has been the minimum perceptible brightness for each color after the eye has been obscured in darkness for twenty-five minutes.

	Values of $\frac{\delta I}{I}$			
Luminous Intensity	Red	Yellow	Green	Blue
6.5 units	0.64	....	....	....
25	0.30	0.36	1.2	1.45
56	0.16	....	....	....
100	0.105	0.20	0.49	0.90
225	0.11	0.14	0.28	0.69
400		0.09	0.25	0.48
900		0.068	0.16	0.36

203. These results are plotted in Fig. 120; one sees, with the unit of luminous intensity chosen, that the curves are very different. But this is no longer the case if one takes as the basis the threshold values of the color sensation, that is the chromatic minimum, rather than the minimum light-sense values. For the photochromatic interval increases from the red toward the violet; the result will be that one will obtain the same curve for all colors within the limits of light intensities commonly employed. Hence the differential sensibility is the same for

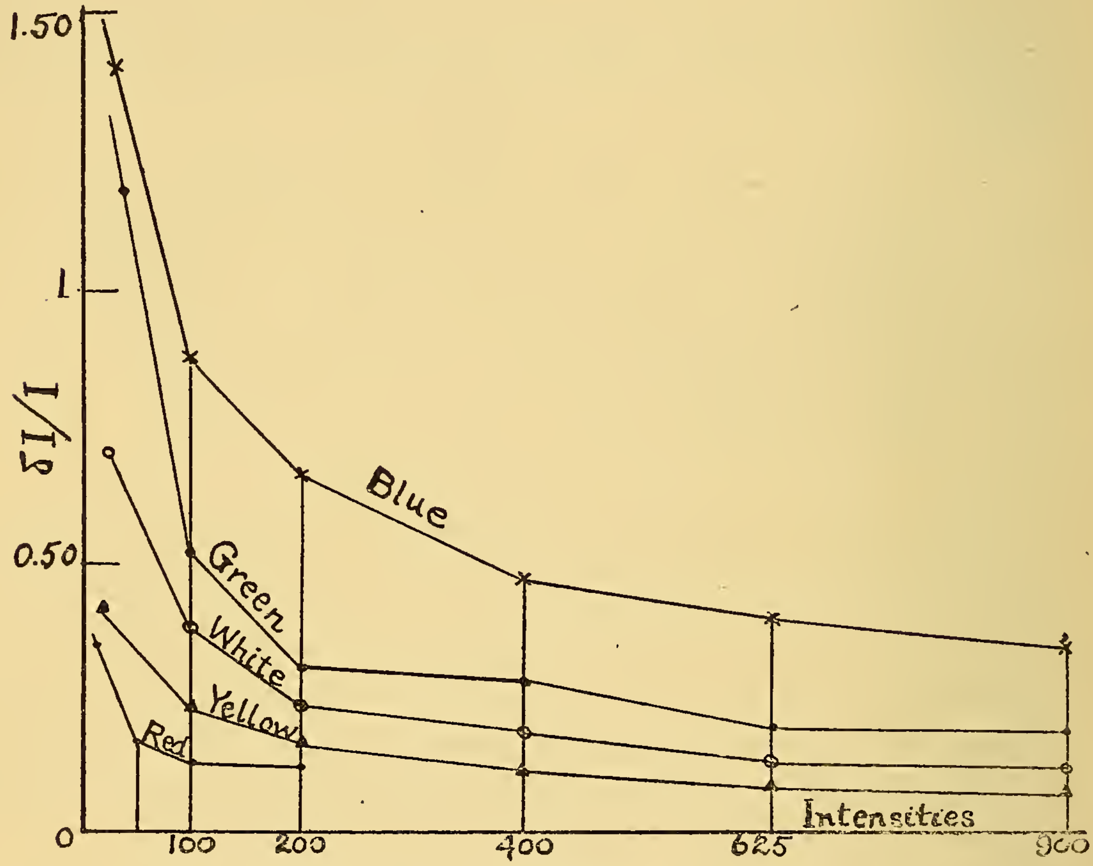


Fig. 120.—Curves Illustrating the Light Sense with Various Colors and Intensities. (After Charpentier.)

all the colors when one compares them under equal intensities, the unit being defined for each of them by its chromatic minimum. With decreasing luminous intensities the sensibility diminishes more rapidly for waves of longer length than for those of shorter length. Koenig and Brodhun have carried out an elaborate series of experiments in this field. They determined the least perceptible brightness increment for lights of various colors, including white, for brightness of a neutral tint (white) surface illuminated to intensities varying from 1,000,000 meter-candles to nearly the threshold of vision using an artificial pupil of 1 square millimeter area.

204. The essential features of a spectrophotometer similar to that employed by Koenig and Brodhun and diagrammed in Fig. 121 are fundamentally as follows: In front of the lens *B* of the collimator is placed a double image prism, *C,* which consists of a prism of calcite or quartz cut with its refracting edge parallel to the optic axis of the crystal cemented to a second prism of equal angle made of the same material but cut with the refracting edge perpendicular to the optic axis. A ray of light *I,* Fig. 121 (b), will pass through the prism *ABC,* in which the refracting edge is perpendicular to the optic axis, parallel to this axis and hence the ordinary and extraordinary rays will coincide in direction. On reaching the prism *ACD,* in which the

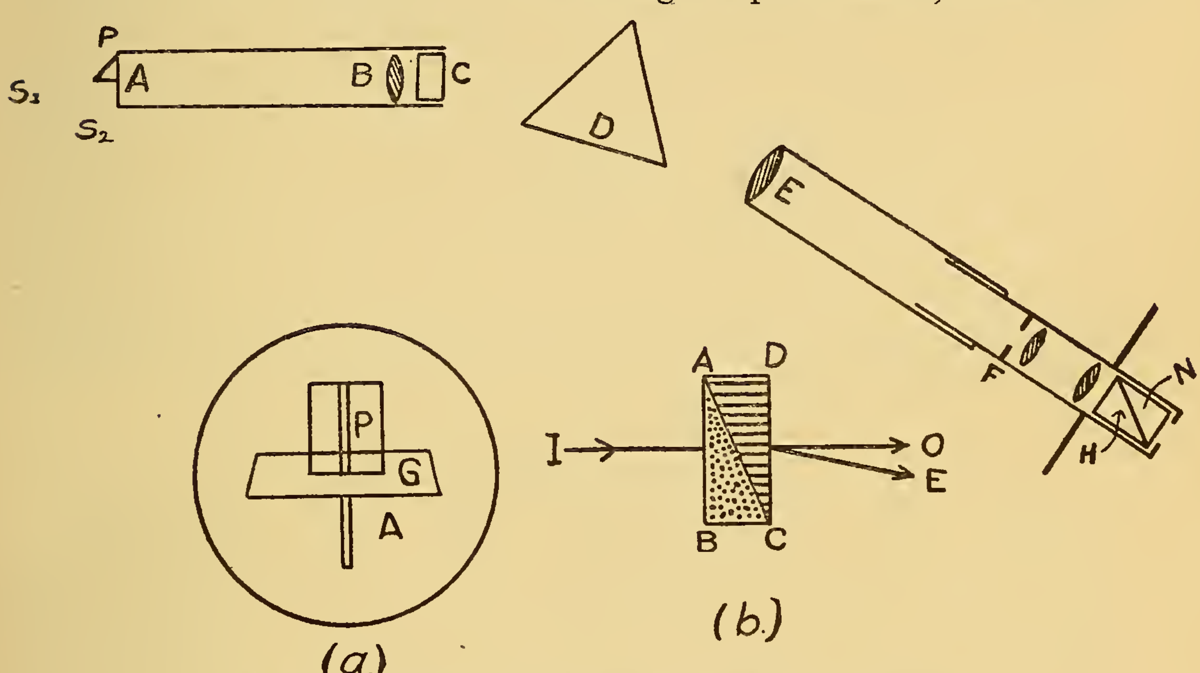


Fig. 121.—Optical Principles of a Spectro-Photometer.

refracting edge is parallel to the optic axis, the ordinary ray, *O,* will be undeviated, while the extraordinary ray, *E,* will be deviated. Hence the ordinary and extraordinary rays will proceed in directions inclined at a small angle. Thus the incident unpolarized light is divided by its passage through the prism into two rays of equal intensity which are plane polarized, the planes of polarization being at right angles. The double image prism is placed in front of the collimator object glass, and on turning the observing telescope to view the slit, the prism *D* having been removed, two images of the slit will be seen. The double image prism is then rotated until these two images form a single line. The two images overlap in the center; a piece of paper, *G,* Fig. 121 (a), must be pasted over the central portion of the slit and its width adjusted so that, of the four images now formed, the two middle ones just touch but do not overlap. Thus in the center of the field there will be two adjacent images of the slit, one formed by the light which has passed through the upper portion of the slit and the other by light which has passed through its lower portion and these will be prolonged in planes at right angles to each other. The Nicol, *N*, with its divided circle, is attached to the telescope and turned until one of the images vanishes, when the reading on the circle is noted. This gives the zero reading; rotation of the Nicol through 90° from this position will cause the other image to vanish. On placing the prism *D* on the table two spectra will be seen, one formed with light from each half of the slit. Suppose, then, that the lower half of the slit is illuminated by a source of light S_1, while by means of a totally reflecting prism *P*, the light from a second source S_2 is caused to illuminate the upper portion of the slit; the two spectra will then be due to the two light sources. To compare the brightness for different color values a diaphragm *F*, with a narrow vertical slit, is placed at the focal plane of the eye-piece of the telescope so as to cut out all of the spectrum except the portions which are to be compared. The Nicol is then turned until the two portions of the spectra appear equally bright. Let the angle through which the Nicol has been turned from its zero position be θ. Then

$$\frac{I_2}{I_1} = \tan^2 \theta,$$

where I_1 and I_2 are the intensities of the two sources of light of the wave-lengths under consideration and comparison. This instrument may be used for comparisons of light sources, for plotting the relative richness or deficiency of a given source in comparison with a standard for various spectral regions and also for the investigations of Fechner's law.

205. Koenig and Brodhun started at 600 meter-candles and extended the illumination above and below by the various steps indicated in the subjoined table. The data for Koenig's eye, after modification by Nutting, are shown in the accompanying table and in Fig. 122, in which the logarithm of the illumination *I* is plotted against the maximum perceptible brightness increment $\frac{\delta I}{I}$. It appears that the increment of brightness difference just perceptible increases as the brightness decreases and more rapidly for rays of longer wave-lengths. At high illuminations the minimum perceptible increment is about the same, 1.6 per cent., for all colors including white. It will be seen that when the illumination reaches 60 meter-candles the curves for different

wave-lengths merge into one curve. In the table, I_0 represents the threshold value of brightness measured as a fraction of the standard light brightness. The data of Koenig and Brodhun has been extended by Nutting by computation to the point where $\frac{\delta B}{B}$ (where B represents brightness) $= \frac{\delta I}{I} = 1$; that is, the threshold value.

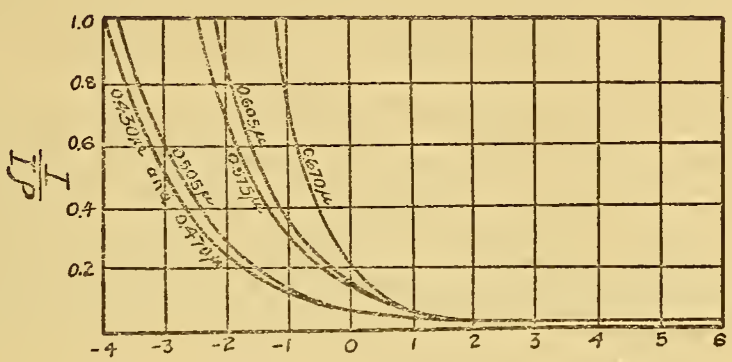


Fig. 122.—Curve Showing Relation of the Logarithm of the Illumination Plotted Against the Minimum Perceptible Brightness Increment.

Table of Data of Koenig and Brodhun on Brightness Sensibility as Recalculated by Nutting

Wave-length	0.670μ	0.605μ	0.575μ	0.505μ	0.470μ	0.430μ
I_0	0.060	0.0056	0.0029	0.00017	0.00012	0.00012
Meter Candles			$\frac{\delta I}{I}$			
200,000.000	......	0.0425	......	......	......	......
100,000.000	......	0.0241	0.0325	......	......	......
50,000.000	0.0210	0.0255	0.0260	......	......	......
20,000.000	0.0160	0.0183	0.0205	0.0195	......	......
10,000.000	0.0156	0.0163	0.0179	0.0181	......	......
5,000.000	0.0176	0.0158	0.0166	0.0160	......	......
2,000.000	0.0165	0.0180	0.0180	0.0175	0.0180	......
1,000.000	0.0169	0.0198	0.0185	0.0184	0.0167	0.0178
500.000	0.0202	0.0235	0.0180	0.0194	0.0184	0.0214
200.000	0.0220	0.0225	0.0225	0.0220	0.0215	0.0245
100.000	0.0292	0.0278	0.0269	0.0244	0.0225	0.0246
50.000	0.0376	0.0378	0.0320	0.0252	0.0250	0.0272
20.000	0.0445	0.0460	0.0385	0.0295	0.0320	0.0345
10.000	0.0655	0.0610	0.0582	0.0362	0.0372	0.0396
5.000	0.0918	0.103	0.0888	0.0488	0.0464	0.0494
2.000	0.1710	0.167	0.136	0.0655	0.0715	0.0600
1.000	0.258	0.212	0.170	0.0804	0.0881	0.0740
0.5	0.376	0.276	0.208	0.0910	0.096	0.0966
0.2	......	0.332	0.268	0.110	0.127	0.116
0.10	......	......	......	0.133	0.138	0.137
0.05	......	......	......	0.183	0.185	0.154
0.02	......	......	......	0.251	0.209	0.223
0.01	......	......	......	0.271	0.189	0.249
0.005	......	......	......	0.325	0.300	0.312
0.002	......	......	......	......	......	0.369

206. The value of the minimum perceptible increment depends upon the method of making the measurements. Usually the brightness of one of the two parts of the photometric field is varied until it appears just perceptibly brighter or darker than the comparison field. The brightness of one portion of the field is varied between certain limits for which it is respectively brighter and darker than the comparison field and these limits are gradually brought nearer together until the middle point is estimated as accurately as possible. P. W. Cobb has recently obtained minimum perceptible increments for white light of less than one-half a per cent. and has come to the conclusion that this increment has a smaller value than that obtained by Koenig and Brodhun.

Charpentier investigated the variations of the photochromatic interval with the retinal adaptation and arrived at the conclusion that, when the eye was fatigued in each case by white light, the chromatic minimum was not modified but that the luminosity minimum decreased in proportions analogous to those which have been observed with white light. The adaptation, then, has essentially to do with the light sense and very little with color-sensation.

207. If one attempts to compare *lights of different colors,* the eye manifests an indecision and uncertainty in its determination of equal brightnesses. For a difficulty is encountered in what is known as the *phenomenon of Purkinje.* If we equalize two sources, one of which is red and the other blue, and then diminish their brightnesses one-half, the blue light will appear much brighter than the red light. As an illustration,—if two papers are selected, one of which is red and the other blue and which by daylight appear to have the same brightness, then by diminishing the illumination the blue will appear brighter than the red paper. In very feeble illumination the red paper will appear black and the blue a pale gray. The papers must be viewed at an angle which is not too small since the phenomenon is not so pronounced for the macula. Likewise Macé de Lépinay and Nicati have shown that the visual acuity falls off much more quickly on diminishing the illumination when red light is used than under blue light. If we select red and blue glasses such that the acuity is the same when viewing the chart by daylight illumination, it will be found that, by reducing the illumination by shutters or curtains, the blue glass permits of the reading of the chart while with the red glass the chart cannot be seen at first; after a little time the larger letters only are readable through the red glass. Hence the acuity through the red is much inferior to that through the blue glass; the latter remains practically stationary.

208. The following experiment, quoted from Tscherning, shows in a striking manner the difference which exists in this respect between the two extremities of the spectrum. "We project the spectrum on the screen *A* pierced by two apertures, allowing the red rays and the blue and violet rays to pass. Behind the screen *A* we place a lens which re-unites these rays on a second screen *B*, forming on it an image of the surface of the prism which is turned toward *A*. This image then shows a pretty purple color. In front of the screen *B* we place a stick which forms thereon two shadows, one red, the other blue, and it is easy to so regulate the apertures of the screen *A* that both shadows may have the same brightness. If we now diminish the width of the slit through which light reaches the prism the purple is diluted more and more with white. The blue shadow becomes grayish and brighter and brighter as compared with the background, while the red shadow retains its color but becomes darker and darker. Finally it is nearly black and alone visible, the other shadow being gray and having nearly the same brightness as the background."

209. The yellow and green rays are in the regions of greatest visual brightness in the ordinary continuous spectrum. This brightness diminishes toward the two extremities of the spectrum but less toward the red than toward the blue end. This difference in the two extremes of the spectrum is largely due to the fact that the prismatic dispersion is greater as the wave-lengths decrease and hence the blue and violet are spread over a much greater space than the colors of greater wave lengths. For if the spectrum is produced by means of a diffraction grating, in which the dispersion is uniform for all colors, it will be found that the intensity is greatest in the middle of the spectrum and diminishes almost equally toward the two extremities. Lessening the intensity of the luminous source causes the spectral colors to change hue. The yellow and blue are the first to disappear; the red, green and violet only remain and these take the places of those which have disappeared. By a further reduction in intensity the blue changes into a blue-gray, the green into a grayish-green, the red becomes brownish. Finally, on still further reducing the luminosity all the colors disappear and only gray remains. It is stated that red alone forms an exception to this statement; it does not appear to change into gray before disappearing. The colors, then, disappear when the luminosity becomes sufficiently feeble; likewise, in turn, when the brightness becomes too strong or rather excessive the impression approaches white. The sun viewed through a red glass allows only red rays to pass through and yet the sun appears of a yellowish-white color. By first passing sunlight through a blue filter and then con-

centrating the light on a screen by means of a lens the image of the sun will be found to be white. A spectrum of sunlight produced by turning a prism toward the sun will be seen as a colorless strip of extreme brightness. According to Parinaud all these phenomena depend upon the adaptation of the eye. The spectrum of feeble bright-

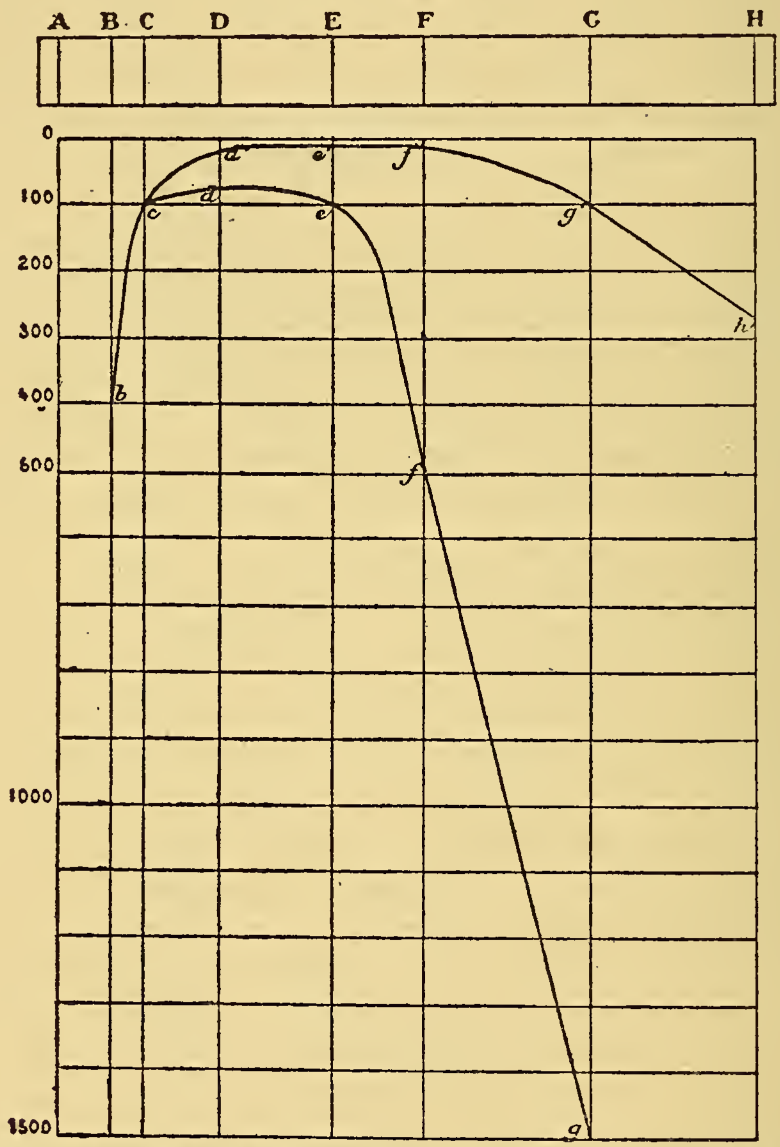


Fig. 123.—Illustrative of the Phenomena Dependent on the Variation of the Brightness of Illumination and Adaptation of the Eye. (After Parinaud.)

ness which appears gray to the adapted eye is invisible to the non-adapted eye. When the intensity increases it becomes visible to the non-adapted eye and appears colored. Parinaud determined the threshold values for different rays of the spectrum and obtained the curves shown in Fig. 123. The upper curve is for the adapted eye, while the lower curve represents the non-adapted condition. The letters *A, B, C* and so forth refer to prominent Fraunhofer lines and indicate roughly the division of the spectrum into the ordinarily named six or seven colors. The ordinates indicate the quantities of light necessary in order that these different parts of the spectrum may be seen. The curves show that the adapted eye requires a quantity of light equal to unity (this quantity being taken as a standard) in order to perceive the green rays in the region *E*, while the non-adapted eye requires a quantity equal to 100 in order to perceive the same rays. Again, in the blue at *G* the adapted eye requires 100 units and the non-adapted eye about 1,500 units in order to perceive them. The two curves by comparison show, therefore, that an eye gains nothing for the perception of red by adaptation but that it gains greatly for the colors of shorter wave-lengths. It gains in luminosity sensibility only, since, with the exception of the part *bc* which is common to both curves, the whole of the upper curve corresponds to colorless sensations only. The fovea apparently gains nothing by adaptation; the rays give color sensation at the same brightness as they evoke light sensations. These results of Parinaud have been criticized by Charpentier according to whom it is incorrect to attribute the colorless sensations which the rays of weak luminosity call forth to the retinal adaptation. However, retinal adaptation must play some rôle in these phenomena. Charpentier gives the following interesting observations. He covered one of the plates of his photoptometer with a black paper pierced by seven small openings in a space of nine millimeters square. The other plate was illuminated by light from different portions of the spectrum. On gradually opening the diaphragm of the instrument he demonstrated that the first impression which is obtained is that of a diffuse luminous area and colorless; let us specify the size diaphragm under these conditions by the symbol "*x*." In order to distinguish the color it was necessary to increase the aperture to a value, let us say, of "*y*." Only by still further increasing the diaphragm to a size "*z*" was it found possible to distinguish the points; in other words, the order of phenomena was *light-*, *color-* and finally *form-sense*. On the other hand, for an eye adapted to darkness, the apertures "*y*" and "*z*" remained about the same as for the non-adapted eye, but the aperture "*x*" could be diminished greatly for the more refrangible rays, demonstrating that the retinal adaptation is an important factor in the sensation of light but that it plays a minor part in color sensation.

210. *The luminosity curves of the eye.* The sensitivity of the eye to radiation obviously changes with the frequency, as it is zero in the infra-red and in the ultra-violet in which regions the radiation is not visible. The sensitivity gradually increases from zero at the red end

of the spectrum to a maximum somewhere near the middle of the visible spectrum and then decreases to zero at the violet end; that is, the physiological effect produced by the same radiation power—as, for example, one watt of radiating power—is a maximum near the middle of the visual spectrum. Inversely, it may be stated, the *mechanical equivalent of light,* or the power necessary to produce the same physiological effect, is a minimum near the middle of the spectrum and increases from there on to infinity at the ends of the spectrum where no power of radiation can produce visibility. It would appear, therefore, that the power equivalent of light is not a constant like the mechanical equivalent of heat but that it is a function of the frequency, or in other words of the color, and that its maximum is not far from 0.01 watt per candle power in the middle of the spectrum.

211. Various methods have been employed for the determination of the sensitivity or luminosity curves of the eyes and various experimenters are not yet in accord in the results which they have obtained nor in the elimination of sources of error and points of dispute in their methods. Obviously, direct comparison photometric methods are open to the objection that one cannot accurately compare lights of different color since the photometer compares by *identity,* and lights of different color cannot be made identical. And yet the spectral luminosity curve may be obtained by this method if it is modified by the so-called ''cascade'' procedure. This method involves a comparison of luminosities by introducing slight hue differences since, if the color difference is small, direct comparison can be made with fair accuracy. The essential point in the method is, then, the comparison of yellow with red, let us say, through several transition steps of small value between these limits. The liability to error in this method is obvious; the more steps there are the less likely perceptible differences in color will arise between steps and the less the errors would appear to be, but in the end it is the summation of percentages of errors which must be taken into account. Another method and one which is in greatest favor at present consists in the use of a *flicker photometer;* this instrument has been fruitful of results in the hands of Nutting and of Ives in particular in the last ten years. In its simplest form it consists of a stationary disc, illuminated by a lamp and a rotating half disc or sector in front of it illuminated by another lamp. At slow rotation a flicker is observable but this disappears if the speed becomes high enough. It is apparent that the more nearly equal the effects of the two illuminants under consideration—that of the stationary disc and of the rotating sector—the lower will be the speed at

which the flicker disappears and hence, by an adjustment of the distances of the two lamps so as to cause the flicker to disappear at the minimum speed, the instrument shows equality of the effect of the two successive illuminations on the eye. Whether these flicker methods give wholly reliable data is a question; the persistence of vision and the general physiological effects of different colors are different and we may therefore postulate a *mixture effect* of such a nature as to be forced to admit that we are not comparing lights of different color values by their illuminating values but by some other feature (this word is used for want of a better term) not directly related to the phenomena.

And, again, another instrument for the determination of sensibilities is the *luminometer,* a very simple device consisting of a black box to screen off extraneous light and carrying an aperture to allow only the light of the source under investigation to fall upon a white card printed in black, the reading matter consisting of a jumble of small letters, capitals and so forth arranged in meaningless order. The method of using it depends upon acuity tests; the observer moves toward or away from the light until a point is found at which the large letters can be clearly distinguished, the small letters remaining indistinguishable. It is claimed by many that this point can be found with considerable sharpness, and that, therefore, the luminometer gives consistent and reliable readings with widely different colors of light. But the error in this method is obvious; any comparisons dependent upon acuity are subject to much uncertainty.

212. In view of these facts we shall content ourselves with the giving of a description of one of the classic methods employed by Abney who used an apparatus of the form schematically shown in Fig. 124. The light from the slit S_1, which is placed at the principal focus of the first lens L_1, falls as a parallel beam of light on the prism P. After refraction parallel rays of each of the different colors fall on the lens L_2 and are brought to a focus on the screen DD. In the screen there is a second slit S_2 through which rays of only one refrangibility pass. These rays fall on a third lens L_3 arranged so as to produce on a white screen at FE an image of the nearer face of the prism. By moving the slit S_2 a patch of light of any required color can be thrown on the screen at FE. Slight tiltings of the lenses L_2 and L_3 must be given in order that a sharp image of the whole of the prism face may be formed on FE.

To apply this device and method to color photometry it is necessary that a vertical stick be placed in the path of this colored beam casting a shadow on the screen, while a second standard light T_2 mounted on a scale, casts a shadow close by. This second shadow is colored, being illuminated by the colored beam from S_2, while the first shadow receives the light from the standard; still, by moving the comparison lamp along the scale a point can be found at which the luminosities

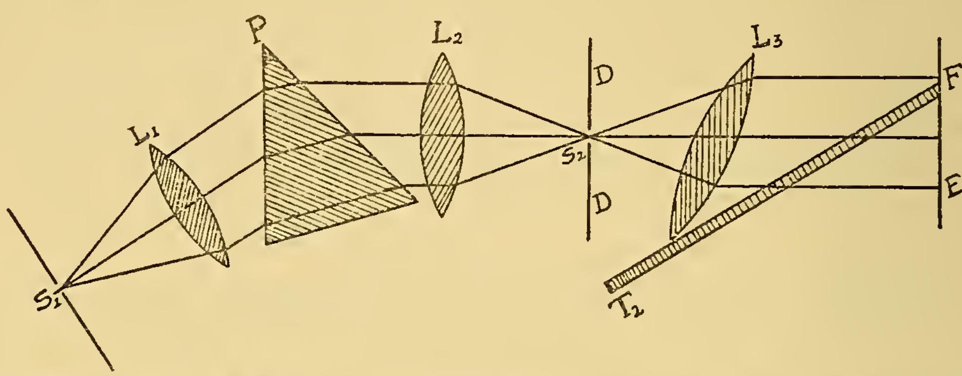


Fig. 124.—Abney's Apparatus for Obtaining Luminosity Curves of the Eye.

over the two appear equal. The determination of this point is, however, attended with difficulty much of which is overcome by the oscillation method described in 1886 by Abney and Festing. Abney found "That the best way of determining the intermediate point

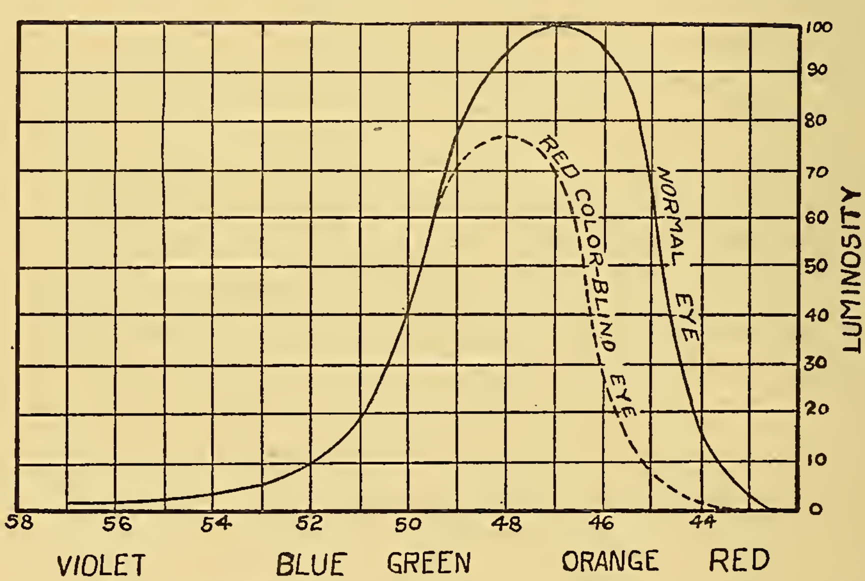


Fig. 125.—Luminosity Curves of a Normal and a Red Color Blind Eye. (After Abney.)

where the shadows balance is by oscillating the slide gently between two points when first one shadow and then the other is palpably too dark; the oscillations become shorter and shorter until the point of balance is determined." By pursuing this method throughout the whole spectrum Abney obtained the curves shown in Fig. 125. The full line curve is that representing the sensitivity curve of the normal eye, the ordinates being the luminosities and the abscissæ the wave-lengths of light; we have also included in the dotted curve a representation of the sensitivity of a red color-blind observer.

213. Experience has demonstrated, however, that the sensitivity curve for different colors of radiation is a function of the intensity of radiation. That is to say, the maximum sensitivity point of the eye is not at a definite frequency or wave-length but varies with the intensity of illumination and shifts toward the red end of the spectrum for high intensity of illumination and toward the violet end of the spectrum for low intensities. For illumination of very high intensity the maximum physiological effects occur in the yellow region while,

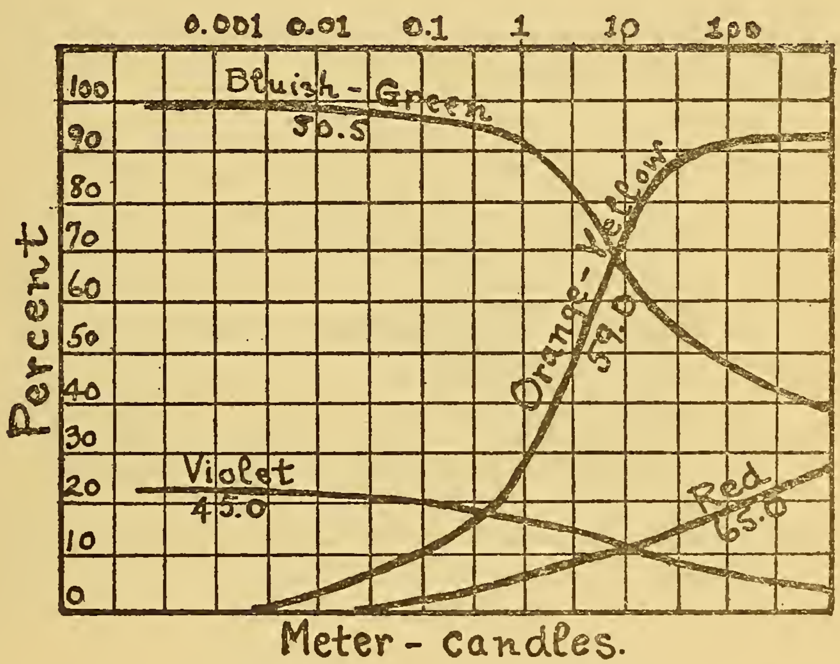


Fig. 126.—Curves of the Variation of the Relative Sensitivity of the Average Human Eye with the Intensity of Illumination for Red, Orange-Yellow, Bluish-Green and Violet Lights.

on the other hand, for very low intensity of illumination it appears in the greenish-blue region. Hence at high intensities yellow light requires less power for the same physiological effect than any other wave-length of radiant energy, while for low intensity bluish-green light requires a minimum power for the same physiological effect. A simple illustration of this is afforded in the following: if an orange-yellow light, as the flame carbon arc, and a bluish-green light, such as the mercury arc, appear of the same intensity at a distance of one hundred feet, then upon approaching the lamps the orange-yellow will appear to increase more rapidly in intensity than the bluish-green. From very short distances the yellow arc will appear "glaring" bright while the mercury lamp will appear much less intense and in fact will appear very dim or weak in comparison. Upon receding from the lamps, however, the reverse phenomenon is observed, for the orange-yellow light fades out more rapidly than the bluish-green and will have disappeared when the bluish-green is still noticeably visible.

214. The accompanying diagrams in Fig. 126 illustrate the change of sensitivity with intensity for the average human eye for red light of wave-length 6,500, orange-yellow light 5,900, bluish-green light 5,050 and violet light of wave-length 4,500 Angstroms. For red and violet the sensitivity is low while for orange-yellow and bluish-green the sensitivity is high. For bluish-green radiation, however, the sensitivity is high at low and moderate intensities but falls off for high intensities, while for orange-yellow light the sensitivity is high at high intensities and falls off at medium and low values. Red light

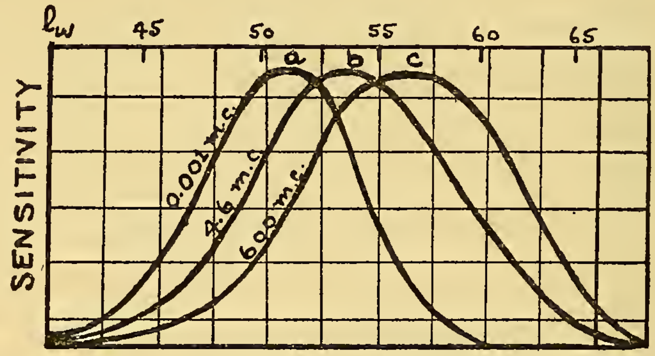


Fig. 127.—Approximate Sensitivity Curves of the Average Human Eye for Illumination Near the Threshold Value (medium illumination and high illumination).

vanishes from visibility still earlier than orange-yellow while violet is invisible even at very low intensities. The intensity of radiation varies inversely as the square of the distance but the physiologic effect of radiation does not vary exactly as the square of the distance but varies faster for the long wave-length end of the spectrum and somewhat slower for the shorter lengths of light.

215. The shape of the sensitivity curves also changes depending upon the intensity of illumination; for low intensity it is more peaked, showing that the sensitivity decreases more rapidly from a maximum towards the ends of the spectrum than it does for high illumination. These statements are substantiated by the curves of Fig. 127 showing the approximate sensitivities of an eye (a) for very low illumination near the threshold value of visibility or 0.001 meter-candle, (b) for medium illumination of 4.6 meter-candles and (c) for high illumination or 600 meter-candles. The maximum visibility region under the in-

tensities of illumination specified in the above three cases is 5,110 Angstroms for curve (a), 5,370 Angstroms for curve (b) and 5,650 Angstroms for curve (c).

216. It is of interest to compare the luminosity curves of the eye obtained when the yellow spot, the regions just outside the yellow spot and the fovea centralis are the subject of investigation. The comparative luminosities of the spectral colors as seen on the yellow spot are obtained by various color-patch and flicker methods either previously described or to be discussed later under color-vision. To get luminosity curves outside the yellow spot the following simple plan is adopted dependent upon the fact that in order that the images of the patches, for example, in the Abney methods, may fall outside the yellow spot

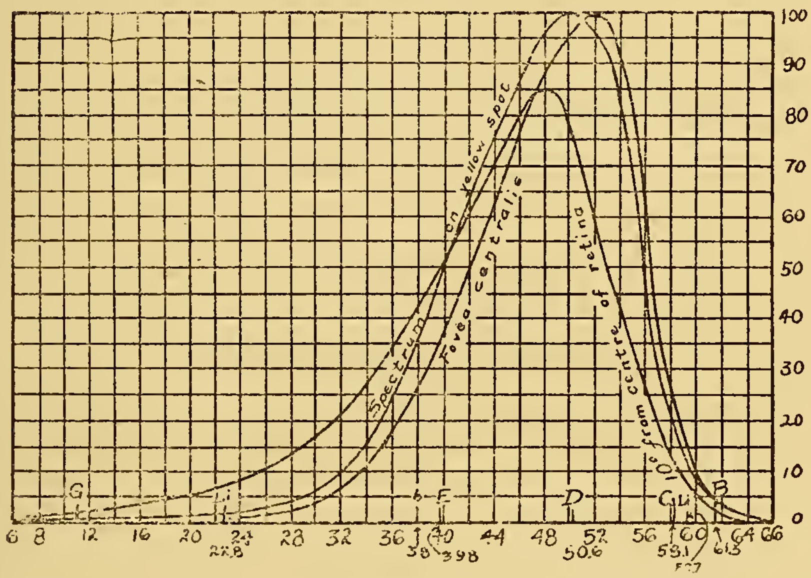


Fig. 128.—Sensitivity Curves of the Fovea Centralis, Yellow Spot and Area Ten Degrees from Center of Retina.

they should be received on the retina at least 5° from the center of the macula. If a spot is marked in a horizontal direction 5 inches away from the outside of the color rectangles used for comparison and the observer's eyes are 5 feet away from the patch and then the spot is looked at, the image of the rectangles will be received outside the extreme edge of the yellow spot. If this outside spot is illuminated by Balmain's paint and the axis of the eye under test, the other being occluded, is directed toward this point, the rectangles of white and color used for comparison will be clearly defined and the luminosities can be compared; they can, as a matter of fact, be compared with even greater facility than when observed with the center of the eye. The sensibility of the fovea centralis can be investigated by using extremely small rectangles; Abney employed a cube of one-quarter inch edge in which the color and the white light each occupied one-half of one of the surfaces and the eye was kept at 5 feet. Observations showed that the fovea is about one-sixth more sensitive to the sodium (D) light than is the macula. The fovea appears less sensitive to the green and the blue than the macula. The results of Abney's experimentation on the fovea, the fovea centralis and ten degrees from the center of the retina are shown in Fig. 128.

XVI. THE COLOR-SENSE

217. We have discussed the light-sense in a considerable number of the preceding pages and have considered some phases of the relations between the light and color sensations in which, however, our interests were centered chiefly in the former rather than in the latter. We are now interested in the consideration of colors according to their hue, their saturation, their combination to produce white, the effects of environment upon them and similar topics. The *hue* or *tone* depends on the wave-length alone; the *saturation* or *purity* depends upon the white which is found added to nearly all existing colors except those of the spectrum. The hue changes constantly in the spectrum; the change reaches its greatest rapidity in the green-blue part of the spectrum, where a variation of ten wave-lengths produces a change of hue, but the rapidity diminishes toward the spectral extremities and in the extreme portions of the red and violet the hue remains the same. Therefore, notwithstanding the fact that the visible spectrum is generally considered as exhibiting only six or seven colors, there are theoretically present an infinite number. According to Koenig we can distinguish about 160 different hues in the spectrum, while according to the same author the eye can distinguish about 600 different degrees of brightness between the threshold and dazzling intensities. However, the number of hues which a person is able to distinguish is strikingly less than these figures would lead us to believe; in fact the number depends somewhat upon the manner in which the experiments are conducted. In Edridge-Green's apparatus, for instance, which is one of the latest we have, the principle involved therein is that of two opaque screens held over a spectrum and slightly separated from each other. One of these screens is then moved until the hue at its edge appears different from that at the edge of the other. Edridge-Green states that he has never met a man who could see more than twenty-nine monochromatic patches in the spectrum. Lord Rayleigh, who could distinguish the difference in hue between the two D lines (λ 5,890 and λ 5,896 Angstroms respectively), could distinguish only seventeen hues on Edridge-Green's apparatus and attributes the small number to the method of testing since he was able

by the use of a color box apparatus similar to that of Maxwell to distinguish many more hues. By the use of spectral apparatus fifty-five distinct spectral hues have been seen. By beginning with papers dyed to represent six spectral colors and by adding various intermediate hues Ridgeway obtained some thirty-six distinct hues. Steindler obtained data on the hue sensibility of twelve subjects and found as a mean of these eyes several maxima and minima in the curve showing the relation between relative hue sensibility and the wave-length. He found maxima at 4,550, 5,340 and 6,210 Angstroms and minima at 4,440, 4,920, 5,810 and 6,350 Angstroms. Calculations from this work as made by Nutting show that there are twenty-two of these colors "just easily perceptible" within the limits given.

218. The data and information which we have on the sensibility of the eye to *changes in saturation* are not very extensive or satisfactory. H. Aubert found that two to three degrees was the smallest sector of color that could be made just noticeable on rotating a white disc; with black and gray discs somewhat smaller sectors were recognized, showing in every case less than one per cent. Experiments on the differential limenal values of color sensitivity showed that on a black background the stimulus-increments for orange, blue and red were 0.95, 1.54 and 1.67 per cent., respectively, in order to produce a noticeable increase in saturation. A few years ago Geissler extended our knowledge along these lines; his experiments involved seven different degrees of saturation ranging from 360° of red to 110° of red plus 250° of gray of the same brightness. His results indicate that the stimulus-increments corresponding to just noticeable saturation differences are constant at about 4 degrees of gray within wide differences of stages of saturation; as an average of several observers it was found that 1.2 degrees of red when mixed with 358.8 degrees of gray caused a just perceptible appearance of color. Other experiments were carried out with red, yellow, green and blue colored papers and their corresponding grays both for each eye separately and for binocular vision. In general the averages for binocular vision were lower than for monocular vision. The results *in toto* when averaged for each of these colors gave as the mean limenal values of color saturation 2.23 degrees for red, 5.81 degrees for yellow, 7.19 degrees for green and 2.99 degrees for blue. This says that these values represent the smallest increments required to distinguish between color and no color. Experiments with a practically color-blind subject showed that his limenal values were high, being 37, 18, 140 and 8.25 degrees respectively for red, yellow, green and blue papers. This problem of saturation sensibility has been attacked from two extremes; one con-

sists in reducing a maximum saturated pigment color and the other of introducing more and more color into a colorless stimulus. Geissler employed the rotating double color disc with the Zimmerman colored and gray papers illuminated with an artificial daylight devised by Ives and Luckiesh. "In the first method he used red beginning maximum saturation, i. e., 360° of red, for both the inner and outer concentric components of the double disc and gradually added small amounts of gray, of the same brightness as the red as measured with a flicker photometer, to the inner or smaller disc until it appeared just perceptibly less saturated than the outer or larger disc. This procedure was then reversed, the outer disc being decreased in saturation until the change was just perceptible as compared with the inner disc whose saturation was kept constant." (Luckiesh; *Color and Its Applications.*)

219. *Color mixture.* The *equations of color* constitute the fundamental method for the examination of the color-sense. Two or three colors are mixed in different proportions until the observer says that this mixture is similar to a fourth given color, generally white. There are two distinct methods of mixing colors, one is by *addition* and the other by the *subtraction* of light rays. A body appears of a certain color because, as a rule, the chemical substance used in staining it has the property of absorbing certain visible rays and reflecting or transmitting others. The integral color of the light absorbed is said to be complementary to the color of the light remaining if the light was initially white. Any two complementary colors can be made to overlap and produce white by means of a simple apparatus in which a spectrum of sunlight is produced and two selected spectral regions can be deviated by means of prisms of small angles, or by means of adjustable mirrors, and combined into one spot by the aid of lenses; if they are complementary they will produce white. The *subtractive* primary colors have been termed red, yellow and blue; in reality they should be more exactly expressed as purple, yellow and blue-green. If the three subtractive primaries are superposed, black will result. If yellow and purple are superposed, red will result. The explanation of this last statement, which also serves as a basis for a superposition of any two or all three of the primaries, is that the blue of the purple is subtracted by the yellow, since yellow does not transmit blue rays, and as purple consists of red and blue rays only, the red rays remain to be reflected to the eye. In the case of the superposition of the three primaries we see that, where the yellow and purple overlap, red results; the blue-green disc, however, does not transmit red rays, hence total extinction of color results. The additive method has as

its primaries red, green and blue (or violet as some people prefer to call it). This method always tends toward the production of white, whereas the subtractive method tends toward the production of black. When red is added to green, yellow is produced and when blue is added to this combination white results. A table of complementary hues and wave-length complements is given below.

Color	*Color Complement*
Red	Blue-green (cyan blue)
Orange-red	Green-blue (bluish cyan)
Orange	Blue
Yellow	Blue-violet
Yellow-green	Violet-purple
Green	Purple (magenta).

Wave-length of Complementary Spectral Hues in Angstroms

6,562	4,921
6,077	4,897
5,853	4,854
5,739	4,821
5,671	4,645
5,644	4,618
5,636	4,330.

220. Very simple apparatus will permit of fairly accurate study and demonstration of these elementary color effects. Maxwell's discs offer a ready means of mixing colors. Colored papers cut in circles and slit along one of the radii can be overlapped to any degree and by the use of circles of various sizes a number of mixtures can be produced upon the same disc, when it is set into motion at a sufficiently high speed, by virtue of the phenomenon of persistence of vision. Lambert's device is a simple contrivance for color mixing; it consists of a glass plate set practically in a vertical position at some little distance from two colored objects lying on a table and at opposite sides of the plate; the glass plate transmits rays from one object while it reflects from one of its faces light from the second body: the eye receiving both stimuli experiences the resultant color sensation. And again, by looking at two colors, placed side by side, through a double refracting prism, they will be seen separated by a strip the coloration of which will be that of the mixture. Painters frequently use mixtures of coloring matter but the colors which are obtained are often not in

accord with those which are obtained by other methods. A mixture of yellow and blue pigments gives green, while with a revolving disc there is obtained a grayish-white. This is explicable on the basis that in a mixture of yellow and blue pigments the superficial molecules send back yellow and blue light; together these produce the impression of white as on the revolving disc. The blue molecules situated deeper in the layer also send back blue light just the same as the superficial layers but it is not pure for the spectroscope shows that it contains green, blue and violet. The deeper seated molecules of yellow in turn return red, yellow and green rays. Generally the molecules send back only rays of the colors which they allow to pass. Hence only green rays, reflected by the deeper yellow molecules, can pass through the superficial blue molecules and likewise the green rays reflected by the deeper blue molecules can pass through the superficial yellow ones; the result is, therefore, a green colored paint or pigment, this green being mixed with the white light reflected by the surface.

221. *Newton's color table.* The "king of physicists" devised a table to give a graphical representation of the results obtained by mixing colors. This table is shown in Fig. 129. Suppose, for example, that we desire to know the result of mixing three parts of green with two parts of blue and one of red. The red and green are first joined in the diagram by a straight line which is divided into segments at the point p such that the distance of this point from the green may be one-third of its distance from the red. The point p is then the place or position of the mixture of the red and green. This point p is then joined to the blue by a second straight line which is so divided at a point q that the distance pq is to the distance qb (where b represents the blue point on the color diagram) in the ratio of 2 to 4; q is, therefore, the point of mixture of the three colors. Drawing, then, the line oq and continuing it until it intersects the spectral curve we find that the color of the mixture is bluish-green diluted, of course, with white since some of the red, green and blue will combine to form white.

The form of this color curve of Newton's is, up to a certain point, arbitrary: there is the necessity of considering the quantity of the colors. In Newton's scheme, therefore, one must consider as equal the quantities of two complementary colors which, when mixed, give white: furthermore, if we take any other two complementary colors, one must also consider as equal the quantities of these colors when, upon mixture, they give a white of the same brightness as the former mixture of complementaries. Maxwell and Helmholtz used other definitions. The table of Newton also shows that, excepting purple, one cannot produce new colors by mixing spectral colors for it is always possible, after having found the position of the point representing the mixture, to draw a straight line passing through this point and the center and, by prolonging this line to meet the spectral curve, to thus find the color of the mixture diluted with white. Newton's table also shows that one can reproduce all existing hues by mixing, two by two, three colors properly chosen. Referring again to Fig. 129, let red, green and blue be selected and let them be connected by straight lines. If any spectral color is selected, it can be joined to the center of the circle by a straight line which must of necessity cut one of the sides of the blue-green-red triangle. At the point of intersection is found the mixture which is similar in hue to the spectral

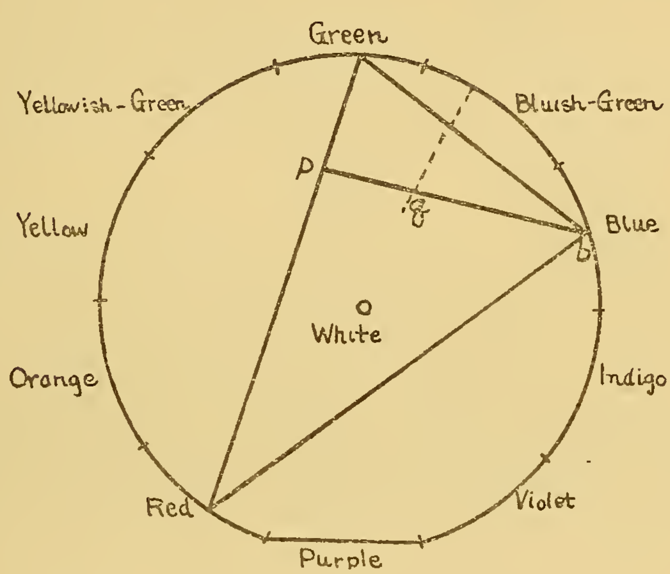


Fig. 129.—Table of Colors. (After Newton.)

color. Because of this peculiarity the normal eye is called trichromatic. It is to be observed that the two colors are said to be alike as to *hue* but they are not generally alike as to *purity,* the color of the mixture being diluted with white. There is a further requirement in this table and that is that the spectral colors must always have a greater purity, for if it were possible to reproduce a third color exactly by mixing two spectral hues these three colors would have to be placed in a straight line and the spectral curve would not be circular. This condition is not, however, fulfilled as we shall see from the work of Maxwell.

222. *Maxwell's color table.* Newton's work was verified by Maxwell; but the latter found that the spectral colors cannot be arranged on a circle because there are portions of the spectrum the colors of which can be exactly reproduced by the mixture of two given colors and which, therefore, must be placed on a straight line. Fig. 130

shows Maxwell's spectral curve. This curve was determined experimentally by Maxwell while Newton's was largely a mental conception. The apparatus of Maxwell consisted of a box, a sectional diagram of

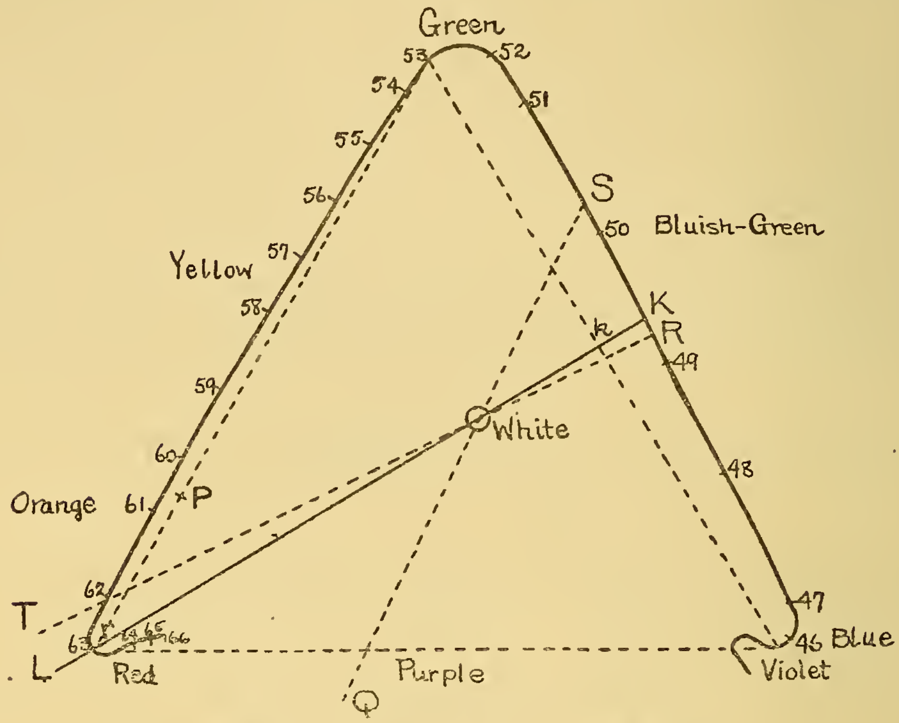


Fig. 130.—Color Table of Maxwell.

which is shown in Fig. 131. At E there is a narrow slit through which passes light. This light is, in turn, reflected by the mirror e to the prisms P_1 and P_2 through which it passes to the concave mirror S.

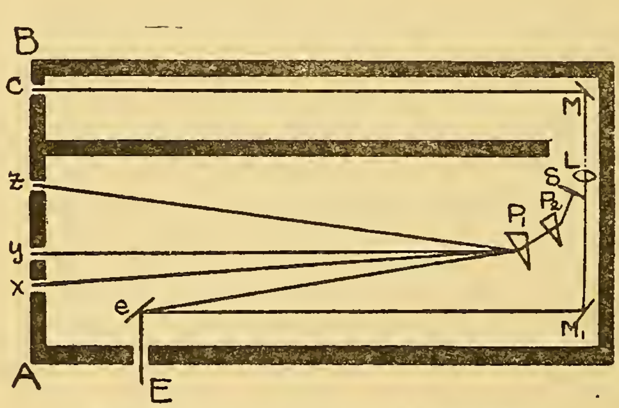


Fig. 131.—Color Box of Maxwell.

The mirror S reflects the light back through the prisms and there is formed a spectrum along the side of the box AB. At this end of the apparatus there are also three movable slits x, y and z which permit of the selection of any portions of the spectral colors which may be desired. This device is optically reversible provided that these slits are illuminated by the same spectral regions as would have, in turn, been found at these positions when the light was sent in at E. Hence it is possible to illuminate the three slits x, y and z by white light; an eye at E then sees the prism P colored by a mixture of the three colors which a similar source placed at E would have projected onto the slits. Through the slit C white light can be allowed to enter and after reflection by the mirror M, concentration by the lens L and a second reflection from a ground-glass plate M_1 blackened at the rear surface, an eye at E will see this plate at the side of the prism and can thus compare brightness and color of the mixture with that of the white light admitted through C. By proper adjustment of the sizes and positions of the slits there can be obtained a spectral mixture which is not distinguishable from the white light reflected by M_1 either as to brightness or color.

223. In the determination of his color table Maxwell selected the three following colors as standards:—

	Red (R.)	Green (G.)	Blue (Bl.)
Wave-length (Angstroms)....	6,300	5,280	4,570

He then gave these radiations access to the three slits respectively of his color box and by regulating the widths of the slits he produced a mixture which did not differ either in hue or brightness from the white introduced through the second lens-mirror system (CMM_1E of Fig. 131) just described. He measured the widths of the slits as $x = 2.36$ mms., $y = 3.99$ mms. and $z = 3.87$ mms. and by designating the white, which remained constant throughout his experiments, by W he wrote as his color equation

$$2.36 \text{ R.} + 3.99 \text{ G.} + 3.87 \text{ Bl.} = \text{W.}$$

By displacing the slit x so as to give access to orange light and by regulating the widths of the slits he obtained as another color equation

$$2.04 \text{ Or.} + 3.25 \text{ G.} + 3.88 \text{ Bl.} = \text{W.}$$

Since the white is equal in both equations, we have, therefore, an equality of equations and a little arithmetic shows that

$$1 \text{ Or.} = 1.155 \text{ R.} + 0.362 \text{ G.} - 0.006 \text{ Bl.}$$

A repetition of the measurements for other colors, always combining two of the standard colors with the color in question to give white, was made by Maxwell and demonstrated that all colors of the spectrum can be expressed in terms of three primaries. The accompanying table gives the results of these measurements.

Color	Quantity	Wave-length		Red	Green	Blue	Sum	Unity
Red	5.63	(6630)	=	2.36	+ 0.05	+ 0.36	2.77	2.032
	2.36	(6300)	=	2.36	+ 0.00	+ 0.00	2.36	1.000
Orange	2.04	(6060)	=	2.36	+ 0.74	— 0.01	3.09	0.662
Yellow	2.79	(5830)	=	2.36	+ 2.45	— 0.01	4.80	0.582
	3.20	(5620)	=	1.55	+ 3.99	— 0.01	5.43	0.589
	3.30	(5440)	=	0.42	+ 3.99	— 0.03	4.38	0.754
Green	3.99	(5280)	=	0.00	+ 3.99	+ 0.00	3.99	1.000
	5.26	(5130)	=	— 0.33	+ 3.99	+ 0.44	4.10	1.282
	7.87	(5000)	=	— 0.43	+ 3.99	+ 2.22	5.77	1.362
Blue	7.83	(4880)	=	— 0.39	+ 2.67	+ 3.87	6.15	1.275
	5.14	(4770)	=	— 0.24	+ 0.98	+ 3.87	4.61	1.116
	4.28	(4670)	=	— 0.14	+ 0.14	+ 3.87	3.87	1.105
Indigo	3.87	(4570)	=	0.00	+ 0.00	+ 3.87	3.87	1.000
	4.10	(4490)	=	0.08	+ 0.03	+ 3.87	3.98	1.032
	5.59	(4410)	=	0.14	+ 0.09	+ 3.87	4.10	1.362
Violet	8.09	(4340)	=	0.04	— 0.23	+ 3.87	3.68	2.197

224. By dividing each equation by the coefficient of the color on the left in each of the above expressions we can obtain the value cor-

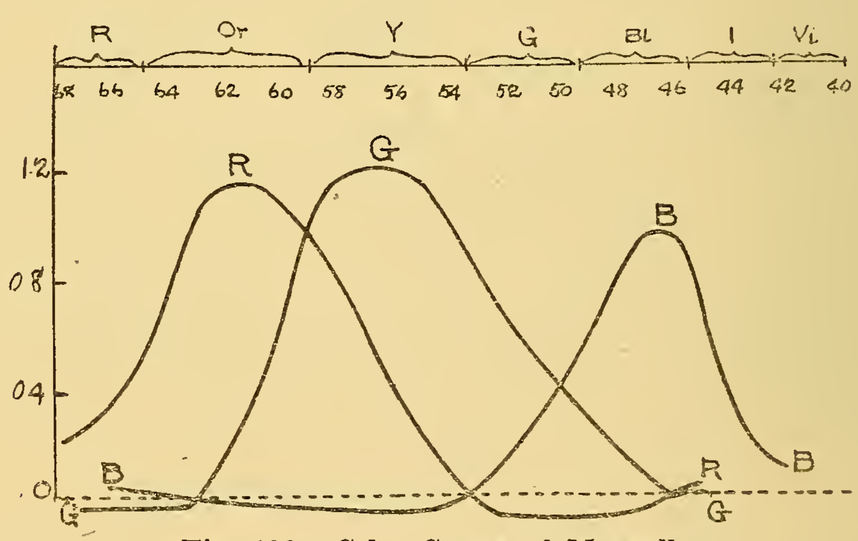


Fig. 132.—Color Curves of Maxwell.

responding to a slit width of one millimeter. The results can be expressed in the form of three curves designated as *R*, *G* and *B* in Fig. 132 corresponding to the three standard colors. The numerals underneath are the wave-lengths of the different colors of the spectrum and the positions of the three points in which the curves cut the vertical axis indicate the quantities of the three standard colors needed to produce the mixture. The significance of the negative signs attached to various colors, in general to the blue or red, is readily grasped. By writing the equation of the orange as

$$2.79\ \text{Y.} + 0.01\ \text{Bl.} = 2.36\ \text{R.} + 2.45\ \text{G.}$$

we see that we cannot, with the three standard colors, produce a mixture exactly like yellow, but must add a little blue to the yellow so that it may be like the mixture of red and green.

If one desires to use the foregoing color table to solve equations of color mixtures one must multiply the quantities found in the table under the heading "quantity" by the figures indicated in the column called "unity" in order to obtain a result expressed by the width of the slit in millimeters. These units are obtained by dividing the numbers expressing the quantity or coefficient of any color by the sum of the component colors. The standard colors have, necessarily, a unit ratio between the coefficient of the specified color and the sum of the component colors; but for all other colors we are obliged to select the units in a different manner. The sum of the three components for *green* is, from the table,

$$2.36 + 2.45 - 0.01 = 4.80,$$

while the width of the slit is 2.79 mms.: accordingly, then, the quantity of yellow passing through the slit of 2.36 mms. is 3.09, hence the unit of yellow corresponds in this case to a slit width of 0.582 mm.

225. To construct the spectral curve we draw initially an equilateral triangle as shown in Fig. 130 and place the three standard colors at the corners thereof. To find the position of the orange we commence by dividing, by virtue of the color equation for orange, the red-green side into two parts in the ratio of 0.74 green to 2.36 red. Let P be the point of division; this point is to be joined to the vertex of the angle of the blue by a straight line of which the length, l, is measured. The color at P is due either to a mixture of 2.36 R. + 0.74 G. or to a mixture of 3.09 Or. + 0.01 Bl. Hence the color in question must be placed on the prolongation of "l" slightly beyond the point P by an amount $\frac{0.01}{3.09}$ l. In the case in hand this amount is so small that the curve almost coincides with the dotted side of the triangle. However, a survey of the color equations for blue will show why the color curves lie outside the standard color triangle. A color which is situated in the interior of the triangle may be reproduced exactly by a mixture of the three standard colors. For a color on the outside, however, this is impossible and it is necessary to mix it with one of the standard colors in order that it may appear equal to the mixture of the other two. As nearly all of the spectral colors have one of the coefficients negative, practically the entire curve lies outside of the standard color triangle. This means that the mixture produced has a little less

purity than the spectral color. By means of the table of Maxwell we can construct the result of the mixture of any colors; if the colors on the same side of the triangular curve are mixed the resultant will have as much purity as the spectral colors but if two colors situated each on a different side are mixed the mixture will be strongly diluted with white.

226. Lastly, the table indicates a large number of pairs of complementary colors; that is, of colors which, mixed two by two in the proper proportions, give white. To find the color complementary to a given color we have only to prolong the line which joins it to the white until it meets the curve again. The point of intersection will be the complementary color and the quantities to be taken of each color are inversely proportional to their distances from white. An inspection of the table shows that the green colors from 5,700 to 4,950 Angstroms have no complementary colors in the spectrum; their complementaries are purples.

227. *Color-blindness.* It is said that about four per cent. of men are afflicted or affected with the form of dyschromatopsia known as Daltonism. There occurs in the spectrum for such persons a region which resembles white (gray) and which is commonly designated as the neutral point. For the Daltonists this neutral point occurs in the green-blue; they, therefore, see only two colors, one of which is usually called yellow and which fills the entire spectrum from the neutral point to the red extremity, and the other, which is designated as blue, extending from the neutral point to the violet end. The hue does not change in either of these two regions respectively; there are differences of purity and brightness only. The color called yellow includes the normal red, orange, yellow and green up to about 5,300 to 5,400 Angstroms. There are in this region differences of brightness only. The red and orange of the spectrum are often so feeble to such color-blind persons that they are not perceived as being present unless the spectrum is very clear. Proceeding, then, from 5,400 Angstroms the color becomes more and more grayish until, as it reaches the neutral point at 5,000 Angstroms, the color is apparently whitish-gray. The brightness also diminishes; the parts situated near the neutral point are usually darker than those situated at some distance from it. This may be due to the fact that this neutral point occurs in the green-blue region where the rays are most affected by the influence of absorption due to the yellow pigment of the macula. After the neutral region has been passed, another color designated as blue makes itself apparent and gains in purity until the 4,600 Angstrom point is reached when the brightness and purity become a maximum. From

this point on there are differences of brightness only. Dichromates see, therefore, only two colors, but it is difficult to tell which they are. Dalton's investigations convinced him that he had only two and at the most three color sensations which he called yellow, blue and perhaps purple; his blue and purple coincided with those of normal color-visioned people. He says that "The part of the image which others call red appears to me little more than a shade or defect of light; after that the orange, yellow and green seem one color which descends pretty uniformly from an intense and a rare yellow, making what I should call different shades of yellow. The difference between the green part and the blue part is very striking to my eye; they seem to be strongly contrasted. That between the blue and the purple much less so. The purple appears to be blue, much darkened and condensed." If, then, we designate the colors as blue and yellow, it is not a surety that these spectral colors give them the same impressions as those which we obtain by yellow and blue.

Ophthalmic literature contains the interesting case of a person whose color-vision was normal in one eye while the other eye exhibited an anomaly analogous to ordinary Daltonism. The case was investigated by Hippel, who found that the neutral point, which was situated at 5,120 Angstroms, divided the spectrum into a yellow and a blue portion. The red and green appeared of the same hue as the yellow but were less bright. From a comparison of the sodium yellow line as seen by each of these eyes it was reported that the appearance was the same for both eyes except that there was a slight diminution of brightness for the dichromic eye. The same was found true in the case of the blues, hence it may be concluded that the sensations which are designated by Daltonists as yellow and blue are identical with those of normal persons.

228. Color-blind persons recognize the equation of the normal eye, hence the colors which are complementary in one case are also complementary in the other condition. Of course the complement to the neutral region must appear to them as either gray or be totally invisible, as well as the colors situated on the diameter of the table which joins them. But, on the other hand, color-blind persons recognize as similar mixtures which are by no means such for normal eyes. The impression of any color of the spectrum can be reproduced for a Daltonist by mixtures of two colors; this is true also of white. Maxwell, by using green and blue, obtained as the color equation for white in a case of dichromasia the following:

$$4.28 \text{ G.} + 4.20 \text{ Bl.} = \text{W.}$$

The position of this mixture color is represented on the Maxwell table given in Fig. 130 by the letter *k;* the letter *K* indicates the corresponding spectral color which is the neutral point. Since the Daltonist recognizes the equation of normal eyes, which is according to Maxwell,

$$2.36\ \text{R.} + 3.99\ \text{G.} + 3.87\ \text{Bl.} = \text{W.}$$

we can equate these two expressions and obtain

$$2.36\ \text{R.} - 0.29\ \text{G.} - 0.33\ \text{Bl.} = 0.$$

This last equation would not, therefore, represent any impression on the dichromatic eye but would represent in a certain way the element which is lacking. This place is marked by the letter *L* in Fig. 130. Since *L* is slightly outside of the spectral curve it represents a color which does not exist and is, therefore, fictitious but which must be

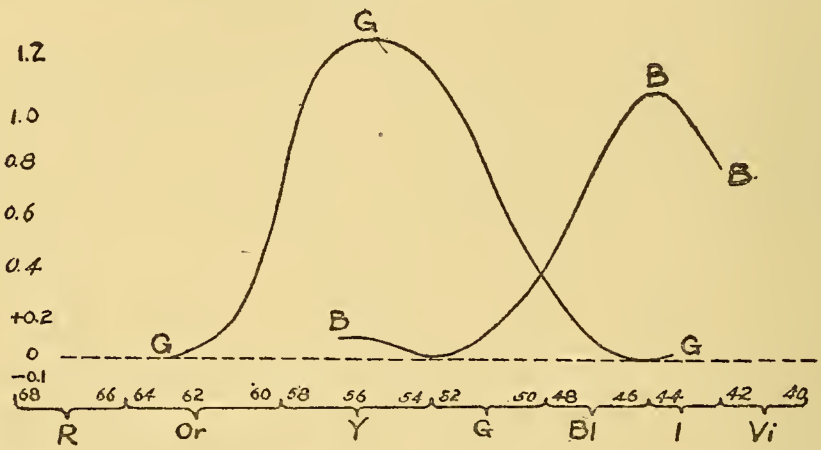


Fig. 133.—Color Curves of a Dichromatic. (After Maxwell.)

supposed to be much purer than the corresponding spectral color on the equilateral triangle which is marked as *l.* Compared with *L, l* is to be considered as a mixture of white; it is not wholly invisible but is very feeble.

229. The results shown in Fig. 133 are the color curves of a dichromatic, after Maxwell. On the table of colors the whole chromatic system would be reduced to a straight line since all the colors which we can produce by mixing two given colors must be placed on the straight line which joins them. An examination of a number of dichromatics shows that the neutral position is not exactly the same in all; it varies between 4,920 and 5,020 Angstroms. In Fig. 130 these two limits are marked as *R* and *S,* consequently the direction of the neutral diameter would vary between *RT* and *SQ.* Therefore, there results a difference between dichromatics whose neutral point is situated nearer *R* or nearer *S.* In the first case the neutral diameter

passes through the bluish-green at one extremity and the reddish-orange or orange at the other end; the spectrum appears shortened in the red end. In the second class the neutral point corresponds to a color situated near the green the complementary of which is purple and not found in the spectrum. As the colors complementary to the gray parts of the spectrum do not correspond to the red end, the red will therefore preserve its ordinary intensity and the spectrum will not be shortened. These two forms are often differentiated, the former being designated as *anerythropsia* or red-blindness and the latter as *achloropsia* or green-blindness. This distinction has been followed by a large number of scientists; there seems to be a reasonable objection to this differentiation on the basis that the neutral diameters, which have been represented by *SQ* and *RT* in Fig. 130, do not represent the only two possibilities since other intermediary forms appear to exist.

230. We have discussed dichromasia because it is the most pronounced or the most frequently found of all color disturbances or abnormalities. But there exist also abnormal *trichromasia* and *monochromasia*. Monochromatic eyes manifest all signs of weakness such as photophobia and diminution of visual acuity: color-blindness, on the other hand, implies no other abnormality. In *monochromasia* differences of color do not exist and the only variations such subjects experience are those of brightness differences. The spectrum appears to them simply as a luminous band of which the maximum brightness appears in the green (5,200 Angstroms) rather than in the yellow as in the normal eye. This abnormality is rare but extremely well established when it does occur. There is, furthermore, a class of eyes, discovered in 1880 by Lord Rayleigh, for which the generally accepted assertion cannot be made that an equation of color which is true for a normal eye remains true for all eyes, as well for dichromatic as for normal color-visioned eyes. Rayleigh produced a mixture of spectral red and spectral yellow which appeared to him identical with spectral yellow and had various observers compare the two fields. For the majority of persons tested the two hues were identical but others declared they saw no resemblance, for the pure color appeared yellow to them while the compound color appeared red. To make this "mixture" color appear like the pure spectral yellow there had to be added a considerable amount of green; in fact so much of this latter color was demanded that the resulting color appeared greenish to the normal eye. The mixture for Rayleigh was 3.13 R. + 1.00 G. while for a person possessing the above form of *abnormal trichromasia* the color equation for yellow was 1.5 R. + 1.0 G. No other abnormali-

ties were discovered in such persons; they were not in any sense of the word dichromatics. This anomaly appears to be as frequent as Daltonism; Koenig and Dieterici discovered three cases of it out of seventy persons examined.

There are other slight differences which occur in the color tables of normal eyes and are doubtless due to the fact that portions of the rays are absorbed by the media of the eye; this absorption is more pronounced in some persons than in others. Thus the crystalline, as it becomes in general slightly yellowish in advancing years, absorbs some of the blue rays. Hence a mixture of blue and yellow which would appear white to the normal eye, must appear slightly yellowish to the older eye. After cataract extraction the patient quite often, at the first moment, sees everything blue.

231. *Topography of color fields.* We have evidence in our own eyes that the color-sense has been evolved. A very simple experiment carried out by the reader will convince him of the facts that the sensation of light exists quite regardless of color and that, in turn, the two do exist together. Let the experimenter put a green button on a sheet of black paper in a well lighted room. Standing some feet away let him close one eye and let the green button be observed in the ordinary manner. The image will fall on the center of the retina, the fovea centralis. Then let the head and eye be turned together so that the image of the green button will fall on a portion of the periphery of the retina. At a certain distance from the axis the green spot will appear white, hence the object will be seen but no notion of its hue would be forthcoming unless the image had been initially received on the center of the retina. It is, of course, to be noted that the brightness of the color and the size of the spot cause variations in the angle at which the color disappears. If the brightness be feeble and the angle which the colored disc subtends on the retina be very small, a shift of the axis of the eye by a very few degrees will suffice to render the spot colorless. This simple experiment is worthy of consideration as it shows that the retina is most sensitive to color in the region which the axis of the eye cuts and that there is a gradual diminution in sensitiveness to color though not necessarily to light as the periphery is approached. This is what would be expected if the eye has followed the laws of evolution. Every individual, therefore, is color-blind though not light blind in the outer retinal regions. The most difficult color (exclusive of white, which should be considered a combination color) to cause to disappear is the blue.

232. These color-blind conditions in the peripheral regions are of considerable interest to the physicist, physiologist and the psy-

chologist; to the ophthalmologist they are of particular interest only when contracted or abnormal color fields are found, for these are an aid in the diagnosis of disease. We shall confine our attention here to the normal eye when pure spectral colors are used, the eye being dark adapted. Sir William Abney describes in his book on *Researches on Color Vision* two forms of special apparatus for plotting the color fields. The first is a *perimeter* of ordinary form but modified for use in a dark room. The perimeter is an instrument consisting essentially of a semi-circular arc, graduated into 5°—10° portions, which can be rotated about an axis piercing the center of the metal arc. The diameter of the arc is usually about 18 to 24 inches. Abney modified this for use with spectrum colors "by fastening a mirror to a ball-and-socket joint placed just below the center of the sphere, that is, the position occupied by the eye. By means of an arm the mirror can reflect along the arc any beam of light falling upon it. The light reflected was so arranged that a circular spot of any desired color could be caused to travel along the arc (which was covered with white) when it occupied any angle with the vertical. The distance of the arc was so arranged that the image of the first surface of the first prism was in focus on it, and the spot was formed by placing a diaphragm against the prism. The intensity of the color could be altered (1) by closing or opening the slit through which the colored ray issued; (2) by placing a graduated annulus in front of the slit; (3) by closing the slit of the collimator and (4) by using sectors in front of either slit." The mode of operation was to cover one eye and keep the other eye directed at the center of the semi-circle marked by a pin point of Balmain's luminous paint. A spot of colored light was caused to travel along the white arc; when the color of the light was judged to have gone the reading of the arc was taken.

233. Making use of this form of apparatus or one of a similar nature the question of the similarity of fields for different colors can be investigated. It is essential to know whether the fields for each color are of the same form when the illumination is adjusted so that one point in a field of one color coincides with one point in the field of a different color. Two sets of experiments were made by Abney using intensities of 4.5 and 0.23 amyl-acetate lamps respectively. The results are shown in Figs. 134 and 135. The diagrams show that the fields for properly selected luminosities are evidently the same, the fields of the yellow sodium and the red lithium being very close to one another. A comparison of the fields for the yellow sodium and red lithium rays in the second of these diagrams with the green (wave-

length 5,085 Angstroms) in the first diagram shows that they are practically identical.

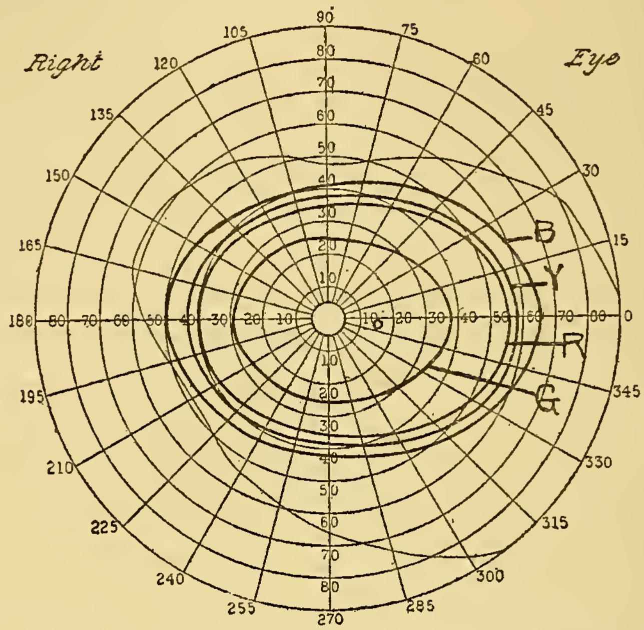


Fig. 134.—Investigations on Similarity of Fields for Different Colors. (Abney.) The intensity was 4.5 units.

An investigation as to the differences in the extent of field caused by differences in illumination was carried out by Abney in horizontal

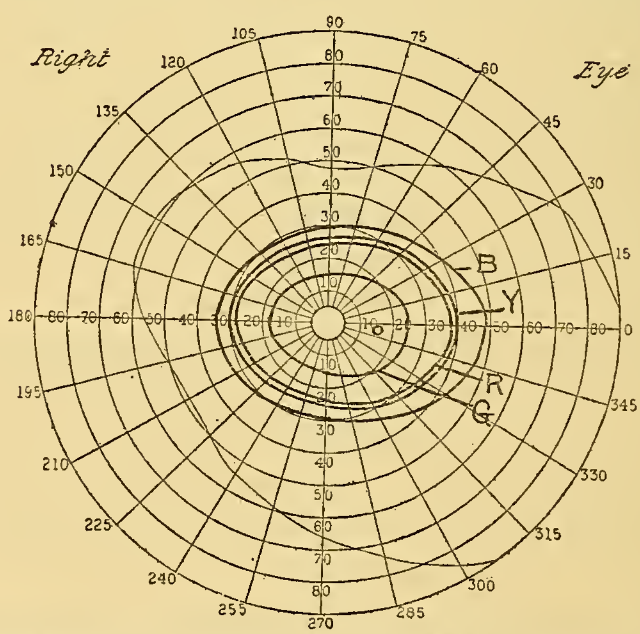


Fig. 135.—Investigations on Similarity of Fields for Different Colors. (Abney.) The intensity was 0.23 unit.

directions only. These experiments showed that the average diminution in field for each reduction of half intensity was 3.75° on the temporal side and on the nasal side about 3°. Curves plotted from his data, using the intensities of the illumination as ordinates and the degrees to the nasal or temporal side of the direct fixation line as abscissæ, show that there is a linear relation existing between the limits of the fields for all colors and luminosities; there is apparently a diminution in the angle of field in an arithmetic progression as the intensity diminishes in geometrical progression.

234. Other sets of experiments were carried out to ascertain the extent of the color fields for all colors when a slit was passed unaltered through the spectrum. When the curves are plotted from the data thus obtained and the distance apart of the nasal from the temporal ordinates is given it will be found that when the latter reads 40°, for example, the former reads 30° no matter what the color may be, and that when the field increases about 7.5° on the temporal side the field on the nasal side increases nearly 6°.

It is known that the loss of light in the center of the retina depends quite largely upon the size of the spot of light viewed. This indicates that the boundaries of a field would contract if the spot of light viewed is diminished. Experimentation has demonstrated that, between apertures subtending 4° 28′ and 10′, the fields decrease in extent and that there is a linear relationship existing between the field in degrees and the diameter of the aperture. For each diminution in aperture to one-half diameter the diminution in field on the temporal side is 5° and on the nasal side 4°.

235. *Growth and decay of color sensations.* The problem of the growth and decay of color sensations as dependent upon the effect of time of exposure and intensity of the stimuli has been the subject of numerous investigations on the part of Bloch, Charpentier, Sulzer, Broca, Ferry, Porter and others. Colors are often produced due to stimuli which have no single color as commonly understood; that is to say, if a disc composed of black and white be rotated at the proper rate—which is moderately slow—colors appear upon the edges of the sectors instead of gray. Fechner in 1838 was probably the first to describe these subjective colors; many have studied this problem and agree as to the experimental results but not in their explanations. In 1894 Benham produced a disc somewhat different from those of preceding investigators; one form of disc which, when rotated, shows the colors in striking manner consists of two half circles of black and white, the white sector carrying arcs of black laid down in a step-like arrangement, each set of arcs having a shorter radius and all having their common center at the center of the disc. When this apparatus is rotated the colors are seen in a very striking manner. In general, when black is followed by white at a moderate speed a sensation of red

is produced, but if white is followed by black there results a blue sensation. By the introduction of various angular intervals, as is done in the Benham apparatus, the sensations of intermediate colors are experienced. By rotation of the disc in one direction a blue sensation is aroused in the inner ring and red in the outer; if the rotation of the disc is reversed the colors are also reversed in their order. The phenomena have not been explained satisfactorily: no doubt retinal inertia and the difference in the rates of growth and decay of the color sensations are important factors in these subjective colors. These colored effects are often observed when the eye is run rapidly over black and white surfaces in the field of vision. A simple device for showing these Fechner colors is one containing black and white sectors; on rotating such a disc at a certain speed it will have the appearance of a greenish hue but at a more rapid rate of motion it appears reddish. Rood employed an opaque disc with four open sectors each of seven degrees; through this rotating disc he viewed a clouded sky; with a rate of nine revolutions per second the sky appeared of a deep crimson hue except for a small space in the center of the visual field which remained constantly yellow, due probably to selective absorption in the "yellow spot" of the retina. When the rate of motion was eleven and a half revolutions per second the field appeared bluish-green. Finnigan and Moore made the lines on a disc a centimeter wide and found that on rotation the band following the black was bordered with a red over the black and on that which came from white to black the band was bordered on the white with a blue to green color leaving the band quite black. Bidwell's explanations as to these effects appear to be borne out by these experiments; his explanation, in essentials, is that the red color of the fine lines following the black are due to sympathetic spreading of the red sensations whilst the blue color of the fine lines following the white is due to the lack of such sympathetic action when the illumination is suddenly shut off, leaving the other sensations exhibited on the black surface on which these lines are practically viewed and which the retina takes as part of the lines.

When a flash of white light is received on the retina there are what are known as *positive recurrent optic images.* These appear to have been first accidentally discovered by Professor W. Young while experimenting with an electrical machine; he noticed that after a strong spark had illuminated any object it was seen at least twice, the second time about a quarter of a second after the first. Sometimes it was seen a third time or even a fourth. This is known as *recurrent vision.* When an object is illuminated by a discharge from a static electrical machine carrying a condenser and the eye is screened from the discharge, Bidwell found that under favorable conditions six or seven recurrent images could be detected.

236. The recurrent image may be shown by means of a device due to Bidwell. This consists of a disc which can be turned about its center and which carries a small hole drilled near the periphery. The light from a projection lantern can be passed on to a screen through this aperture. When the disc is rotated so that the spot travels round the screen with a slight elongation in the line of travel, if the eyes are kept steadily fixed on the screen, there will be found a faint violet spot traveling behind the white oval separated by an interval of darkness. If the speed of rotation is increased the interval between the two spots will increase. Bidwell repeated his experiments with spectrum colors and found that one color gave no ghost, namely, red. The ghost to every other color is of a violet tinge. The time of rota-

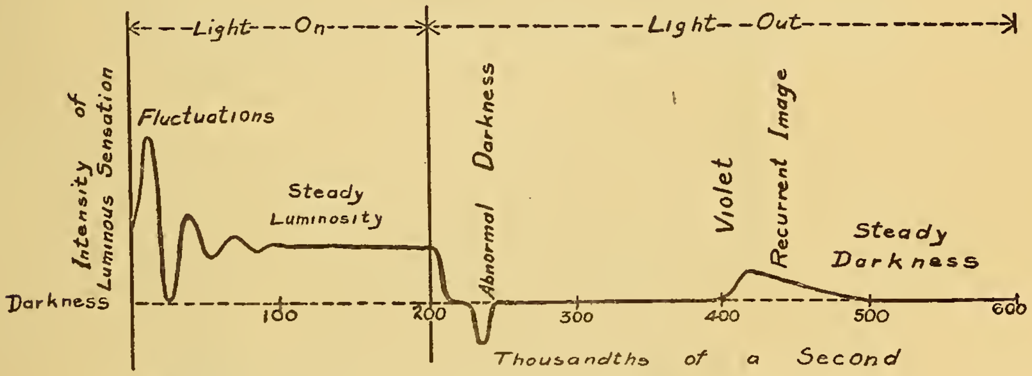


Fig. 136.—Growth and Decay of Luminous Sensations.

tion being known and the interval between the original spot and the ghost being measured, we have a means of calculating the interval that elapses between the first image and that caused by recurrent vision; Bidwell puts this at about one-fifth of a second.

237. Charpentier made many observations on the impressions received on the retina due to light. Charpentier's law can be stated in the following words:—"When darkness is succeeded by light, the stimulus which the retina first receives and which causes the sensation of luminosity is followed by a brief period of insensibility resulting in the sensation of momentary blackness. It appears that the dark period begins about one-sixtieth of a second after the light has first been admitted to the eye and lasts for about an equal time." Charpentier's apparatus for demonstrating and measuring the duration of this effect is simple: it consists of a blackened disc carrying a white sector. When the disc is illuminated by sunlight and turned rather slowly, the gaze being fixed upon the center, there appears upon the

white sector, close to the leading edge, a narrow but conspicuous dark band. The portion of the retina which at any moment is apparently occupied by the dark band is that upon which the light, reflected by the leading edge of the white sector, impinged one-sixtieth of a second before. A graphical representation of some of the results upon growth and decay of luminous sensations, abnormal darkness and recurrent images is given in Fig. 136.

238. The work of Broca and Sulzer is especially comprehensive in the field of investigation on the growth and decay of color sensations. The complete account of their apparatus and methods of experimentation is to be found in the *Journal de Physiologie et de Pathologie*

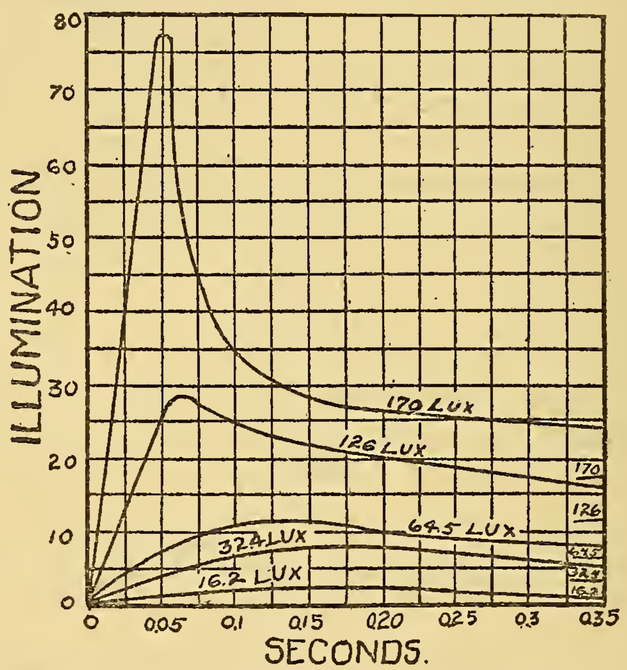
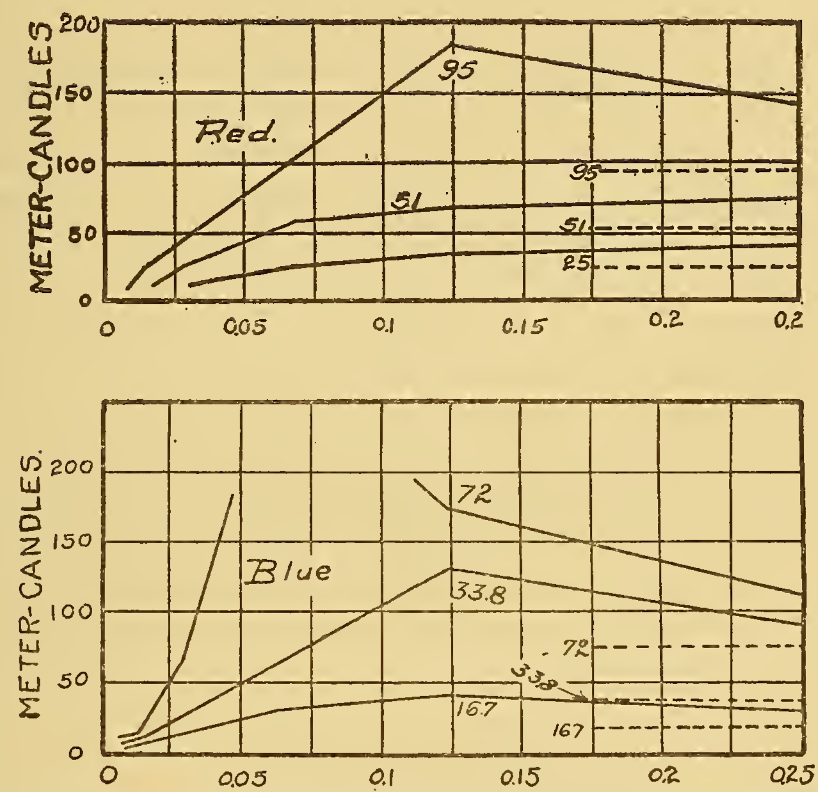


Fig. 137.—The Growth and Decay Curves for White Light Sensation. (After Broca and Sulzer.)

generale, 1902. They compared the brightness of a white screen illuminated by light of short duration produced by a disc, carrying a small opening and driven by an electric motor at speeds which could be definitely determined, with that due to a standard steady light. Some of their results for white, red and green lights are shown in Figs. 137, 138 and 139. These show that, except for lights of low intensity, the luminous sensation overshoots its final value. By this we mean that the maximum luminous sensation is passed a comparatively short time after the beginning of the exposure and that the luminous sensation reaches a steady value which is less than the maximum only after the lapse of an appreciable fraction of a second (of the order of 0.05 to 0.1 second) dependent more or less upon the intensity. The numbers on the curves indicate the final steady values of the stimuli. The data obtained with colored lights indicate that under the stimulation from blue light the luminous sensations overshoot very much more than in the case of red or green lights. The luminous sensation, then, increases at first but soon commences to decay due to fatigue under the higher intensities; these effects are negligibly small at very low intensities. Working with red, green, blue and white lights Broca and Sulzer found that with blue the maximum sensation was at least five times the final and occurred about + 0.07 second after the initial exposure. Red and green overshoot to about double



Figs. 138, 139.—The Growth and Decay Curves of Color Sensations. (Red and Blue.)

the final intensities after about 0.13 second. Green overshoots scarcely at all, indicating either a very slight fatigue or else a very small lag in the fatigue behind the impression.

239. According to Talbot's law, a periodic illumination, such as would pass through a rotating sector, will produce on the eye the same luminous sensation as the mean constant illumination provided the period is below that producing flicker. This law has been quite rigidly proven experimentally for white light by Hyde but no satisfactory theoretical foundation is as yet forthcoming.

Exponential functions of time satisfactorily represent visual impression and fatigue, but we have not sufficient data to determine the constants of these functions. The persistence of vision as determined by critical frequency has been employed by Allen with success in investigating color-blindness.

Luminosities of very short duration are perceptible if intense enough. A lightning flash of a millionth of a second is visible and by rotating mirrors flashes of light of one eight-millionth of a second have been perceived. Blondel and Rey studied the perception of lights of short duration at their range limits. Bloch had previously laid down the law that the excitation necessary to produce minimum sensation was constant and proportional to the product of the brightness and the duration; Charpentier verified this law for luminous durations between 0.00173 and 0.058 second. Blondel and Rey concluded that Bloch's law is applicable only to intense lights of very short duration; they deduced after a considerable amount of experimentation a single law of the form $(B - B_0)\ t = aB_0$ in which B_0 is the minimum perceptible brightness of the field, t the duration of the stimulus in seconds and a is a constant of time equal to 0.21 second.

240. *Effects of environment on the appearance of color.* It is known that the intensity, spectral character and distribution of the illuminating source, the adaptation of the retina for light and color, the duration of the stimulus and the character of the stimulus preceding the one under consideration, the size and position of the retinal image, the surface character of the colored medium and its surroundings all affect the appearance of a given color. The sensitivity of the various retinal zones explains why the size and position of the colored object affect its appearance. It has been found, using squares of one to sixteen square centimeters in area viewed from a distance of a meter, that the larger areas appear more saturated than the smaller ones. This saturating effect is greatest for violet and least for red, the remaining colors being affected as per their order in the color scale. Another phenomenon, presumably connected with the growth of color sensations and with chromatic aberration, is found when one views a red piece of paper on a blue-green background held at a meter or so from the eyes and under moderate illumination. If the paper be moved forward and backward while fixation is kept at a point in the plane in which the card is moved, the red area will appear to shake or oscillate and will not be apparently in the same plane as the blue-green paper.

It has been previously pointed out that the maximum spectral sensibility of a normal eye shifts toward the shorter wave-lengths at low intensities. Hence colors will shift in hue under low luminosities; for instance, a green pigment appears to be more bluish as the illumination is greatly decreased. Colors appear more saturated at low than high intensities of illumination; intense illumination causes colors to appear very much less saturated. The spectral character of the light affects the appearance of the color of an object; a red fabric, for example, appears red because it has the ability to reflect chiefly the red rays, hence such a colored fabric would appear black under a light source possessing no red rays as in the case of a mercury arc. A colored fabric cannot, except in special cases, appear the same under two different illuminants; in other words, the eye is not capable of analyzing a color spectrally and it is, therefore, possible to produce colors which appear the same but whose spectral compositions differ. A purple, for instance, under noon sunlight appears a blue-purple while when illuminated by ordinary artificial light of continuous spectral character it appears a red-purple for the reason that artificial lights are proportionately richer in the longer wave-lengths while the maximum energy regions in natural light are in the green-blue region. The brightness or value of a pigment is also affected by the spectral character of the illuminant. Experiments on a series of Zimmerman papers by means of a reflectometer have been carried out under illumination from an overcast sky and from a tungsten lamp; the results of this work show that papers which have the ability to reflect the rays of light of longer wave-length predominantly appear relatively brighter under artificial light, while colors which reflect the shorter wave-lengths are relatively brighter or have a greater reflection coefficient under daylight illumination. Finally, the distribution of light is of some importance; when the light is so distributed that an appreciable amount of it is specularly reflected into the eye of an observer the color appears less saturated. A striking illustration of the effect of light distribution is found in the case of so-called changeable silks. When light strikes such material in certain directions it is more or less specularly reflected; in other directions the light penetrates the fabric which is then colored by multiple selective reflections. These two colors are roughly complementary.

241. *After-images.* Retinal excitation requires an appreciable time to decay after the stimulus has been removed. If the filament of an incandescent lamp is viewed for an instant and the eyelids are then closed a *positive* image of surprising distinctness will be seen which will persist for some time. If the eyes are kept closed, the less illuminated parts of the image disappear, while the more illuminated parts change color, becoming bluish, violet, orange and so forth in turn. The image finally disappears only to reappear with a repetition of the foregoing series of color changes. The image will then reach a

state of decay when it appears darker than the surroundings, coupled with these changes in color or hue. Helmholtz explained the colored after-images obtained in the above manner by assuming different rates of decay of the three hypothetical color sensations which are the basis of the Young-Helmholtz theory of color-vision. The *negative* after-image is the complement of the original color; if the object we look at is white the negative after-image is black. This after-image is explained as being due to retinal fatigue produced by the original bright image of the white or colored objects. On stimulating the whole retina with white light the portion previously fatigued does not respond in the same degree as the unfatigued portions. The question of after-images is intimately connected with the retinal conditions as influenced by stimulation; after-images are very quickly and pronouncedly obtained when objects are viewed after the eye has been rested in darkness, as when one, for instance, views an object on a wall for a second just after awakening and then fixes his gaze upon the clear wall at some distance from the original object. It is difficult to reconcile all the facts obtained from studies on after-images with the fatigue hypothesis of Helmholtz. Hering proposed an explanation of these phenomena on the basis that the retina is not fatigued but that a metabolic change is aroused which is opposite in character to that produced by the original stimulation. After-images are, of course, produced by fixing colored objects; they usually appear approximately complementary in hue to the original stimulus. If a colored triangle of red, green and blue is viewed, for instance, approximately complementary colors will be seen.

After-images play an important part in vision, especially in viewing paintings and many other colored objects. If a blue sky-line is viewed in a painting in juxtaposition to a green landscape, there will be sufficient shifting of the eye, even though it is fairly definitely fixed, to cause a shifting of the dividing line upon the retina. There will result, then, a pinkish after-image due to the green as well as to the blue which will, in shifting above and below the dividing or horizontal line give an effect of vividness or "life" to the picture. As a general rule the color appears to become less saturated and often a change of hue results after steady fixation upon a colored object. If a background of red, carrying a patch of black, is fixed for a few seconds and then, without changing the fixation, the black patch is removed it will be found that the red occupying this spot will be more luminous than its surroundings and of a more saturated reddish appearance.

The longer the time of fixation the longer does the negative after-image exist. Purkinje states that there is an exact proportion; each additional second of fixation increasing the duration of the after-image some twenty seconds. Aubert found that when the sun was regarded for three seconds the after-image persisted about forty seconds, while if the time of gaze was five seconds the image lasted about five minutes. The brighter the object the longer will be the duration of the image.

242. *Successive contrast* further complicates the appearance of colors. If the retina is stimulated with red and the eye is suddenly fixed upon a green color this latter will appear more intense or saturated in color than if the previous stimulation under red had not taken place.

243. *Simultaneous contrast* greatly modifies our judgments of colors. On viewing a gray pattern on a black background it appears brighter than when viewed upon a light background. The effect is so marked that a much darker gray can be placed on the black background and still appear brighter than the one on the white ground. If a series of gray papers of different shades are placed edge to edge it will be found that the edge of a lighter gray strip which is adjacent to a darker one will appear brighter than the outer edge of the brighter gray strip. The intensity of the contrast effect diminishes rapidly as one passes away from the point of maximum contrast. When two colors, such as red and blue, are in juxtaposition they appear more saturated and deeper in hue. If the colors are separated the contrast effect nearly disappears. If, for example, a disc of green is placed on a larger disc of red the contrast is very effective, but if the smaller disc is surrounded with a black circle the effect is reduced. If a gray figure is placed upon a green background, the gray figure will appear of a pink hue; the contrast hue thus induced is approximately complementary to the exciting color. Hering devised a striking demonstration of binocular contrast: red and blue glasses were placed in front of the two eyes respectively; the glasses sloped away from the eyes from the nasal to the temporal sides. A white image introduced from the sides by reflection permitted a control of the saturation. A black stripe on a white background is doubled by increasing or decreasing the visual divergence. The stripe seen through the red glass appears green and through the blue glass appears yellow; the observed background appears spotted, alternately blue and red and at times a purplish white. Helmholtz, Brücke and others contend that the contrast effects are not of a physiological nature but are due to errors of judgment; that is to say, through the influence of an adjacent color our "standard white" is modified so that our mental judgment is also affected. The whole effect would then be of a psychological nature

according to these notions. There appears to be no agreement as to the true explanation at the present time. Contrast may be due to unconscious eye movements, to fluctuations and incipient retinal fatigue, to errors of judgment or to some cause not yet discovered. Mayer conducted experiments in which the contrast color could be matched by means of rotating color discs thus obtaining quantitative measurements; he found that the subjective contrast colors were perceptible when viewed through a small opening for exposures as short as one one-thousandth of a second; they were also perceptible with instantaneous illumination from an electric spark in which the duration of illumination was of the order of one ten-millionth of a second. From these experiments he concluded that fluctuations of judgment could not be entertained as a satisfactory hypothesis for the explanation of subjective color contrast because of the extremely short period of time of exposure. But, in support of the hypothesis of errors of judgment, Edridge-Green contends "That all our estimations of color are only relative and formed in association with *memory* and the definite objective light which falls upon the eye. In many of the most striking contrast experiments the color which causes the false interpretation is not perceived at all; for instance, if a sheet of pale-green paper be taken for white, a piece of gray paper upon it appears rose-colored, but appears colorless when it is recognized that the paper is pale-green and not white."

244. *Irradiation.* There yet remains for brief consideration the phenomenon of irradiation: this name is applied to the apparent increase in size of objects as they are increased in brightness. We know, for instance, that the filament of an incandescent lamp appears to increase as the temperature of the wire is raised from a dull red to its normal temperature and, yet again, the crescent of the new moon appears larger than the remainder of the disc. This effect has been attributed by many to what is known as a spreading of the retinal image on account of a stimulation of nerves outside of the actual geometrical boundaries of the image, while others attribute the effect to the aberrations in the optical system of the eye. The phenomenon of irradiation may be easily illustrated by constructing two background cards of the same size, one of white and the other of black, and placing upon each of these a much smaller card so arranged that the small black square shall rest upon the outer white card in the one case and the small white square shall lie upon the black background in the second case. The inner white square appears larger than the inner black square under high illumination, yet both are identical in size. The phenomenon of simultaneous brightness contrast is also pres-

ent, for the white square in the black surroundings appears brighter than the larger white square.

245. *Theories of color-vision.* The process of vision involves the physical causes, the physiological retinal processes and the psychological elements in the experience of visual sensations. Color-vision is largely physiological and psychological. To explain the mechanism of color-vision various hypotheses have been presented. The older theories were without any, or at best but little, anatomical basis. None of them is satisfactory in character; any theory of vision to be satisfactory must explain the physiologic process of vision, color-vision and the nature of perception, and thus far no one has been able to present anything more than a working hypothesis on any of these three most important factors.

246. *Young-Helmholtz theory.* Thomas Young is credited with the conception of the three-color theory. This lacked experimental foundation until after the work of Helmholtz. Young explained his hypothesis as follows:—"It is certain that we can produce a perfect sensation of yellow and blue by a mixture of red and green light and of green and violet light. There are reasons for supposing that these sensations are always composed of a combination of separate sensations. We shall proceed, therefore, to consider white light as composed of a mixture of these colors only, red, green and violet."

In this theory it is postulated that each nerve fiber of the retina is composed of three sub-members or fibers each of which is provided with a special terminal organ (a photo-chemical substance). An irritation of the first fiber supposedly produces a violet sensation, an irritation of the second fiber a green sensation and of the third, a red sensation. These three are the primary or principal sensations giving rise to the principal colors. An irritation, then, of the red and green sensation fibers would produce yellow, and so on through the color scale. White is produced by the simultaneous irritation of all three fibers; no irritation of any of the fibers gives the sensation of black. Young explained color-blindness as due to the lack of one or more of the fibers, the remaining process being assumed to be "redistributed" to some extent. This theory has some advantages in explaining cases of red and green color-blindness by assuming the absence of the corresponding process and, if necessary, a slight modification of the two remaining ones. It fails to explain total color-blindness, however. It likewise must meet the demand that by the proper mixture of three spectral colors all existing hues and degrees of purity can be reproduced; this is found to be impossible. Likewise, according to Young, the color table must be triangular in shape but Maxwell's observations have shown that this cannot be the case.

247. Helmholtz modified Young's original hypothesis by assuming that each spectral color irritated all these fibers at once but in a different degree. Thus the red rays would irritate one fiber strongly and the other two feebly. The impression produced by spectral red would, therefore, also contain white and hence this impression is not the purest sensation which we can have. It has been found possible by Koenig, Maxwell, Abney and others, by studying the color sensations of normal eyes and of color-blind persons, to draw three curves showing the sensitiveness of the three primary sets of nerves to stimulation

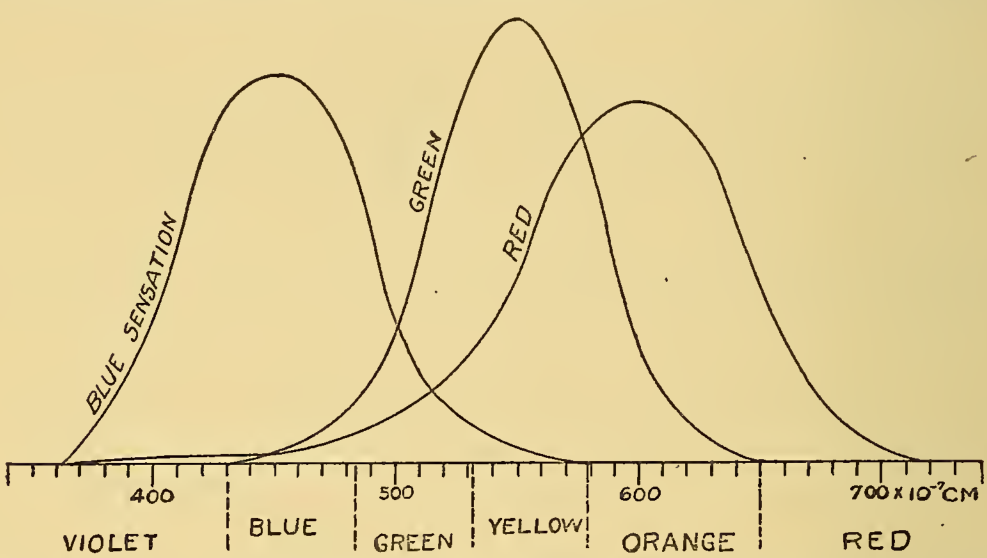


Fig. 140.—Curves Showing the Sensitiveness of the Three Primary Sets of Nerves (according to Young-Helmholtz theory) to Stimulation of Light of Different Wave-lengths.

by light of different wave-lengths. Such a set of curves as obtained by Abney is shown in Fig. 140. The scales of the curves are so chosen that when the ordinates intercepted on all three curves are equal the colors of wave-lengths corresponding to the abscissæ produce the sensation of white in the case of a normal eye. It will be noticed that the sensation of red can be stimulated by light of all wave-lengths between 4,000 (violet) and 6,900 (red) Angstroms. The sensation of green is stimulated by light of wave-lengths between 4,300 (blue) and 6,600 (orange-red). The sensation of blue is stimulated by all radiations between the extreme violet end of the spectrum to 5,900 (yellow). These results have afforded strong support to the Young-Helmholtz theory. This theory, then, explains the main facts of color-vision; there is as yet, however, no anatomical evidence of the existence of the three substances or sets of nerves; many of the observed facts in the study of after-images are only approximately reconcilable with this theory; the problem of simultaneous contrast is not satisfactorily explained.

248. *"Duplicity" theory of von Kries.* The name of von Kries is chiefly associated with the duplicity theory which attempts to differentiate colorless and color-vision. This theory is based upon the anatomical evidence which we possess of the existence of rods and cones in the retina. The rods are assumed to be largely responsible for the sensation of light at twilight illumination and to be more responsive to the shorter wave-lengths of light; that is, they are responsible for our achromatic colorless sensations. The cones presumably respond only under stimulation by brightnesses represented by the range between maximum luminous conditions and twilight illumination and are not greatly increased in their sensitiveness by dark adaptation; they are responsible for both achromatic and chromatic sensations. Anatomical investigations show that the cones alone exist at the very center of the retina, the fovea centralis, and that the rods are present just outside this region and predominate in the outer retinal regions or zones. The chief facts which this theory successfully explains (since, parenthetically, it may be said that the theory was built in the main from these facts) are: (1) decreased sensitivity of the fovea in twilight, (2) colorless vision over the whole retina in dim light such as moonlight, (3) the shift in the maximum of the luminosity curves of the eye (the Purkinje effect) at low illuminations, (4) the absence of such a shift for foveal vision, (5) no achromatic threshold for red light is found for any region of the retina, (6) no achromatic threshold is found for any light in foveal vision and (7) colorless vision occurs over the whole retina in cases of total color-blindness. Other supporting evidence is the similarity of the luminosity curve of a totally color-blind person at ordinary illuminations to the curve obtained for a normal eye for twilight vision.

249. *Hering theory.* Hering assumed that there are six fundamental sensations, coupled in pairs; white and black, red and green, yellow and blue. To account for these six fundamental sensations he assumes the presence in the retino-cerebral apparatus of three distinct substances. Red light, for example, acts on the red-green substance causing a katabolic change or disassimilation which produces the sensation of red; the green light on the contrary would cause an anabolic change in this substance by its action, or assimilation, which would produce the sensation of green. The same phenomenon takes place in the case of the yellow and blue rays in relation to the yellow-blue substance. Intermediary rays act on the two substances alike. The building up of the black-white substance causes a sensation of black-

ness and the breaking down of the substance gives a sensation of whiteness. A favorite argument in support of this theory is the observed fact that yellow appears to be a primary color because there is no simultaneous suggestion of both red and green in a yellow made by mixing these two colors. Many phenomena of after-images agree with this theory. If the eye be stimulated, for instance, by blue rays, anabolism will take place in the yellow-blue substance and an accumulation of the substance results. If, then, yellow light stimulates the same retinal area the breaking down of the yellow-blue substance proceeds at a greater rate and the sensation is greatly augmented. On the other hand, yellow decreases the amount of substance and increases the rate of anabolism under the subsequent stimulation of blue rays. Positive after-images are explicable by assuming that the process of anabolism or katabolism continues for a brief period owing to chemical inertia.

250. *Koenig's theory.* Arthur Koenig exploited a theory which may be considered a development of that of Young-Helmholtz. Red, green and blue are his primary colors. The decomposition of the retinal purple into yellow produces the weak sensation of gray, which causes any color when it is sufficiently weak. Further decomposition produces the sensation of blue. Perception of the two principal colors, red and green, is affected by the agency of the pigment cells. The cones are considered as dioptric instruments for concentrating the light on the epithelial layer.

251. *Ladd-Franklin theory.* This is one of the more modern theories. A primitive, photochemical substance, which is composed of numerous gray molecules, is assumed as being responsible for the colorless sensations of white, gray and black. These molecules exist in the primitive state only in the rods but upon dissociation they cause the colorless sensations. The gray molecules in the cones undergo development and only a portion of the molecule becomes dissociated by rays of a given wave-length or color. Three stages are postulated in the evolution of the gray molecule and are shown in Fig. 141. In the first stage the gray molecule is so constituted that it becomes broken up or disintegrated by light of all colors, thus producing a white or gray color sensation. In the second stage the molecule is more complex and contains two groupings. The dissociation of one or the other of these causes a yellow or blue sensation respectively. Their simultaneous dissociation produces white or gray. This stage is assumed to exist in the peripheral regions of the retina where red and green cannot be perceived as being such. In the third stage the yellow grouping is divided into two new combinations, the dissociation of one of which

produces a red sensation while a similar process in the other gives a green sensation. If, then, the red and green are dissociated together simultaneously, a yellow sensation results while the red, green and blue stimulated together produce gray.

252. The essentials of this very important development theory of color are given by C. Ladd Franklin in an essay on "Color Vision, Theories of," in Volume IV, page 2499, *et sequ,* of *The American Encyclopedia of Ophthalmology.* (For quotation from this section see Appendix A.)

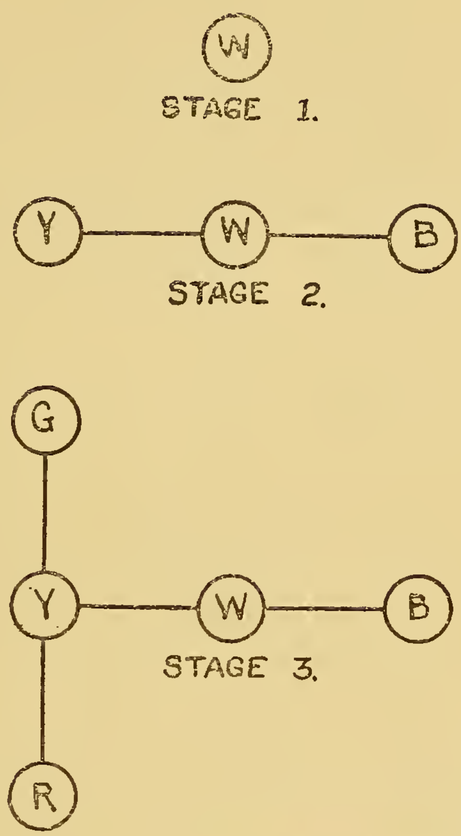


Fig. 141.—Illustrating the Ladd-Franklin Theory of Color Vision.

253. *Edridge-Green theory.* The retinal purple was discovered by Boll in 1876. This discovery gave rise to hopes that a photochemical theory of vision would explain the observed facts inasmuch as it was known that the visual purple was sensitive to light. If one examines the eye of an animal which has been left in darkness for some time before enucleation it will be found that the external segment of the rods has a purple color which vanishes quickly under the influence of light, assuming a yellow tint. The cones do not have this color and the fovea, which is composed of cones only, is without color. Kuehne labored with the question of the function of the visual purple, studying particularly the chemical properties of the retinal purple and

yellow; after his elaborate work the visual purple lost much of its significance in explaining the phenomena of vision. The yellow appearance which the purple retina assumes under the influence of light to which reference has just been made is supposed to be due to the formation of another pigment, the visual yellow. Many attempts have been made to find a relationship between the retinal purple and the vision of certain colors and with the adaptation of the retina to feeble light. Edridge-Green has recently done so; he assumes "That the cones of the retina are insensitive to light but sensitive to the change in the visual purple. Light falling upon the retina liberates the visual purple from the rods and it is diffused into the fovea and other parts of the rod and cone layer of the retina. The decomposition of the visual purple by light chemically stimulates the ends of the cones (probably through the electricity which is produced) and a visual impulse is set up which is conveyed through the optic nerve to the brain." Edridge-Green further assumes that "The visual impulses caused by the different rays of light differ in character just as the rays of light differ in wave-length. Then in the impulse itself we have the physiological basis of the sensation of color." It is also assumed that "The quality of the impulse is perceived by a special perceptive center in the brain within the power of perceiving differences possessed by that center or position of that center. According to this view the rods are not concerned with transmitting visual impulses but only with the visual purple and its diffusion."

254. *The Troland hypothesis.* Two of the most recent and noteworthy attempts to explain visual response in general and color-vision in particular are due to Troland (*American Journal of Physiology,* Vol. XXXII, 1913) and to Houstoun (*Proceedings of the Royal Society,* London, Series A, Volume 92). We shall in the succeeding paragraphs present some of the essential features of these two hypotheses.

Troland argues that most of the extant theories of visual response,—e. g., those of Hering, Donders, Mrs. Ladd-Franklin, etc.—err in a quantitative rather than qualitative way; "They are on the right track but have failed in progressiveness" since they involve vaguely or else not at all "The fundamental concepts and principles of modern theoretical physics and chemistry. . . . Most of these are not only vaguely formulated and contain no distinct reference to general physics and chemistry—to say nothing of the special physical chemistry of light and of nervous response—but they often flatly contradict both physical and physiological principles. The physical conception of *resonance* lies at the bottom of practically all of the hypothetical accounts which have been given of the processes of visual stimulation; but it requires only a very simple calculation to show that if any microscopically observable structure is to resonate in tune with even the largest light waves the material substance involved must possess a modulus of elasticity two hundred million times greater than that of hard drawn steel. This is a *reductio ad absurdum* of all theories of mechanical stimulation which depend upon resonance." A second very vital objection is that light can act directly only upon electrical and not upon neutral mechanical structures; this leads us from mechanical to chemical hypotheses and from molar to molecular systems in which it is possible that the systems are of the right order to make possible selective response. We may, therefore, believe with Troland that the "Ultimately successful doctrine as to the nature of the visual mechanism must involve the concept of electricity due to the fact that light is an electromagnetic process and consequently can react only with electrical or magnetic systems, as well as by the facts of retinal and general nerve physiology, which all point to electrical factors in stimulation." We shall have occasion to point out in some detail, under the succeeding caption, the general explanation of the mechanism of visual stimulation and visual impulses as outlined by Troland.

255. This investigator comes to the conclusion that there are in the retina *five* distinct visual substances; M_r, M_g, M_b, M_y and M_w: these are designated as *molecular resonators* because they are selectively ionized by lights of specific and differing wave-length or frequency and their intrinsic positive ions, I_{r+}, I_{g+}, I_{b+}, I_{y+} and I_{w+} are the exact psycho-physical correlatives to the visual qualities R, G, B, Y and W respectively. A study of the modes of occurrence of the fundamental attributes of *S*, the elementary visual sensation, reveals the following correlations: (1) if $g > o$, $r = o$; (2) if $y > o$, $b = o$ and conversely (3) if $r > o$, $g = o$ and (4) if $b > o$, $y = o$. In other terms, the hues R and G, Y and B are mutually exclusive or "antagonistic". The hues when arranged in the cyclic order

B
R G
Y

are such that adjacent qualities will fuse while opposite ones exclude or cancel each other. The attribute W (white) can be added to any possible combination of hues, while B is present in strict proportion to the absence of R, G, Y and W. In order, therefore, to take into account the "antagonistic" relations it is necessary to postulate the existence of what may be called the *complementation* substance within the large ganglion cells of the inner stratum of the retina. The complementation molecules are made up of a nucleus and two side-chains. These side-chains are each a negative ion, the nucleus being doubly charged and positively. "Of these molecules there are two varieties. The first is so constructed chemically that its two negatively charged ionic side-chains are capable of combining simultaneously, but not separately, with the two positive ions of the visible impulse: I_{r+} and I_{g+}. The second reacts in a similar way with the visual ions: I_{y+} and I_{b+}. The result is that in each case the positively charged nuclei of the molecules are set free and become a part of the impulse

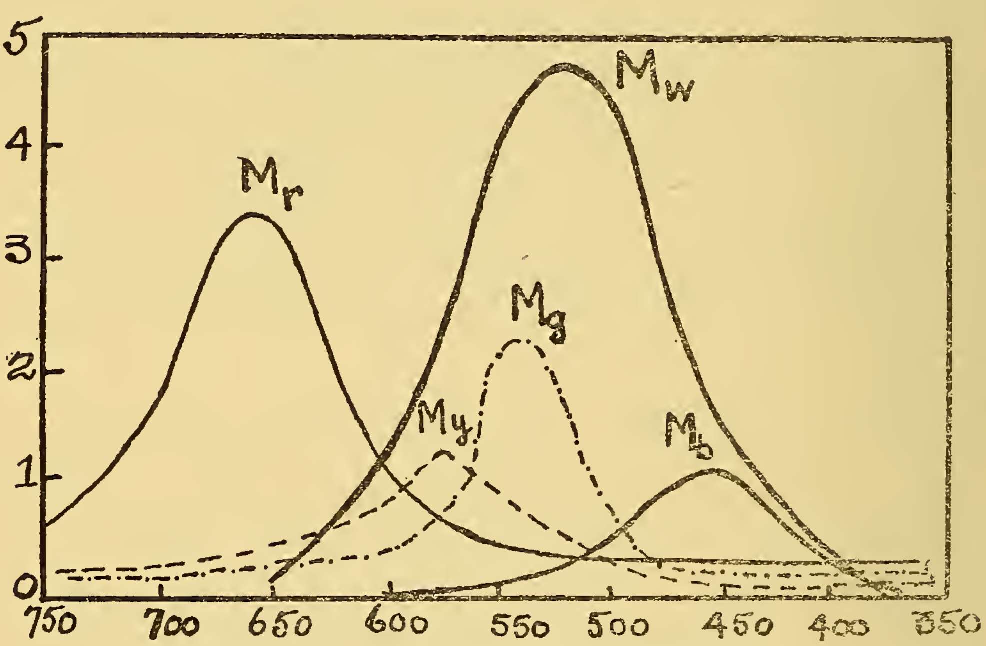


Fig. 142.—Curves Illustrating the Troland Theory.

as it is passing through the ganglion cells." These two substances may be spoken of as the R — G — complementation substance and the Y — B — complementation substance.

Fig. 142 contains five curves which represent the maxima into which the five specific molecular resonators are broken down by light waves of varying frequency. They are theoretical curves and represent what Troland speaks of as resonance functions of the specific substances M_r, M_g, M_b, M_y and M_w; the exact shapes may vary widely under alterations in the concentrations of the resonators, the intensity of the light and so forth. For stimuli of high intensity these curves will all be flattened owing to the concomitant action and influence of the forces expressed in Fechner's law.

256. It is very reasonable to suppose that the number of ions leaving

a retinal element via the neuro-fibrillæ per second is proportional to the number present or to the concentration, and it is quite apparent that what may be designated as the intensity of the impulse, or the number of ions passing through any cross-section of the nerve fiber per unit of time will depend upon the number leaving the retinal element and the number lost in the process of conduction. Let the impulse intensity be represented by the letter "i." "If, then, the impulse before passing through the ganglion cells has the constitution: $i_r + i_g$ where $i_r > i_g$, the constitution after passing these cells will be: $(i_r - i_g)$ of $I_{r+} + 2i_g$ of I_w" (in which I_{r+}, and so forth, represent the intrinsic positive ions which are the exact psycho-physical correlatives of the fundamental visual qualities R, G, B, Y and W respectively). "The corresponding sensation, S, will be a pink, not a greenish-red." Likewise it is obvious that when $i_r = i_g$ and $i_y = i_b$ the only impulse component reaching the cortex will be i_w; these are the conditions for complete complementation. All of the familiar effects of "color mixture" are represented in Fig. 142. For example, suppose the same cone is stimulated with lights of $\lambda = 6{,}500$ and $\lambda = 5{,}500$; with appropriate intensities the two elements i_r and i_g will cancel each other leaving only the i_y (and i_w) which is also introduced by both lights. The constitution of the resultant sensation will be $S = Y + W$, although red and green lights have been mixed. The diagram (Fig. 142) also explains, according to Troland, the fact established by J. J. Müller and von Kries that when a heteronymous light stimulus is made up of two (or more) lights having wave-lengths falling between the limits of $\lambda = 7{,}600$ to $5{,}670$ or $\lambda = 3{,}900$ to $4{,}920$ the *chroma* of the induced sensation does not differ from that of a sensation induced by a homogeneous wave yielding the same hue. It also explains the location of points of least chroma and greatest luminosity in the visible spectrum at $\lambda = 5{,}750$ (approx.) and $\lambda = 5{,}000$; at these points the M_r and M_g, and M_y and M_b curves, respectively, intersect and hence with these lights the complementation reaction finds its maxima.

Another important phenomenon this theory accounts for is the disappearance of hue with increasing light intensity. Every light stimulus supposedly acts upon every molecular resonator but at low intensities a light of $\lambda = 6{,}550$ (say) acts very strongly only upon M_r and very weakly upon M_g, M_y and M_b; but as the intensity is increased the increase of the several components of i (ions at the retina) follows Fechner's law (q. v.) and hence each of these components approaches a definite maximum. The net result would be that, whatever the wave-length of light may be, its effect upon the several resonators at very high intensities is the same. The curves of Fig. 142 also account for the repetition of the hue R in the violet end of the spectrum; extant visual hypotheses—except that of Hering—are at fault because of their inability to explain the repetition of the hue R in two widely separated parts of the spectrum.

257. *Houstoun's theory.* In his paper on a Theory of Color Vision Houstoun has attempted to explain the facts of color mixing by a theory which does not depend upon primary color sensations. The theory is, in part at least, a mathematical formulation of Edridge-Green's views; the portion which deals "with the retinal process applies ideas already more or less familiar, while the second part, which deals with the cerebral process, uses an idea quite original in its applications to color vision."

Fig. 143 (A) represents by the full line the sensitiveness of the eyes for light of different wave-lengths as determined by H. E. Ives from observations on about twenty persons viewing a surface having an illumination of 25 meter-candles with the pupils at normal apertures. The eye has, therefore, a maximum of sensitiveness in the green and falls off rapidly on both sides toward the red and violet. To what is this due? "The most obvious explanation is to *suppose* that there exist in the eye a very large number of vibrators, *with a free period in the green,* and that these execute forced vibrations under the influence of the light wave. The amplitude of the forced vibrations is a maximum when the free period of the vibrators coincides with the period of the incident light." The motion of one of these vibrators may be represented by the equation

$$\frac{d^2x}{dt^2} + h\frac{dx}{dt} + n^2x = E \cos at \quad \ldots\ldots\ldots\ldots (1)$$

in which x represents the displacement of a typical vibrator from its position of rest and $E \cos at$ is the force per unit mass exerted on it by the incident light wave. The symbol "a" represents the frequency and is equivalent to $\frac{2\pi c}{\lambda}$ in which "λ" represents the wave-length involved and "c" the velocity of light. The rate at which energy is absorbed by the vibrator is given by

$$E\frac{dx}{dt} \cos at \ldots\ldots\ldots\ldots (2)$$

The solution of equation (1) consists of two parts, the free vibration and the forced vibration: when a vibrator is left to itself the free vibrations die down but are renewed under excitation. It can be mathematically shown that when the part of $\frac{dx}{dt}$ due to the forced vibration is calculated and substituted, equation (2) becomes ultimately as to its mean value

$$\frac{E^2 h \alpha^2}{2 \quad (n^2 - \alpha^2)^2 + h^2 \alpha^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

"The intensity of the incident light is proportional to E^2. If it is assumed that the luminosity is proportional to the energy absorbed, and omit a constant factor, the ratio of absorbed to incident energy or, in other words, the visibility of radiation, is proportional to

$$\frac{\lambda^2}{(\lambda^2 - a^2)^2 + b^2 \lambda^2}$$

This expression is graphically represented by the dotted line in Fig. 143 (A) for $a = 0.10\mu$. As a whole we may say that (3) represents the visibility curve roughly."

258. Equation (3), however, shows that, as E^2 is increased, the value of the equations remains constant. This is contrary to experience as expressed in Fechner's law that

$$\frac{dI}{I} = \text{constant}$$

(in which I represents the intensity of the light stimulus). Hence it is necessary to assume that the energy absorbed is proportional to $\frac{d(E^2)}{E^2}$: under this assumption it can be mathematically shown that the number of vibrators diminishes as E increases except for small values of E. Hence, this assumption implies that some of the vibrators go out of action when E^2 is increased. "When the energy of the vibrator reaches a critical value, the force attaching the vibrator to its center snaps, the latter then ceases to absorb light energy and a chemical change takes place. This critical value is not the same for all the vibrators, but varies from vibrator to vibrator.'' We have here, it seems to the writer, the fundamental principle of the so called ''Quantum theory'' applied to explain visual phenomena. (See Planck's *Radiation Theory,* trans. by Masius.) According to this conception energy, E, is radiated in discrete or definite units and is connected with the frequency, v, and the universal constant, h, through the equation

$$E = n\,h\,v.$$

The detachment of one, two, three and so forth (n) electrons will give rise to varying quantities of radiation, which are multiples of a definite energy unit.

259. By way of further explanation Houstoun writes:—''Of course we are not to suppose that if E is constant the same identical vibrators remain in action all the time, but that there are two processes going on in opposite directions which balance one another, visual purple being bleached and constantly restored. When E increases, the point of equilibrium is shifted. Owing to the bleaching and restoration of the visual purple we must suppose the vibrators to be in a perpetual state of agitation. Their free vibrations are constantly being renewed. If the intensity of illumination is reduced, it is found that the visibility curve undergoes a change. Its maximum is gradually shifted toward the green, reaching the limiting position of 0.50μ when the intensity is very small, the curve becoming at the same time narrower near the maximum. The phenomenon is known as the Purkinje effect.'' * * * ''According to von Kries, the effect can be explained by assuming that the rods in the retina are chiefly responsible for vision at low intensities and the cones for vision at high intensities. I do not, however, think it necessary to assume two different mechanisms. We can explain the effect with one system of vibrators in either of two ways. First, we may suppose the vibrators embedded in a medium with a yellow color, something like potassium chromate. At low intensities the energy is absorbed near the surface of the medium: at high intensities most of the vibrators near the surface will have become bleached. Consequently a larger proportion of the energy is absorbed at a greater depth, but the energy on the blue side of the maximum suffers a greater absorption by the medium on the way in, and there is not so much left for the vibrators to absorb at this depth. Thus the maximum of absorption is shifted toward the yellow. I do not know the exact color of the yellow spot in the eye and am unable to say whether it would produce the required effect. The alternative explana-

tion is to assume that all the vibrators have not the same period, but that the value of a varies from vibrator to vibrator, $a = 0.55\mu$ being only a mean value. Then, if those that decompose more easily have a smaller value of a, the shift is explained. This explanation seems to me to be the better one. The assumption of different values of a follows naturally from the assumption that the vibrators decompose at different intensities. Also, Ives' visibility curve [the full curve in

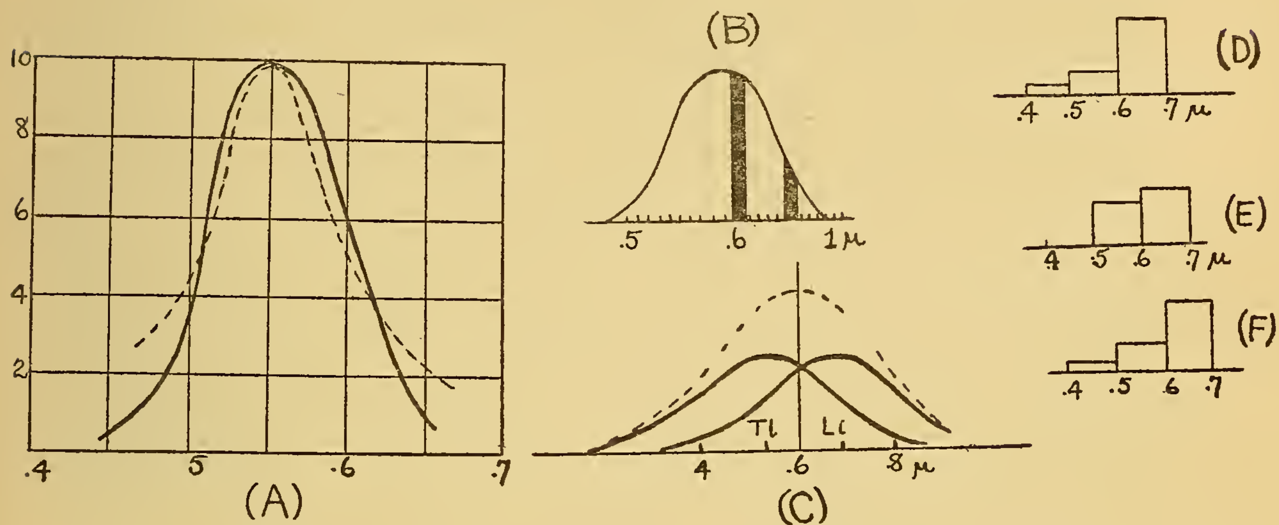


Fig. 143.—Curves Illustrating the Houstoun Theory.

(A) Representing the sensitiveness of the eye to light of different wave-lengths.

(B) Distribution of energy over the different waves set up in the nerve, produced by yellow light of wave-length 5900 t.m.

(C) Energy curves of equal luminosity to represent lithium red and thallium green and their sum as shown by dotted line.

(D-E-F) Luminosity curves representing respectively a sodium yellow, a lithium green and a white light.

Fig. 143 (A)] is very like a probability curve, i. e., it looks as if it could be represented by the expression

$$e^{-k(\lambda - 0.55)^2}$$

where k is a constant. If the different values of a are distributed around the mean value according to the law of error, the visibility curve will be the sum of a great number of small curves of the type represented by (3). If these component curves have the same value of b, their individual peculiarities will not appear in the resultant, but only the law according to which they are grouped about their mean, and hence we shall obtain an expression similar to (4) for the resultant. By proceeding in this way we can obtain a much better agreement between theory and experiment than in Fig. 143 (A)."

"There is no decided evidence in favor of three classes of vibrators,

i. e., the visibility curve has not three separate maxima. It has a simple form, probably simpler than the three component visibility curves into which it would have to be resolved to meet the views of those who believe in the existence of three independent primary sensations with an independent mechanism for each.''

260. Houstoun supposes in regard to the cerebral processes that the vibrators set up waves in the nerves and that these nerves carry these waves to the brain somewhat after the analogue of the telephone. There is one important difference, however, and this is that the vibrator in vision does *not* reproduce the light wave exactly owing to its being subject to too many disturbances. A monochromatic wave is not, therefore, transmitted along the nerve in a manner such as to preserve its monochromatic character. Or again, the vibrator may start from rest having its free and forced vibrations superimposed. ''The free vibration may die down, then be renewed again by an impact: the period of the vibrator may change slightly. The vibrator may then decompose and, after a short rest, reunite and start again. Now an irregular motion of this sort may be regarded as due to a superposition of sine waves, i. e., the displacement of the vibrator may be expressed as a Fourier integral.''

261. We may suppose that, with yellow light ($\lambda = 5{,}990$ Angstroms) falling on the retina, the distribution of energy over the different waves set up in the nerve is represented by Fig. 143 (B). The whole area of the curve represents the energy received by the brain or the luminosity of the sensation. The maximum of this curve will coincide with the wave-length of the incident light but the curve will not in general be symmetrical, since more of its area will be on the same side as the maximum of the free vibrations. Furthermore, the area of the rectangular strip at 0.60μ shows twice as much energy included as in the area 0.65μ. If the vibrator executed a pure forced vibration for a very long time, the curve would be infinitely narrow. The breadth of the curve is thus a measure of the degree of disturbance to which the vibrator is subject. Three qualities are associated with a light impression, namely luminosity, hue and saturation: the area of the curve represents the luminosity, the position of the maximum the hue and the narrowness of the curve the degree of saturation.

262. The phenomena of *color mixing* may be readily explained by such curves. Fig. 143 (C) shows two energy curves of equal luminosity representing lithium red and thallium green compounded as one as shown in the dotted line with a maximum at 0.60μ.

263. ''But, it will be asked, how does this theory explain the apparent trichromatism of our ordinary sensations? We here must fall

back upon the reason given by Edridge-Green, namely that the color-perceiving center in the brain is not sufficiently developed to discriminate between the character of adjacent curves. Two curves must be widely different in shape and position, before the color-perceiving center can detect the difference. A curve has an infinite number of points on it. The color-perceiving center is so badly developed that, as far as it is concerned, the curve is sufficiently specified by three points on it, provided that these points are distributed over the spectrum. We can therefore represent our energy curve by three points. Since the sensation of luminosity is better developed than that of color, I have preferred to represent the curve by three rectangles as in the Fig. 143, (*D*), (*E*) and (*F*), which represent respectively a sodium yellow, a lithium green and a white light, all of equal luminosity. The area of the rectangles may be regarded as the amount of stimulation of the three primary sensations of the Young-Helmholtz theory: indeed, I took the value of the ordinates from one of Sir William Abney's curves, merely exaggerating the size of the third component in diagram (F) to make it visible. The diagrams may therefore be regarded as a means of representing the results of the Young-Helmholtz theory. I believe, however, that they are more than this: that they actually are energy curves—crude ones, it is true, but sufficiently representative for the discriminating power of the color-perceiving centers. * * * The diagrams are, to some extent, a connecting link between the Young-Helmholtz theory and that of Edridge-Green."

XVII. LIGHT STIMULUS AND RETINAL CURRENTS

264. The effect of the stimulus of light on the retina is perceived by the brain as a visual sensation. The process or processes by which the ether-wave disturbance causes this visual impulse is still very obscure. As a matter of fact the whole of the field of photo-chemical action is still in its infancy. The process of making an ordinary negative by exposing a dry plate in a camera to light and the manner of developing and fixing such a plate are mechanically easy of accomplishment. But ever since the discovery that an invisible light effect could be developed into a strong image by the application of suitable reducing agents, the constitution of the invisible or so-called "latent image" has been the subject of study and controversy and no wholly satisfactory explanation of the effects of radiation upon silver salts has been presented. The process by which the ether disturbance causes a visual impulse may be ascribed to (1) chemical action, (2) molecular strain and (3) electrical action.

265. According to the *chemical theory* it is presumed that certain visual substances in the retina are affected by light and that vision originates from the metabolic changes produced in these visual substances. It is supposed that the metabolic changes consist of two phases; the upward, constructive or anabolic phase and the downward, destructive or katabolic change. These anabolic and katabolic changes in various visual substances are supposed to produce the variations of sensation of light and color. This theory is complex. Numerous objections have been urged against its acceptance; for it is difficult, for instance, to see how this very rapid visual process can be due to a comparatively slow chemical action consisting of the destructive breaking-down of the substance followed by its renovation. Support was at first furnished the chemical theory by the bleaching action of light on the visual purple present in the retina, but it has been discovered that the presence or absence of visual purple is not essential to vision and that its presence is of only secondary importance. For it is well known that the visual purple is lacking in the fovea centralis and it is also found to be completely absent from the retinæ of many animals possessing keen sight.

Writing in the *Ophthalmic Review* during 1916 Edridge-Green states his belief as to the nature of retinal stimulation. He says:— "A ray of light impinging upon the retina liberates the visual purple from the rods and a photograph is formed. The rods are concerned only with the conveyance of the light impulses to the brain. The ends of the cones are stimulated through the photo-chemical decomposition of the visual purple by light, and a visual impulse is set up which is conveyed through the optic nerve fibers to the brain. The character of the stimulus and impulse differs according to the wave-length of the light causing it. In the impulse itself we have the physiological basis of the sensation of light and in the quality of the impulse the physiological basis of the sensation of color."

266. The *mechanical theory* depends, in large measure, upon the theory of resonance in connection with chemical action. It is readily conceivable that a ray of light can cause a chemical decomposition of a substance in which the rhythmic excursions of an atom or atoms from the center of attraction in a molecule are in exact tune with the waves of light falling on such atoms. The excursions may be so increased in extent by the rhythmic energy supplied by the light waves that the atoms will leave the parent molecules and produce new molecules. It is not as easy to see why the rhythmic excursions of atoms in the same molecule are also increased to the point of molecular rupture when the wave-motion of the impinging rays is not exactly

in tune. But some photographic and mechanical examples help us out. For if a sensitive salt, such as silver chloride, is exposed to the action of the spectrum, we can plot a curve showing the sensitiveness of this particular salt to the different spectral rays. Such a plotted curve shows a rise in sensitiveness to a maximum followed by a decline; the maximum of such curves shows the place in the spectrum where the vibrations causing the ray are in tune with the vibrations of the chlorine atom in silver chloride for example, the chlorine being that part of the molecule which is swung away and annexed to some other adjacent molecule. We have also mechanical examples of the effects produced by vibrations which are not in tune with, but which act upon, a vibrating body. A simple apparatus consists of two different pendulums which can act upon one another through a proper communicating medium; such would occur, for example, when the pendulums are connected to a taut piece of rubber tubing fastened horizontally. When the pendulums are of equal length and one is started into vibration, the second one also begins to swing and, since it is in tune, the amplitude constantly increases. But if one pendulum is a little longer or shorter than the other, experimentation shows that one pendulum causes the second one to swing with increasing amplitude and by degrees the two will swing in opposite directions; the amplitude of the first pendulum will decrease and finally come to rest when the motion starts out again as at first. Thus, if the mechanical analogy is applicable here, it is seen that if the waves causing a ray of light are out of tune with the vibrations of the atoms the amplitude will still be increased and the increase can be such as to swing the atom beyond the sphere of molecular attraction and so decompose the molecule but with less ease than when the waves are in tune.

267. We have written of the *resonance theory* as a correlation and interaction between radiant energy and the atom of the molecule and have thus followed and outlined the theory as it is generally presented. However, in the light of modern physics, we should presumably have written of *resonance* and *electrons*. In fact, the essential points in the mechanical theory of retinal stimulation as consisting of resonance effects coupled with chemical action fit in with many of the physical phenomena known as "*photo-electric* actions." This term includes phenomena due to the action of light in liberating negative electrons from various metallic substances. It is known, for example, that there is a considerable influence of the wave-length or frequency of the light upon the number of electrons emitted and that curves plotted between frequency and rates of "leak" of negative electricity from metals such as sodium, potassium and rubidium show maximum or

resonance effects. Likewise, salts which undergo decomposition in the light, such as silver chloride, are strongly photo-electric. We are, therefore, presumably dealing under the tenets presented to us in this theory with the expulsion of electrons due to resonance; the electrons are set in resonant vibration by the incident light and acquire sufficient velocity to enable them to escape from the atom. The student of physiologic optics who is interested in this great unsolved problem of the connection between light and retinal stimulation will do well to peruse such a work as H. Stanley Allen's *Photo-Electricity.*

268. The *electrical theory* supposes that the visual impulse is the concomitant of an electrical impulse; that an electrical current is generated in the retina under the influence of light and that this is transmitted to the brain through the optic nerve. It is an undoubted fact that light gives rise to retinal currents and that, on the other hand, an electrical current suitably applied causes the sensation of light. Holmgren, Dewar, McKendrick, Kuhne, Steiner, Waller and others have shown that illumination produces electric variation in a freshly excised eye. The currents are very small in value, hence a very sensitive dead-heat galvanometer (having a figure of merit of about 1×10^{-10}) must be used. Currents of injury or contact potential differences arise when the galvanometer terminals are connected to the cornea and to the cut end of the nerve respectively. These may be compensated for by means of a potentiometer device. When a freshly excised eye, thus connected, is illuminated it is found that the current of response due to the action of light on the retina is always from the nerve, which is not directly stimulated by light, to the retina. Such currents have been designated as *positive* when flowing from the less excited to the more excited. The normal effect of light on the retina as noticed by the observers already mentioned is a positive variation during exposure to light of not too long duration. Cessation of light is followed by recovery. Deviations from this are regarded as due to abnormal conditions of the eye, rough usage, mechanical pressure and so forth. Unlike muscles, successive retinal responses exhibit little change; for, generally speaking, fatigue is very slight and the retina recovers quickly even under strong light if the exposure is not too long.

The *general experimental method* adopted in investigations on the retinal currents due to light stimuli is as diagrammed in Fig. 144. Contacts are made with the galvanometer *G* through non-polarizing electrodes with the cornea *A* and the cut optic nerve *B*. On making contacts and closing the circuit through the galvanometer a considerable current of injury will be found which must be compensated for by the use of an auxiliary circuit *CD* attached at the points *A* and *B*. This auxiliary circuit consists of a battery short-circuited through a high resistance so arranged that any desired potential difference may be tapped off and applied to the eye at *A* and *B* in such a direction as to counteract the effects due to the current of injury. The eye is enclosed in a black box with an aperture through which light can be admitted when desired.

Historically, Holmgren is accredited with the initial experiments on the electrical response of the eye to stimulation by light. His work was published in 1866 (*Physiol. Untersuch, Heidelberg,* Bd. ii, page 81 and Bd. iii, page 358). He was able to demonstrate that when light was allowed to fall upon the eye of a frog that had been kept in the dark and, again, when the light was removed, there was an increase in

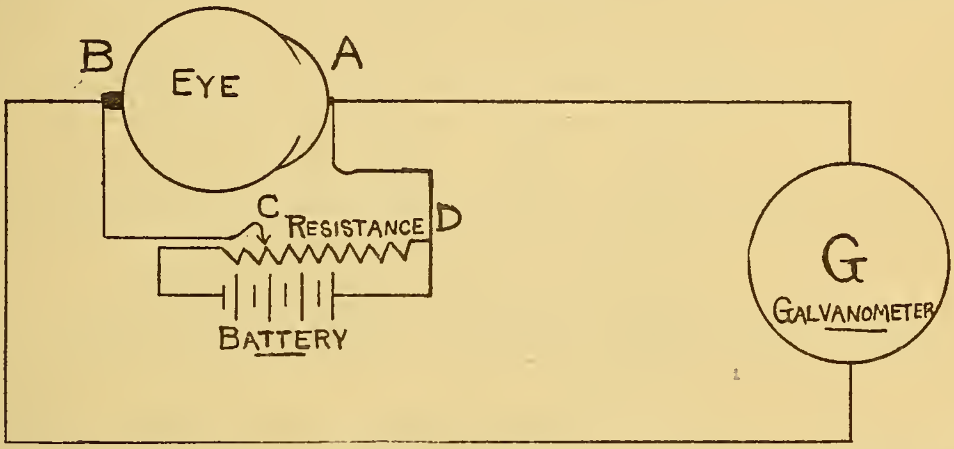


Fig. 144.—Scheme of Arrangements of Eye, Galvanometer and Compensating Potentiometer Device Used in Investigating Retinal Currents Due to Light Stimuli.

the positive direction of the current present during darkness. The strength of the current was, within certain limits, proportional to the intensity of the light. Likewise, the onset and removal of light was attended with the same electrical changes when the posterior half of the eyeball only was employed.

Independently of Holmgren, experiments on the physiological action of light were conducted by Dewar and M'Kendrick: their joint papers appear in the *Transactions of the Royal Society of Edinburgh,* Vol. XXVII, 1873. The conclusions to which these experimenters came are: (1) That the impact of light on the eyes of mammalia, aves, reptila, amphibia, pisces and crustacea, produces a variation of from three to ten per cent. of the normal electromotive force existing between the corneal surface and the transverse section of the nerve; (2) this electrical alteration may be traced to the brain; (3) that the rays which we regard as most luminous produce the largest variation; (4) that the alteration of the electrical effect with varying luminous intensities follows the law of Fechner; (5) that the electrical alteration is due to the action of light on the retinal structure itself and (6) that it is possible to discover by experiment the physical expression of what is physiologically called fatigue.

Kühne and Steiner (*Physiol. Untersuch, Heidelberg,* Bd. iii, 1880) investigated the reactions of the isolated retina. They found that the electrical change on lighting and darkening is a complex one, the variation being positive, then negative, and finally again positive. The reaction is divisible, according to these observers, into two parts: the first, due to the onset and continuance of illumination, consists of a negative variation preceded by a positive; and the second, caused by the disappearance of the light, consists of a simple positive variation.

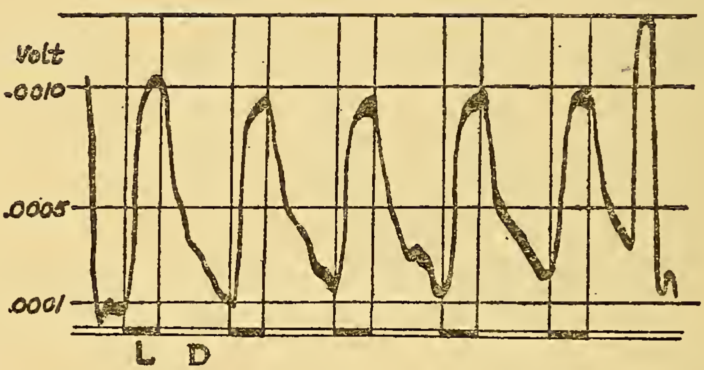


Fig. 145.—Photogram of Response to Light (and Recovery) in Frog's Retina. (After Waller.)

269. Waller has carried out a number of important researches in this field of investigation. A series of photograms of responses to light in the frog's retina is given in Fig. 145. These maxima represent five normal responses due to a candle at 2 feet with illumination for 1 minute and obscurity for 2 minutes respectively. The abscissæ represent the time in minutes and the ordinates the absolute electromotive forces. These results indicate rapid rises in electromotive force under illumination and less rapid recovery.

270. These retinal current effects can be imitated and their counterparts observed in non-organic substances. Considerable work on this subject has been done by Bosé. He took, for instance, a rod of silver which he beat out into the form of a hollow cup and sensitized the inside of this cup with bromine vapor. The cup was filled with water and connected through a galvanometer by non-polarizable electrodes. A current arose due to differences between the inner and outer surfaces of the cup; this was balanced by a compensating electromotive force.

This gave an arrangement somewhat resembling the eye, with a sensitive layer corresponding to the retina and a less sensitive rod corresponding to the conducting nerve-stump. The apparatus, being enclosed inside a black box, was illuminated through an aperture at the top; on exposing the sensitive surface to light the balance was at once destroyed and a responsive current of positive character produced. Upon cessation of light there was a fairly quick recovery. It is of interest to compare the response and recovery curves of the frog's retina as obtained by Waller and given in Fig. 145 with similar phenomena obtained by Bosé with his sensitized silver cell as shown in Fig. 146.

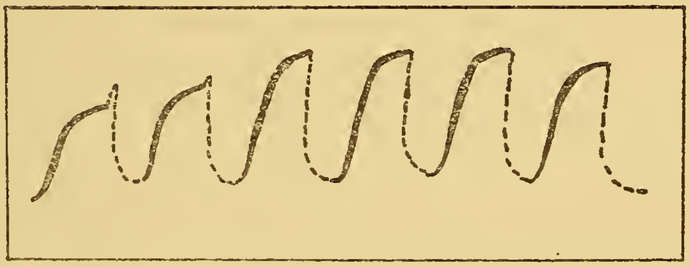


Fig. 146.—Curves Obtained with Sensitized Silver Cell Analogous in Form to Retinal Current Curves. (After Bose.)

271. The main conclusions to which Waller arrived in his work are:—

(a) A fresh normal eyeball manifests a positive current which gradually declines to zero and becomes reversed.

(b) On exposure to light the normal current, whether positive or negative, undergoes a positive variation.

(c) The magnitude of the response to light increases with the duration of illumination.

(d) The magnitude of the response to light increases with the strength of the illumination.

(e) Fatigue is less pronounced in the case of the retina than in that of muscle.

(f) Colored lights act in the same direction, and in accordance with their luminosity. No electrical evidence is obtained of antagonistic influence.

At the conclusion of one of his papers Waller says:—"I believe it to be proven by these observations that the retina is the seat of a double electrical movement, a simultaneous positive and negative effect, but whether the double effect is the expression of duplex change in one substance or of two changes in two different components cannot be strictly demonstrated. I am of the opinion that we have to do with a duplex change, constructive and disruptive, in one substance.''

Gotch (*Journal of Physiology*, Vol. XXIX, 1903 and Vol. XXXI, 1904) made use of a capillary electrometer which recorded the rapid alternations of current present in the response of the eye. This observer divided the electrical reaction of the eye to light into three portions: (1) The rise due to the sudden illumination, or the ''on-effect'': (2) the continuous change occurring during the continuance of the illumination, and (3) a second rise due to the sudden change from light to darkness, or the ''off-effect.'' The photo-electric changes, he concludes, are all of the same general type, giving rise to monophasic effects and appear to be due to processes occurring in the posterior part of the eyeball. The results seem to indicate the localization of two photo-chemical substances in the posterior half of the eyeball, these being a substance reacting to light and a substance reacting to darkness. Each reaction is a change of the same type, but for the change to occur the eye must be previously adapted, i. e. the substances must undergo phases of metabolism under conditions opposite to those which evoke the reaction effects.

Einthoven and Jolly (*Journal of Experimental Physiology*, Vol. I, 1908) conclude that the form under which the photo-electric reaction manifests itself gives ground for the supposition that there occur in the eye three separate processes and that each of these may be dependent upon a separate substance. The *first* substance reacts more rapidly than the other two. On lighting, it develops a negative, on darkening a positive potential difference. The *second* substance reacts less rapidly than the first and in an opposite direction. Hence, on lighting, it develops a positive, on darkening a negative potential difference. The *third* substance reacts in the same sense as the second but much more slowly. For each of these substances the rule holds good that with moderate and strong lights the energy of the stimulus increases more quickly than the energy of the reactions. Furthermore, the latent period of the photo-electric reaction is in a high degree dependent upon the intensity of the stimulus. With strong stimuli it is of the order of 0.01 second, while with very weak stimuli it may be lengthened to more than 2 seconds. These values are in agreement with the latent periods of light perception in the human eye.

272. Some investigations, as yet unpublished, now being carried on by Sheard and McPeek upon the retinal responses to light of varying wave-lengths using enucleated dog eyes indicate that there is a positive increase of potential established by wave-lengths of light ranging from the extreme red to the green, reaching a maximum in the yellow

to yellow-green region and that there is a decrease or relatively negative potential established between the retina and the cornea under stimuli from green-blue to the lower visible limit. The form of curve established corresponds in a general fashion to the luminosity curve of an eye, positive potentials of increasing values being established after the cessation of stimulation up to the maximum in the green region and negative potentials resulting after the cessation of stimuli in the green to violet region. In all cases the zero of comparison was taken as that established under no light excitation and with such compensating electromotive force established across the eye from cornea to cut nerve as to produce a balance at the initial zero of the galvanometer scale. The current responses are, as stated by Waller, always initially positive in direction (from retina to cornea). This increase and decrease, or positive and negative, potential effect could be carried through a series of changes such that the retina was exposed in succession, followed by periods of rest of six minutes, to a certain wave-length and then to its complement. Interesting results have been obtained showing that practically no retinal potential changes occur, after the initial period of excitation by light of a particular spectral character, unless exposures are made in a region very closely approximating the complementary color and that when so exposed the potential is carried back approximately to the value which it possessed previous to the dual exposures. The writers are inclined, therefore, to the view that there is electrical evidence of antagonistic influences and of anabolic and katabolic processes such as those demanded in the Hering theory or in the hypothesis as to the mechanism of visual stimulation and the exact mechanism of the visual impulse as promulgated by Troland and outlined in the following paragraphs.

273. Troland bases his physico-chemical theory of visual response upon the fact that the mechanical hypothesis of visual response is untenable because the mechanical systems are not of the right order of magnitude to vibrate in unison with light and because molar systems do not carry the free electronic charges which are essential in order that the forces of light rays shall act upon them. There is, however, evidence that many, if not all, chemical molecules are electrical dyads made up of positively and negatively charged atoms or radicles which can be separated from each other by electrical forces. Consider a certain molecule *M* which is composed of positive and negative parts: I_+ and I_- respectively. Then, since the system is not rigid, the I_+ and I_- will be capable of vibrating with respect to each other with a certain frequency, *n*. If then *n* is also the frequency of some light ray impinging upon M, the molecule will resonate with

respect to this ray so that under proper conditions the constantly increasing amplitude of vibration will result in a final disruption of the molecule. I_+ and I_- in the free state are ions and the process initiated by the light is one of ionization. Hence, from these premises and from the general physical point of view, it is probable that *the immediate effect produced by light upon the retina consists in an increase in the ionization of certain specific chemical substances there present.* These substances, in general designated by M, are supposedly enclosed in the terminal segments of the rod and cone cells. The most successful hypothesis to explain the nerve impulse is that of W. Nernst (*Archiv. für die ges. Physiologie,* Vol. CXXII, 1908) as elaborated by Hill (*Journal of Physiology,* Vol. XL, 1910) and Lillie (*American Journal of Physiology,* Vol. XXVIII, 1911). In accordance with this hypothesis the stimulation of nervous tissue is conditioned by an increase in the ionic concentration of its native dissolved substances. Certain substances, then, contained in the terminal segments of the rods and cones suffer increased ionization under the influence of light and this increased ionization, via the mechanism of the Nernst hypothesis (*vide infra*) initiates the visual impulse. Troland, independently of Nernst, makes the following definite assumptions concerning the mechanism of the visual impulse. "(1) The visual impulse consists in the actual propagation of the positive ion, I_+, from the rod and cone cells along the optic nerve and tract to the cerebellum (especially to the neurons of the cuneus in the cerebral cortex). (2) This propagation takes place with the speed of the visual impulse and occurs within the neuro-fibrils which are thought of as *molecular tubes* within which it is possible for even single ions to travel without encountering great resistance. (3) The manner in which an individual ion may be imagined to be propagated is as follows: The non-fibular portion of the nerve fiber is made up of a mixture of substances, certain of which are ionized and others of which are capable of constituting an osmotic membrane which is normally equally permeable to positive and negative ions. However, when a positive ion comes into contact with one of the neuro-fibrils the surrounding neural substance acquires a slight differential permeability, so that the negative ions are capable of moving within it more readily than are the positive ions. This being the case, the loss of negative ions into the surrounding tissues—say into the myelin sheath when this is present—results in the development of a positive charge within the core. The original positive ion thus finds itself placed within the influence of a positive field. Since this is a state of disequilibrium, if the ion is free to move—and if, as will be the case, its charge is much smaller than that produced in the nerve—it will travel in one direction or the other along the neuro-fibril. If we suppose the ion to have had an original impetus in the afferent direction, it will move in this sense. The resulting process is obvious. As soon as the ion has moved into a new region of the nerve fibril the permeability of the neural substances about it for negative ions will be altered as before, a new state of disequilibrium will be produced and the process will be repeated, the ion moving continuously in one direction within the fibril.'' The general nature of the visual cerebrosis, *C*, follows at once from the account of the mechanism of the impulse. *The cerebral state corresponding with any condition of retinal stimulation consists simply in the presence in the cerebral cells of the specific ions which are liberated in the retina by the action of the light.*

274. Writing on the electrical phenomena in the stimulated and non-stimulated eye, Troland says:—''We have supposed that the rods and cones of the retina are the seats of the production of an equal number of positive and negative ions and that, of these ions, the former are propagated along the optic nerve in the form of the visual impulse. It follows that the negative ions remain unneutralized in the bacillary layer. Since the state of ionization is not quantitatively zero even in the absence of all light stimulus, it follows that if we examine a fresh and even unstimulated eye we shall find the cut surface of the optic nerve to be positive with respect to the layer of rods and cones, the latter being negative. Experiment shows this to be the case. (See Rivers, W. H. R.: *A Text-book of Physiology.*) The fact that the inner layers of the retina are normally positive with respect to the cut surface of the optic nerve may be explained by supposing that there is a large *impulse loss* (of the positive ions) in the synapses of these layers. This corollary also accounts for the negativity of the outer as compared with the inner strata, and of the nerve as compared with the ocular media and cornea. When light falls upon the retina, our postulates demand an immediate increase in the ionization of the molecular resonators in the rod and cone layer, the consequence of which is an increase in the impulse intensity, an increased impulse loss in the synaptic strata and an increased positivity of the optic nerve endings. If the entire retina is illuminated, the first electrical effect will be an increase in the negativity of the bacillary layer, owing to the departure of a larger number of positive ions per element of time. The second electrical effect will be an augmented positivity of the synaptic layers, owing to the discharge of the above mentioned positive ions into this region. These ions will be in part picked up by the fibrils of the optic nerve fibers, with the

result that an increased positivity of the cut surface of this nerve will ensue. Coincident with this, however, there will be a still greater enhancement of the positivity of the ocular media and hence of the cornea, by virtue of the increased impulse loss. We expect, therefore, that the incidence of light at the retina will result in a positive variation of the current normally established between the cornea of the eye and the cut surface of the nerve and that with an injured retina this will be immediately followed by a negative variation. Both of these expectations are fulfilled by experimental data. When the stimulus is removed the flow of positive ions along the nerve will immediately decrease, but on account of its relatively large mass the

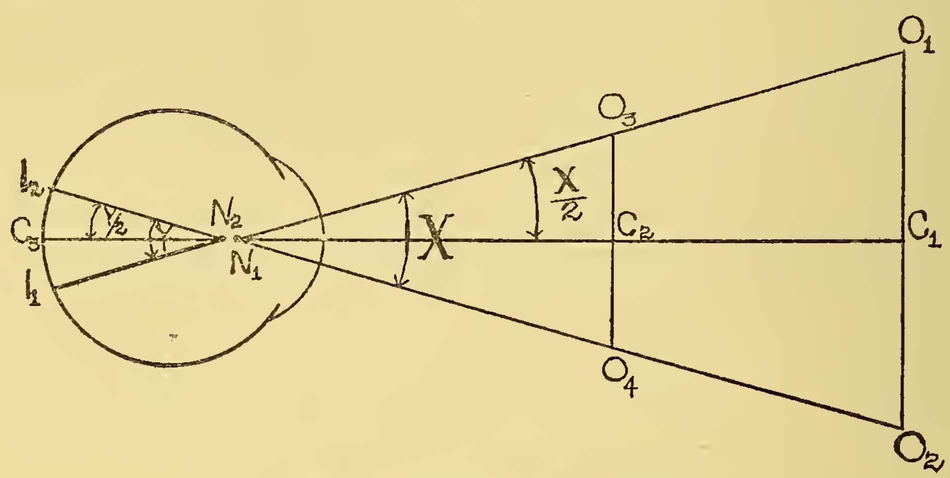


Fig. 147.—The Visual Angle.

charge of the ocular media will be only slowly lost; consequently the removal of the stimulus will effect a second 'positive variation' as shown by experiment."

XVIII. THE FORM SENSE

275. *Central visual acuity.* The perception of form or the visual acuity, properly speaking, is measured by the smallest angle under which the eye can distinguish the form of an object, or it is measured by the lowest angle under which two points can be distinguished from each other. The visual angle, *X*, Fig. 147, is comprised between the two lines which connect the two extremities of the object to the anterior nodal point of the eye; this angle is also equal to the angle *Y* which is formed by the lines connecting the extremities of the image with the posterior nodal point. By reference to the figure it will be seen that

$$\text{tangent}\ \frac{X}{2} = \frac{C_1O_1}{C_1N_1} = \frac{C_2O_3}{C_2N_1} = \frac{C_3I_2}{C_3N_2}$$

The principle of Giraud-Teulon states that "The acuity of vision of a subject is inversely proportional to the size of the minimum visual angle by which it can be impressed or affected." This minimum angle is modified and influenced chiefly by the luminous intensity, by the pupillary diameter, by the state of adaptation and by the retinal region which receives the image.

276. Astronomers have devoted considerable attention to this question of the visual acuity minimum. Hooke said that in order that a double star be recognized as such by an eye it is necessary that the interval correspond to one minute and that it would demand good eyes in order that such a recognition be made. Physiologists have made investigations using small gratings the bars of which were of the same size as the intervals; in a general way these were used for determining the maximum distances to which they could be removed from the eye before the bars and interstices became confused. The numerical results of these experiments made by such men as Tobias Mayer, Volkmann, Weber, Helmholtz and Bergman, in which gratings with various sized bars and intervals were used, vary from 51 to 94 seconds. It is, however, to be observed that it is neither the width of a bar nor that of the interval but the sum of the two which corresponds to the minimum visual angle as determined in these grating experiments. The minimum visual angle separating two points is commonly taken as one minute: a mathematical calculation (based upon the theory of the limit of resolution of a telescope) in which the radius of the pupil is taken as 2 millimeters, gives an angle of 42 seconds. If the calculation is made for the size of the retinal image corresponding to 1 minute there is obtained the number 0.0044 mm.,

since $\text{tangent } 1' = \frac{I_1I_2}{C_3N_2} = \frac{I_1I_2}{15 \text{ mms.}}$ (Fig. 147). The bacillary layer of the fovea centralis is made up of cones of about 0.002 mm. diameter. If, then, the least angle of visual distinction should correspond to the size of a cone we should have

$$\text{tangent } X = \frac{0.002}{15}$$

or the minimum visual angle would be about 30 seconds. Hence in the experiment of Hooke we may suppose that two stars can be distinguished if there is found a third cone between the two cones on which their images are formed; this third cone must evidently receive no impression. It may, therefore, be concluded that the angular size of a cone must be smaller than the angular distance separating the two stars. In the experiment of Helmholtz, however, it cannot be said that the size of the cone must be smaller than the angular size of the black bar; but we can conclude that the cone must be smaller than the angular distance separating the centers of the two neighboring luminous intervals or smaller than the sum of the black bar and the luminous interval, for if the size of the cones were the same as this distance all the cones would receive the same quantity of light and the bars would be confused. At any rate, the visual acuity does not appear to reach the degree which would be expected according to the retinal structure and the reason is doubtless to be assigned to optic irregularities, sizes of diffusion circles and other similar sources of optical inefficiency. It is rarely likely, then, that a luminous point forms its image on a single cone; the varying results obtained from the various experiments on the minimum angle of visual distinction are, without doubt, partially dependent upon the perfection or imperfection of the eye in optical details. It might seem permissible to measure the form sense by finding the *least angle of visibility;* this would involve the determination of the smallest visual angle under which an object could be seen. But it is evident that this latter angle depends almost solely on the luminous intensity of an object for we see, for instance, fixed stars very well when they are sufficiently luminous in spite of their minimum angular size. If an eye were optically perfect so that the image of a star could be formed on the surface of a single cone, then the object would be made visible if the luminous impression were sufficiently strong even though the image did not occupy the whole of the surface of the cone. But stars are not seen as points; their images are circles of diffusion composed of more and less luminous portions; when the illumination is feeble these less luminous parts disappear and the stars appear smaller. The image, therefore, generally covers several cones; if the light diminishes the image may be formed on a single cone and the visibility then depends upon the brightness only.

277. The following is a description of an apparatus for the measurement of the central acuity of vision due to Burch. He says:—"Two fine wires about 0.1 millimeter in diameter are stretched across a pair of sliding frames furnished with a screw adjustment so that they can be set parallel to each other at any desired distance from contact up to 3 centimeters. The frames are mounted in a ring which can be rotated behind a circular aperture in a screen so that the operator can alter the direction of the wires without affording any

clue to the observer. A sheet of white blotting-paper is fixed at an angle of 45° a foot or more behind the screen so as to receive the light of the sky and serve for a white background against which the wires appear as black lines. Ascertain the maximum distance at which the observer can distinguish the wires when they are 2.5 to 3 cms. apart. To make sure that he really does see them the operator alters the direction two or three times. Call the distance *l* and the diameter of the wire *d*. Then $\frac{d}{l}$ equals the chord of the smallest angle of black upon white that produces a visible effect.'' But Burch goes on to remark that ''This does not measure the visual acuity. Owing to inevitable irregularities of refraction in addition to the errors of spherical and chromatic aberration, the image of a point is not a point but is spread over an appreciable area. And the wire is visible, not when its geometrical image is large enough to be discerned, but when the black of it mixing with the white, and spread over that area, makes a perceptible shade of gray.''

278. The visual acuity is measured in practice by the charts of Snellen or some similar device. The letters are constructed so as to be under an angle of 5 minutes when viewed at the distance for which they are marked; the lines which form the letters and the intervals which separate various portions of a letter (such as the letter **E**) are seen under an angle of 1 minute. The basis of the Snellen charts is, then, double the normal acuity as determined by the grating in which each bar and each interval corresponds to about a half minute. In order, therefore, to have a standard with which we can compare, with the least trouble, the vision of an eye with that of the assumed normal we use letters that are known to be recognizable at the proper distance by eyes whose $V = 1$; in other words, at such a distance that their lengths appear to fill the angle of 5 minutes and their breadth (each limb) the angle of 1 minute; this distance expressed in meters or feet is called *D*. The formula for visual acuity then becomes the one generally used, $V = \frac{d}{D}$ or, in words, the vision of an eye is equal to the distance *d* of the smallest recognizable letter divided by that distance *D* at which a *standard* eye ($V = 1$) ought to read the same letter. It is obvious that the letters which are intended to be seen at a distance of 12 meters, for example, must have double the linear size of those which are seen at 6 meters. If the former are seen at a distance of 6 meters only we have $V = 6/12 = 1/2$. In some respects,

however, this is illogical; Javal and other writers have pointed out that we should, in the above case, say that the acuity is $\frac{1}{4}$ since the surface of the letter in question is four times greater than that which corresponds to an acuity equal to unity. And, again, although it has been supposed that a normal eye has $V = 1$ as a minimum on the standard adopted, this statement needs modification to some extent for it is obvious that all healthy eyes cannot have the same acuity of vision. As the acuity depends not only upon the quality of the dioptric apparatus but also upon that of the retina, of the optic nerve and of the brain, certainly differences must arise, hence V is not always unity in the normal eye. In youth it is frequently better than unity; $V = 9/6$ to $12/6$ is not uncommon and a case has been reported in which $V = 42/6.5$ or six and a half times the normal. The writer has seen a case of a man forty-two years of age, slightly presbyopic and possessing a distance error of not over a quarter of a diopter of hyperopia read the 13-foot letters with comparative ease at 40 feet, thus exhibiting $V = 3$.

279. Visual acuity gradually declines with increasing years. Donders and de Haan assert that from the thirtieth year on visual acuity sinks one-tenth for every ten years and that between the fiftieth and sixtieth years as much as two-tenths till in the eightieth year it may be only one-half normal. This *"de Haan law"* has not, however, stood the test of further investigation. H. Cohn examined one hundred persons more than sixty years of age and found that the decrease in visual acuity was extremely slight. Boerne and Walther examined over four hundred persons and found that in the healthy eye there was a slight and uniform decrease in acuity from the fortieth year on, but that the acuity in the eightieth year period was of the order of $6/9$.

It is also a well known fact that some letters are much more easily read than others on the same test line. The readability of a letter is, indeed, a very complex affair and does not appear to depend altogether on the size of the intervals separating the different lines of the letters. Various attempts have been made to remedy this by constructing the test-letters less readily seen on a slightly larger scale. Likewise, the manner and degree of illumination of a chart are questions of importance; at the present time various cabinet devices, with electric lamp and metallic reflectors within, ensure a fairly uniform and sufficiently high illumination. And, again, the background and surroundings of the black test letters should be as perfectly white as possible; the writer is convinced from his experiments along this line that the readability and hence the visual acuity is considerably affected by a cabinet frame of black as compared with one of white set upon a white wall. There has been recently pointed out the desirability of having a single small light source in a large uniform background in muscle testing; it seems, therefore, desirable that there should be some device invented whereby all these various tests may be made with the objects under inspection, and which serve as the basis of various tests, *rid of irradiation, color effects or presence of other objects in the field of view.*

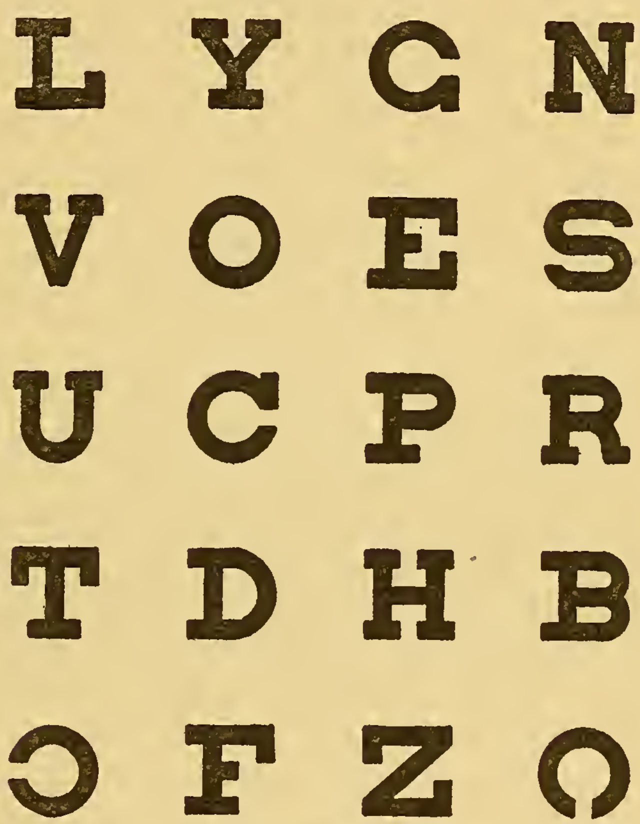


Fig. 147 (A).—Letters Subtending the Same Visual Angle: the Average Visual Acuity Required for the Recognition of Each of These Letters Varies from 0.71 for L to 1.00 for B. (From Report of Committee on Standardizing Test Cards for Visual Acuity, by E. Jackson, 1916.)

In the second Report of the Committee on Standardizing Test Cards for Visual Acuity of the section on Ophthalmology of the American Medical Association, of which committee Dr. Edward Jackson was chairman, we are told that three members of the committee made comparative tests on different individuals, noting the distance at which each letter was visible in the same light: the other two members compared the visibility of the letters at a fixed distance. The results of all tests and methods showed that B was the hardest letter to see and L the easiest. The letter B has been known for some time to be the letter which most nearly conforms to the standard five minute angle requirement of the Snellen test. Taking, then, the ability to recognize B at the five minute angle as standard vision, or V = 1, and comparing other letters with B, the average visual acuity required for the recognition of each is as follows:

L	0.71	D	0.82
T	0.74	Z	0.84
V	0.78	N	0.85
U	0.79	E	0.85
C	0.79	R	0.88
O	0.80	S	0.89
Y	0.80	G	0.92
F	0.81	H	0.92
P	0.81	B	1.00

The foregoing list necessarily gives only the order of the various letters as the averages of the different persons tested. It is perfectly obvious that two persons with full visual acuity and fully corrected ametropia will differ in the order of the visibility of these letters. "Apparently there is an individual difference in distribution of retinal elements, or in the connections and sensibilities of cerebral elements which prevents rigid exactness in the measurement of form vision." This is, therefore, an important reason for putting on test cards a fairly wide variety of letter forms.

"The positive suggestions for the use of these letters on test cards," says the Report of the Committee of 1916, "are as follows: The eighteen letters may be divided into four groups:

L T V U C
O Y F P D
Z N E R
S G H B

"Each group successively contains letters more difficult to recognize. Each line of the test card should contain at least one letter from each group; and the lines of smaller letters (the more used and more important lines of the card) should contain two letters from each group."

"The most readily visible letters have a visual acuity value of about three-fourths as great as that of the letters most difficult to see. If the interval between successive lines be not greater than this a continuous series is obtained, the easiest letter on one line being an appropriate test, when the hardest letter on the line preceding it has been recognized. When, however, the interval between successive lines represents a change of 20 per cent. in the visual acuity, the letters may be so chosen that reading half of an additional line will indicate a gain of one-tenth in vision, and the reading of the whole line two-tenths. A sample of a test card arranged to embody these suggestions is as follows:

Five Minute Distance, Meters	Letters Used	Vision if Half are Read	Vision if All are Read
50	B	...	0.1
25	U H	...	0.2
16.67	T Z S	...	0.3
12.5	L Y N B	...	0.4
10	V D Z R G	...	0.5
8.33	C F N E S H	...	0.6
6.25	T U Y O R Z G S	0.7	0.8
5	V C D F E N S B	0.9	1
4.17	L U O Y Z E S G	1.1	1.2
3.57	U T D P Z R B H	1.3	1.4
3.12	T L Y P N E H S	1.5	1.6

280. The only test objects whose dimensions can be exactly stated in terms of a visual angle are dots and gaps, for under constant conditions of contrast and illumination the visibility of a round dot depends solely upon the size of the visual angle it subtends. The dots are arranged in groups and the person under test is required to tell the number of dots in each group. Fridenburg has used these in what he calls a *stigmometric card test;* the dots are arranged in groups and the visual acuity is shown by the ability to count the numbers in the group. In the distance tests, instead of dots, squares are used, each square and the intermediate spaces corresponding to the Snellen *"minimum separabile"* of one minute. L. Wolffberg points out that in using a dot as a test object one complies with the principle that visual acuity is to be tested by the determination of the power of perceiving an interruption of continuity; for a dot is nothing but an interruption in the continuity of the surface upon which it is printed. Wolffberg constructed a table on which the test-object is a cross consisting of four black squares arranged upon a white square of equal size. In one of the black squares a white dot is placed whose diameter is one-third that of the square. Guillery, in 1891, proposed to measure the visual acuity by the distance at which one can distinguish a black point on a white background. By comparison with the Snellen chart this experimenter found that a black dot seen under an angle of 50 seconds corresponded to the normal acuity and that at five meters it should have a diameter of 1.2 mms. This point was designated as

No. 1; No. 2 had a surface twice as large as No. 1 and the patient who saw No. 2 only at 5 meters distance had an acuity of one-half. Javal constructed a small portable scale on the same principle; it is composed of small black squares such that the side of a square is always equal to the diagonal of the preceding one; in this way the area of a square is always double that of the preceding square.

281. A method for the expression of the degrees of acuteness of vision which can be used in all languages and which aims to give an official international standard of acuity was adopted by the International Ophthalmological Congress of 1909. The test type thus officially adopted is the invention of E. Landolt and is known as the *split ring test* (Fig. 148). The following are the six general principles laid down by the Congress of 1909 relative to the principles underlying the expression of the visual acuity:—(1) the test is based upon

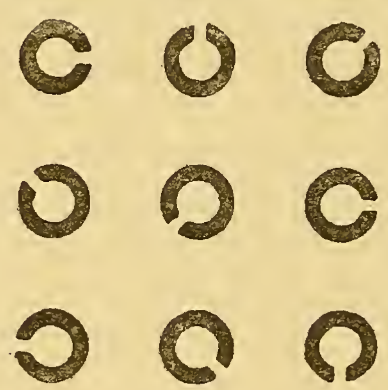


Fig. 148.—Landolt's Broken Ring Test.

the "minimum separabile" or the capacity to perceive an interruption; (2) the test is to be made by means of a black ring on a white ground, the ring to be broken at one place for a space equal to the width of the limb of the ring, which is one-fifth of its diameter; (3) the visual acuteness is to be expressed in relation to the smallest angle under which this can be deciphered, that is, to the maximum distance at which this can be done; (4) the visual angle of one minute is to be the standard of comparison; (5) the mode of expression is to be either in decimals or as a fraction ($V = \frac{d}{D}$), and (6) the eye is to be tested only at a distance from the test object.

Many believe that the charts in common use in consulting-room practice are admirably adapted to the subjective determinations of errors of refraction but that, as a test for visual acuity, they give the poorest and most inexact standards.

282. Various forms of type or arbitrary symbols all suffer from several defects; chief amongst these are that the transition from one size to another is not gradual and that the element of recognition enters in to assist the eye with certain details and not with others of the same size. Such defects are not overcome by variations of illumination or changes in the distance of the object. The ideal test object is, then, presumably one in which the size of the detail is the only variable. H. E. Ives has devised a simple apparatus in which he claims that this ideal is obtained. "It consists of a pair of oblique line gratings on glass with the lines so close as to be indistinguishable.

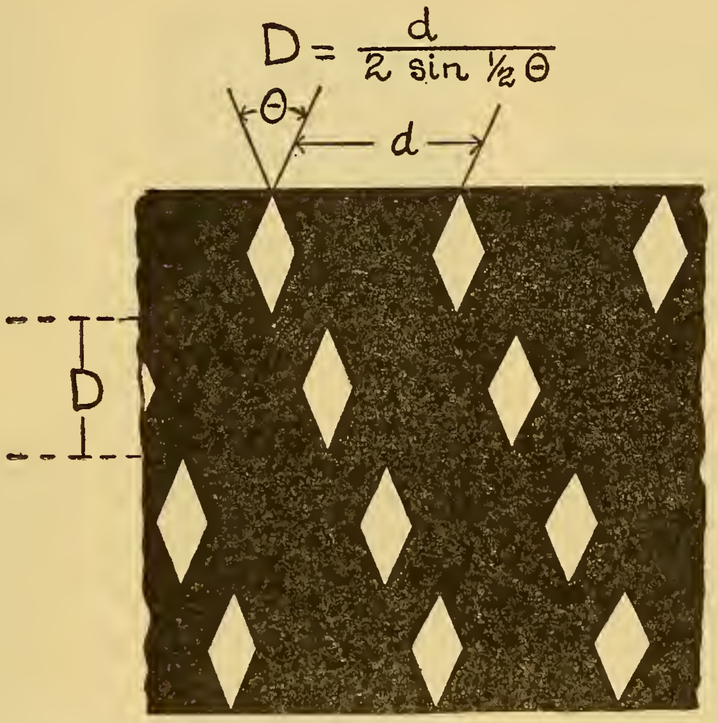


Fig. 149.—Illustrative of the Principles Involved in the Crossed Test Gratings of Ives.

These are superposed and rotated about an axis perpendicular to their surfaces. The result is the production of dark bands on a gray field. The separation of these bands is altered continuously by the rotation of the gratings. The dark bands occur where the opaque portion of the superposed gratings is continuous from side to side, the gray portions where the compound grating is alternately clear and opaque." As a result of the crossing of the lines, a set of parallel black bands appears in a direction perpendicular to the bisector of the acute angle formed and of variable width, separated by comparatively clear spaces. Mathematically it can be shown that the visual acuity varies in any case directly as the angle made by the two gratings when the resultant bands are just at the point of visibility. Fig. 149(A) shows photographs of the gratings. The upper set are microphotographs (reproduced) of the cross-gratings set at three different angles; the lower set are photographs showing the corresponding appearance of the gratings when held at a sufficient distance.

283. *Retinal topography of the visual acuity.* A study of the visual acuity as a function of the distance of the fixation point shows a very rapid diminution. The earliest experiments along this line were carried out by Volkmann and E. H. Weber. They employed the instantaneous flashes due to the discharges of a condenser in order to

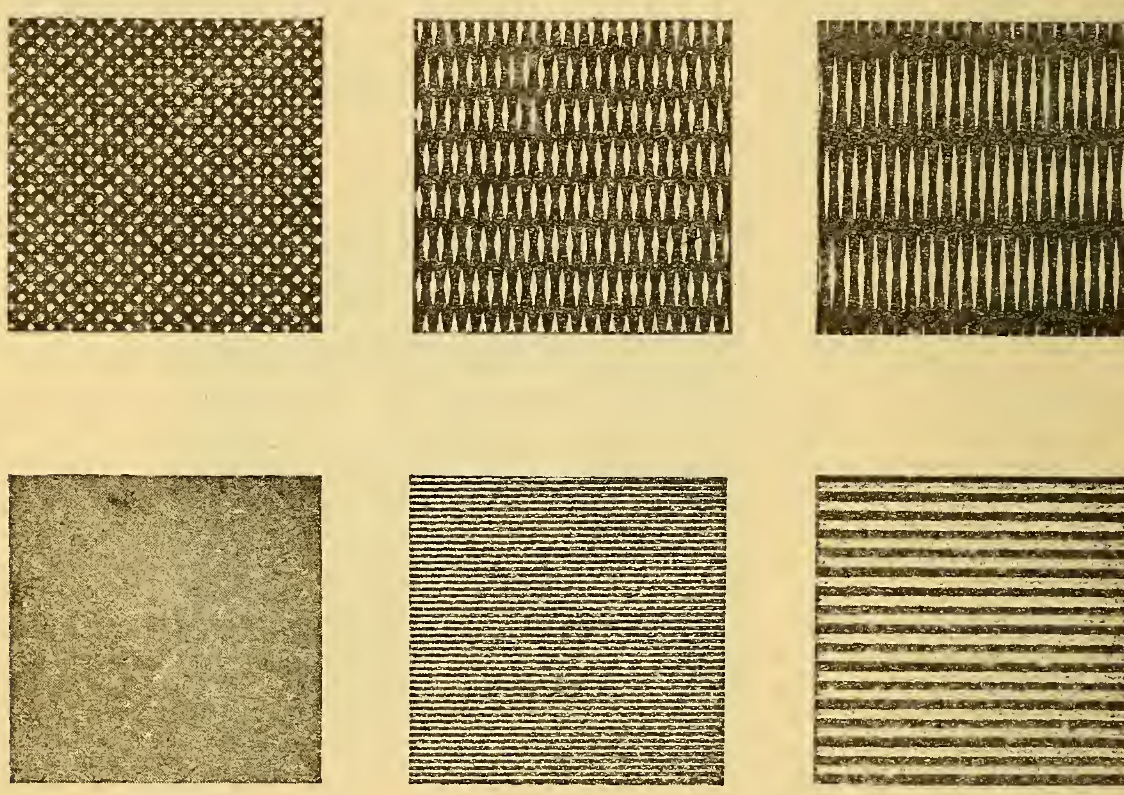


Fig. 149 (A).—Test Gratings Superposed at Various Angles. (After H. E. Ives.)

avoid involuntary movements of the eye which vitiate the results. Volkmann found the dimensions of the disassociable retinal image at various angles, measured from the fovea, given in the following table:—

0°	0.003 mm.	40°	0.193 mm.
10°	0.014	50°	0.301
20°	0.033	60°	0.442
30°	0.117		

Aubert and Foerster attempted to accurately determine the visual acuity of the various retinal zones by using different sized letters and figures from the Snellen charts which were momentarily illuminated by an electric discharge. In another set of experiments they employed a perimeter to the carrier or cursor of which they attached a card bearing two black squares separated by an interval of white. They then determined the angular distance at which these squares ceased to be distinctly separated. The position of the arc of the perimeter gave the azimuth in which they were operating. The results obtained by

Fig. 150.—Test Object Used by Landolt and Ito.

these investigators for the horizontal meridian of the visual field are briefly tabulated below.

Angle from the Retinal Center	*Corresponding Visual Acuity*
0	1/1
2° 52′	1/5
3° 13′	1/6
3° 51′	1/7
4° 17′	1/8
7° 14′	1/12
8° 32′	1/16
10° 13′	1/19
14° 37′	1/24
16° 17′	1/45
30° 20′	1/100

Landolt and Ito extended the investigations of Aubert, making use of four sets of squares represented in Fig. 150. The results of their observations are given below, the object number I, II and so forth referring to the set of squares used while the azimuths 0, 1, 2, 3, 4—7 indicate positions of the perimeter arc 45° apart starting in the vertical position.

		Azimuths							
Object		0	1	2	3	4	5	6	7
I	Landolt	12° 30′	11°	12° 30′	12°	18° 30′	16°	10° 30′	12°
	Ito	9° 30′	10°	11° 30′	10°	12°	13° 30′	10°	10°
II	Landolt	14°	13°	17°	16° 30′	22°	21°	13°	16°
	Ito	11°	12°	18°	12°	19°	17°	13°	13°
III	Landolt	21°	19°	23° 30′	22° 30′	24°	25°	21° 30′	20°
	Ito	16°	15° 30′	20°	20°	20°	22°	17° 30′	18°
IV	Landolt	25°	22°	30° 30′	28°	28°	30°	25°	26°
	Ito	20°	20°	27° 30′	25°	26°	27°	20° 30′	21°

Having given the dimensions of the perimeter it can be readily calculated that the visual acuity for the distinction of object I $= \frac{1}{14}$, object II $= \frac{1}{22}$, object III $= \frac{1}{35}$ and object IV $= \frac{1}{48}$.

284. Aubert and Foerster have given a graphical representation of the extent of the visual field in which they observed as separate two black dots of 2.5 millimeters diameter placed upon a white background at a distance of 14.5 millimeters the one from the other, the carrier being situated some 20 centimeters from the eye. Fig. 151 gives the

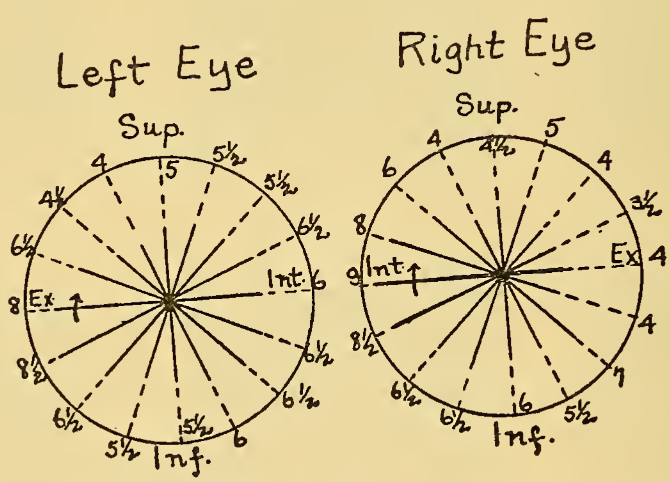


Fig. 151.—Curves Representing Extent of Visual Fields. (After Foerster.)

results obtained by Foerster; the numbers attached at the circumferences of the circles represent the distances in centimeters from the center of the visual field at which the two spots commence to be separated. It will be seen that the field for each acuity does not resemble a circle but rather an ellipse with its longer axis horizontal.

285. Fick and Koester determined the topography of the visual acuity due to the cones and that due to the rods. The acuity of the cones is the ability to distinguish forms when these are illuminated; the visual acuity of the rods is the ability which an eye, adapted to darkness, possesses of distinguishing the approximate form of objects very feebly illuminated. This latter property of the eye is a function of the visual purple and of the rods. Fig. 152 gives a survey of the retinal activities, the dotted lines representing the visual acuity due to the cones and the heavy line the visual acuity due to the rods.

286. Various hypotheses have been advanced to account for the variations of the visual acuity with the distance from the point of fixation. The cause may be attributed to a diminution in the clearness or sharpness of the image. But observations made upon beef eyes have shown that the images received upon the retina are perfectly sharp up to 70°—75° from the optic axis. These images are correspondingly smaller, it is true, at the periphery but this diminution is insufficient to explain the wide range of acuity values. One naturally turns, then, to the retina as the seat of these variations and it

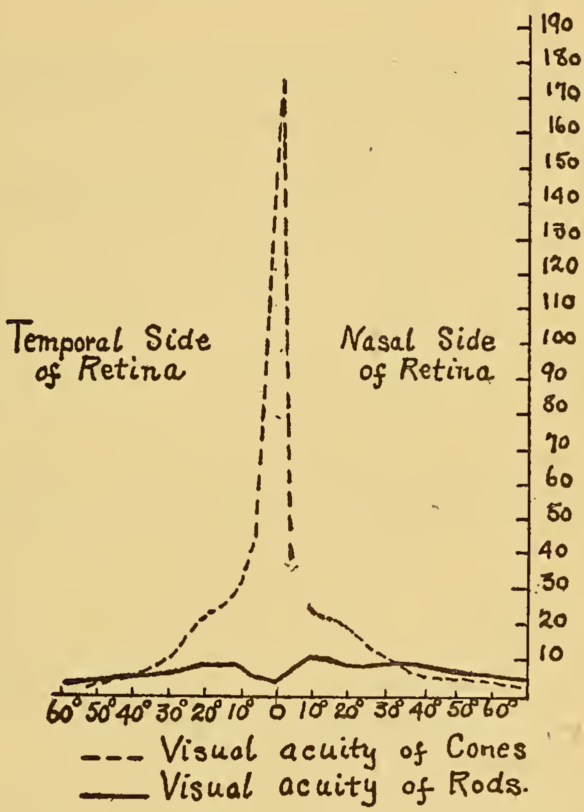


Fig. 152.—Curve (— — —) Shows Plot of Visual Acuity Due to Cones and Curve (———) Shows that Due to Rods.

is probable that the explanation is to be found here since the cones, as found in the macula, are replaced in the periphery by rods producing a sparse distribution of cones amongst the rods. It is doubtless a question of the nerve connections in the retina: the researches of Ramon y Cajal have shown that these nerve connections play an important but variable rôle. It is known, for example, that in certain cases of strabismus a new fovea or new center of fixation can be developed and that the visual acuity at this point can be augmented by exercise.

287. *The field of vision.* The field of vision of an eye is that portion of space from which an eye at rest can receive impressions of

light. A diagram of this portion of space may be projected upon any desired spherical surface described about the nodal point of the eye. The extent of the visual field is modified by the anatomical and optical structure of the bulb and by the surroundings of the eye itself. Investigation has shown that with a wide pupil the field of vision is somewhat larger than with a narrow pupil, other conditions being equal. Likewise, the field of vision becomes larger when the surface of the iris advances during accommodation for a near object; this advancement of the iris is, however, connected with the contraction of the pupil which reduces the field of vision. The extent of the retina must also be considered; in myopia of high degree, for instance, the rays entering very obliquely reach the fundus but are not perceived. In such a case as this the border of the field of vision would not be defined by the obliquity of the ray as it enters the eye but would be limited by the distance the retina extends toward the front of the bulb. Furthermore, it must be remembered that the fovea centralis does not lie exactly at the center of the retina but somewhat to the temporal side of it. The nasal side of the retina is larger than the temporal side when reckoned from the fovea centralis, hence the field of vision from the point of fixation extends farther toward the temporal side than it does toward the nasal side for, according to the laws of projection, the temporal side of the field of vision is referred to the nasal side of the retina.

288. The surroundings of an eye will affect the form of the visual field. A prominent nose or a protruding arch of the temporal bone will modify the field; deep-set eyes will be affected by the maxillary part of the socket. A droop of the upper lid will effect a noticeable reduction in the extent of the upper visual field.

289. The limits of the visual field can be determined with a perimeter or campimeter. The person under examination fixes the center of the arc of the perimeter or the center of the plane of the campimeter. The operator then proceeds to find the limiting position at which an object can be seen in indirect vision in various meridians. The object employed is usually a white square (or a colored one when the color fields are under investigation), the side of which is about one centimeter. The absolute limits of the field are found using a white object; using larger or brighter white objects does not give in general more extended limits. The reverse is true in the examinations with colors; by taking sufficiently large and bright objects one obtains larger limits than by ordinary examination. White, blue, red and green are usually employed in obtaining the fields of vision and it is found, as a rule, that the field is less extensive in the reverse order to

that in which the colors have just been named. The normal limits of the field are commonly given as tabulated below:—

	Outside	Inside	Below	Above
White	90°	60°	65°	55°
Blue	80°	50°	48°	45°
Red	65°	30°	32°	38°
Green	50°	25°	20°	28°

Deviations from these generally accepted normal limits are valuable factors in the diagnosis of and prognosis in various pathological conditions.

290. Bjerrum modified the ordinary procedure in perimetric examinations and made all tests at two meters, placing the subject in front of a black curtain and using small ivory discs of different but decreasing sizes fixed on black rods a meter in length. In this manner Bjerrum found as the limits of the normal field the following values:—

	Outside	Inside	Below	Above
With a disc of 3 mms...	35°	30°	30°	25°
With a disc of 6 mms...	50°	40°	40°	35°
Normal limits	90°	60°	70°	60°

It is said that Bjerrum's method reveals considerable contractions in cases of atrophy of the optic nerve, whereas the field as examined by the ordinary methods may appear normal.

291. There is only one interruption in the normal field and that is the blind spot which corresponds to the papilla. The form of the Mariotte's blind spot can be determined by the ordinary methods with the perimeter; the spot has an elliptical form. The internal border of this spot is about 12 degrees from the point of fixation and its diameter corresponds to about 6 degrees.

292. *Plotting the disc and macula.* It is of interest to actually locate by projection the position of the observer's disc relative to the macula and the posterior pole. The following description of the method of accomplishing this is copied from Laurance's *Visual Optics* (edition 1912).

"To do this place the eye, say the left, a certain fixed distance, e. g., 10 inches, above a sheet of paper, the other eye being closed or occluded. On the paper, mark a dot *M* and through *M* draw a horizontal straight line 'as in Fig. 153'; now take a pencil and, carefully fixing its point, cause the eye to travel slowly inwards away from *M* by moving the pencil to the right along the line. At a certain point *A*, *M* will disappear, showing that its image has reached the nasal side of the disc, where the sensitivity is nil. Continue the movement of the pencil until the dot M reappears, when the pencil has reached the spot B. AB is then the lateral (projected) diameter of the disc, on a horizontal drawn through the macula. In the same way, by moving the pencil upwards and downwards on a line bisecting AB, the upper and lower extremities C and D of the disc will be located, and a rough ellipse described through the four points will represent the approximate shape of the blind spot.

"From actual measurements it is easy to calculate the distances between the macula, posterior pole P, and the disc for any given eye. For example, we know that the distance MP between the macula and the pole represents 1.25 mm. if we assume that MP subtends an angle of 5° at the nodal point, i. e., that the angle alpha is 5°. Further, by measuring PA, it is a matter of simple proportion to find the length

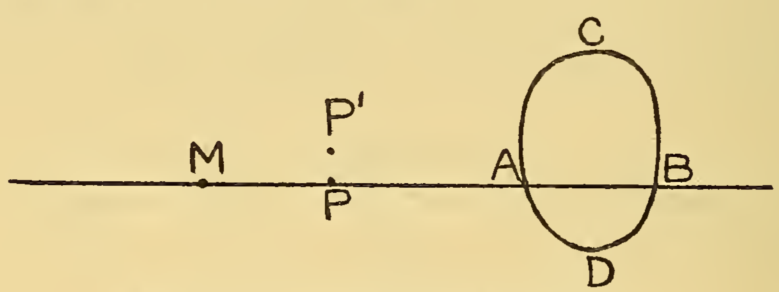


Fig. 153.—Plotting the Blind Spot.

of the eye to which PA corresponds. Thus, supposing the experiment were made at 10 inches or 250 mms. we have—

$$d : PA :: 15 : 250$$

where d is the actual distance from pole to disc required, and 15 the distance in mms. of the nodal point to the retina, whence

$$d = \frac{PA \times 15}{250}$$

"If PA be 40 mms., d is then 2.5 mms., which is about the value found in practice. It will be noticed that the horizontal line passing through M cuts the disc near the lower edge, proving that the macula is below the center of the disc; thus the maximum width of the latter is not represented by AB. Assuming the disc to be level with the posterior pole, the true position of P will be in the neighborhood of P^1, although for practical purposes we assume that the macula, posterior pole and center of disc, are all on the same horizontal line. The method of sketching indicated is easy and convenient, in that it gives an *upright* representation instead of an inverted one; the latter is obtained by fixing the point M and moving the pencil about to locate, as in the perimeter, the confines of the disc. This, however, is not so accurate unless a small object, such as the head of a large pin, be substituted for the pencil."

Factors Affecting the Visual Acuity

293. I. *Influence of age and of sex upon the visual acuity.* De Haan determined the average acuities for each decade as follows:—

Age	*Visual Acuity*
10	1.18
20	1.15
30	1.1
40	1.03
50	0.94
60	0.83
70	0.70
80	0.55

These results have been criticized and it is claimed by several investigators that the acuities as given by de Haan in old age periods are

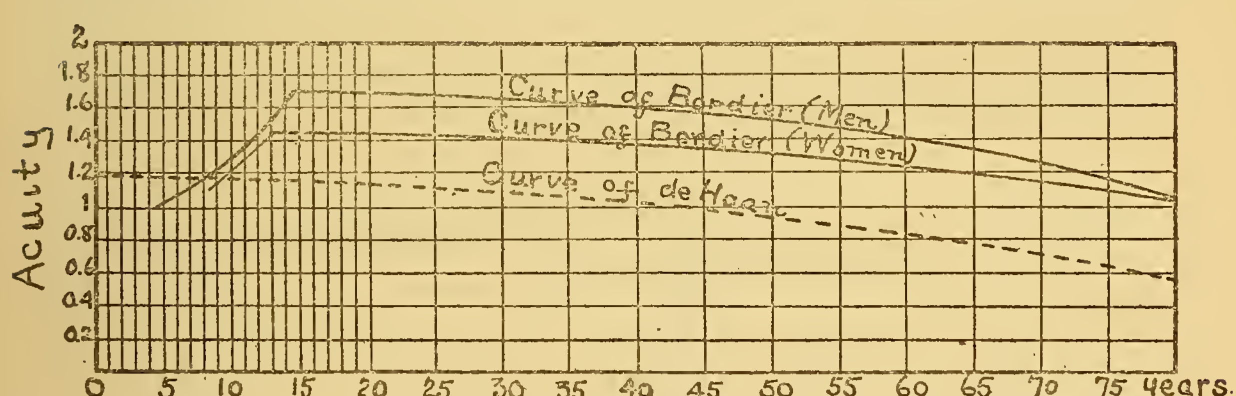


Fig. 154.—Curves of the Variation of the Visual Acuity with Age. (After Bordier.)

too low. Bordier has carried out extended researches on this question and has given his results on the effects of age and sex upon the acuity in the accompanying diagram (Fig. 154); de Haan's results are shown by the dotted line. These investigations show that, on the average, the normal acuities are higher for men than for women and that the maximum point is reached at an earlier age in life for females than for males; the visual acuities for the two sexes approach equality at about the 80-year point showing an acuity about equal to unity. It is a fact well known to all practitioners upon the eye that young emmetropes and hyperopes of low degree invariably possess, according to the commonly used Snellen charts, an acuity in excess of unity, ranging from $V = 20/16$ to $V = 20/10$.

294. II. *Influence of the retinal adaptation upon the visual acuity.* De Haan and Pickema found that there were noticeable differences in the visual acuities of their eyes under the same lighting conditions which apparently depended upon variations in the retinal adaptation. Broca has since investigated this phenomenon in some detail. He used an apparatus carrying either a black screen, in order that darkness adaptation conditions might be produced, or a white screen, suitably illuminated by an auxiliary light source the distance of which from the screen could be varied, which served as an adaptation center. The brightness of this source was then compared with that of the aerial image of a test object of known brightness. An artificial pupil of 2 to 2.5 mms. was employed to eliminate the influence of the pupillary diameter and thus obtain the true effects of retinal adaptation upon the form sense. Broca's results indicate:—(1) for low intensities of illumination the visual acuity increases with darkness adaptation, (2) for average luminosities the visual acuity remains the same whether the retina is adapted or non-adapted, (3) for high intensities of illumination the visual acuity decreases with darkness adaptation. The accompanying table gives a digest of some of the experimental evidence as to the relations between adaptation conditions, luminous intensities and visual acuities.

Brightness	*Visual Acuity*	
(Lux)	Eye Exposed to Light	Eye Adapted to Darkness
2	0.52	0.81
17	0.86	0.97
34	1.00	1.00
170	1.55	1.15

295. III. *Influence of the pupillary diameter upon the visual acuity.* The variations of the size of the iritic diaphragm exercise two effects, which are opposed one to the other, upon the visual acuity. The pupillary dilatation increases the visual acuity in that it re-inforces the luminous intensity of the retinal images; but, by virtue of its participation in the formation of these images in the eccentric portions of the retina, it diminishes their sharpness and therefore reduces the acuity. Hence the pupillary contraction which is favorable to the sharpening of the retinal images diminishes the luminous intensity; it follows, therefore, that the pupillary contraction will increase the visual acuity if the surrounding luminosity does not fall below a

definite amount. Likewise, if an ametropic eye is not corrected, the influence of the pupillary diameter is considerable, for the diameter of the diffusion circles is proportional to the diameter of the pupil. The chief difficulty in determining the influence which the size of the pupil of an emmetropic eye has upon the visual acuity lies in the exact experimental determination of the apparent size of the pupillary diameter, for if this can be found the real dimensions thereof can be obtained by calculation. Bordier, by a series of ingenious experiments, determined the apparent pupillary diameter by the aid of instantaneous photography; the visual acuity was monocularly determined for a given value of the size of the pupil. Various devices were employed to obtain large pupils; darkened rooms with charts illuminated by light reflected from external sources carefully screened from the observer were used. The results of Bordier's experiments are given below:

Apparent Diameter of the Pupil	*Real Diameter of the Pupil*	*Corresponding Visual Acuity*
2.06 mms. (daylight)	1.8 mms.	2
4.48 mms. (average illumination) .	3.9	1.85
4.65	4.04	1.8
6.9	6	1.75
9.58	6.8	1.70

It is obvious, therefore, why objects which are seen dimly become more distinct when viewed through a stenopaic disc, since the aperture, being smaller than the pupil, diminishes the circles of diffusion. This is the reason why myopes see better at a distance through a small opening. Such an opening can be, as a matter of fact, used as a magnifying glass; the object which is to be examined can be moved close to the eye and large retinal images thus obtained. The more the diameter of the aperture is diminished the sharper the image becomes but it loses at the same time in brightness; a certain limit cannot, however, be exceeded as the distinctness of the image becomes blurred by diffraction effects.

296. Percy W. Cobb has recently published (*American Journal of Physiology*, 1915) the results of some investigations upon the influence of pupillary diameter on visual acuity. One of the functions of the pupil is that of compensating for the defects of the refracting system of the eye. By a reduction of the size of the light-pencil the area of the diffusion circle is decreased and the image is sharpened, the effects of chromatic and spherical aberration and regular or irregular astig-

matism being partially eliminated or diminished. It is, however, a well known fact of physical optics that, owing to diffraction at the margins, the aperture of an optical system places a limit upon the resolving power of a system according to the expression

$$\theta = 1.22 \frac{\lambda}{D}$$

where θ is the angular separation in radians of two just resolvable bright points, λ the wave-length of the light considered and D the diameter of the circular aperture. The constant 1.22 is empirically determined. Hence, increasing the diameter of the pupil should bring greater clearness of image. Cobb's investigations attempted to weigh the relative importance of these two factors as they apply to the human eye.

The test-object was the Ives' cross-grating standard (previously described) which was so arranged that while observing it the observer could change the width of the lines seen upon its face and while he was so doing, keeping the lines always as near as possible at the point of just-visibility, the movements of the test-object were recorded on the drum of a kymograph. Artificial pupils of various sizes were used and placed close to the observer's eye. Since the visual acuity is a function of the illumination upon the retina, other factors remaining constant, and the object of these experiments was to investigate the clearness of optical images, the retinal illumination had to be equalized by varying the brightness of the test-objects inversely as the area of the artificial pupil and thus making the flux upon the eye constant. From his experiments Cobb concludes: (1) By the use of circular diaphragms before the eye it is shown that an aperture for optimal visual acuity exists somewhere between the limits of 1 and 5.6 mms. for brightness of test-object between 5.9 and 189 candles per square meter. (2) When the illumination of the test-object is compensated for the size of aperture to give equally illuminated images upon the retina, the optimum is somewhat less than when the test-object is constant and the illumination of the image depends on the pupillary area. In the former case it falls on the average at 2 to 4 mm., in the latter at 4 mms. Similar differences are shown in the case of each of the observers, but those having on the whole the highest visual acuity show also a larger optimum pupil. (3) With an aperture of 1 mm. diameter the observers give almost identical results, which agree closely with calculated results. (4) The optimum pupil corresponds on the whole with the size of pupil accepted as normal for all except extreme conditions: 2.8 to 4 mms. From this lower limit up to 5.6 mms. the variations in visual acuity with size of aperture are not large enough to be of practical importance. (5) The tabulated experimental results obtained from Cobb's curves show the following:—

Brightness Compensated		Constant Brightness of 189 candles per square meter		Constant Brightness of 5.92 candles per square meter	
Diameter of Artificial Pupil	Relative Visual Acuity	Diameter of Artificial Pupil	Relative Visual Acuity	Diameter of Artificial Pupil	Relative Visual Acuity
1	4	1	4	1	3.5
1.4	5.3	1.4	5	1.4	4.45
2.0	6.7	2.0	6.05	2.0	5.25
2.75	7.1	2.75	6.05	2.75	6.00
4	7.2	4	6.10	4	6.15
5	7.0	5	5.8	5	5.80
5.60	6.8	5.6	5.75	5.6	5.70

297. Broca has recently given the results of some investigations on the contraction of the pupil and the resulting loss of illumination caused by placing various sources in the direct range of vision. The well known effect of lamps placed in the field of view is to cause the pupil aperture to contract with the result that the luminosity of other moderately bright surfaces is enormously reduced and they appear very dark in comparison with the source from which their brightness is derived; indeed, for this reason, a bright lamp placed between the observer's eye and the object illuminated may even render it impossible to distinguish the latter at all. The following, for example, represent the contraction of the pupil and the resulting loss of illumination when various light sources are put in the direct range of vision.

Lamp	Pupillary Diameter	Fraction of Light Used
No source in field of view............	12 mms.	1
Incandescent lamp	8	0.43
Mercury lamp	6.8	0.32
Arc lamp in globe....................	6.7	0.25 — 0.34
Naked arc lamp......................	5.7	0.22

It will be seen, therefore, that the apparent gain in illumination following the use of a high illuminant is soon lost owing to the fact that it causes a marked contraction of the pupil; hence the use of bright sources in the direct line of sight is not only of marked inconvenience to eyesight but is not even economical.

298. IV. *Influence of the luminous intensity upon the visual acuity.* In general the visual acuity increases when the illumination is in-

creased and diminishes when its intensity is decreased. In investigating the visual acuities which correspond to different luminous intensities it is indispensable to obtain a "dark adapted" retinal condition. The visual acuity depends directly upon the illumination of the chart, but it is difficult to determine the relation in a precise manner since there are many factors which may affect it. The influence of the adaptation of the eye has been mentioned; the pupillary size, the manner of contraction of the pupil and degree of optic perfection all exert an influence. Aubert, in 1865, determined the surface of the aperture in the window of a blackened cabinet which would permit of the reading of the different lines of Jaeger type at a distance of one meter; from his results he was able to calculate the relation between the luminous intensity (I) and the visual acuity (V). Carp used smoke glass of various depths of shade of known absorption power. He found that ordinary daylight can be reduced to 0.05 of its normal amount without causing a diminution in the visual acuity of emmetropes; in the case of myopes and of the aged the visual acuity diminishes more rapidly, for a reduction to 0.12 of diffuse daylight produced a diminution in the acuity. Druault utilized artificial illumination in his measurements; his method consisted in moving a candle toward the test-chart and noting the distances at which the light would allow each line to be read, the eye being in a state of medium adaptation. For higher degrees of illumination he replaced the candles by a lamp equivalent to 54 candle power. The unit of illumination was taken as one candle at the distance of one meter, or the so-called "meter-candle." The experimental results of Carp, Druault and Uhthoff are as follows:—

Carp		*Uhthoff*		*Druault*	
I	V	I(m.c.)	V	I(m.c.)	V
0.12	1	0.0015	0.0015	0.016	0.075
0.07	0.96	0.0014	0.004	0.020	0.15
0.05	0.87	0.01	0.043	0.028	0.21
0.04	0.74	0.1	0.07	0.047	0.30
0.02	0.61	0.6	0.21	0.12	0.37
0.008	0.51	1.5	0.34	0.25	0.50
0.004	0.35	6	0.74	0.67	0.75
0.003	0.23	15	0.93	1.50	1.00
		36	1.14	16.7	1.25
		144	1.59	540	1.50
		1175	2.0		

All methods of observation show conclusively that the visual acuity increases at first quite rapidly with small changes in the illumination but that finally a very large increase in luminosity is demanded in order to change the acuity by a small percentage.

299. Piekema and Laan, two pupils of Snellen, investigated the influence of illumination upon the acuity by taking every possible precaution to exclude perturbing influences due to retinal adaptation. They reduced their observations to curves in which the abscissæ represent the acuities and the ordinates the luminous intensities expressed in meter-candles. These curves are reproduced in Fig. 155. It will be observed that the visual acuity of one experimenter was practically double that of his collaborator, yet both curves show the same general

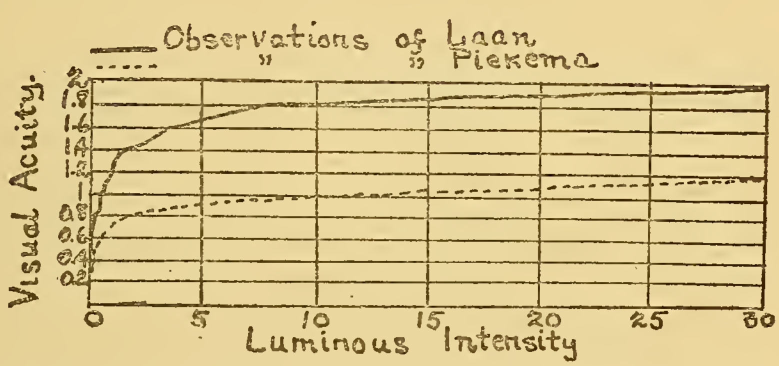


Fig. 155.—Variations of the Visual Acuity Dependent upon Variations of the Luminous Intensity.

effects of the intensity of illumination upon the visual acuity. Experiments made to demonstrate the effects of retinal adaptation show that such curves as those presented in Fig. 155 are modified in a manner which indicates that the increments in the acuity corresponding to small changes in the illumination are very much less for the non-adapted retina.

300. These relations between acuity and illumination for a single eye are altered somewhat when binocular vision is enjoyed. Nicati and Macé de Lépinay, and Snellen, have investigated this question as well as the influence of diaphragms upon monocular and binocular vision. The gist of their results is shown in the curves of Fig. 156 in which curve *A* shows the average results obtained binocularly without diaphragms, curve *B* monocularly without a diaphragm, curve *C* binocularly through diaphragms of 2.75 mms. diameter and curve *D*

monocularly with the same sized diaphragm. These curves show that the acuity-luminosity curves for binocular vision are of the same shape as those obtained for one eye only but give, as is commonly known, higher acuities binocularly than monocularly. The influence of apertures is seen to be most marked when the intensities of illumination are feeble; this is obvious since the pupils will be large under low illuminations, while the apertures of the diaphragm will be small in comparison.

301. We may, therefore, conclude in a general way that different methods and test-objects yield different absolute results but the relation between visual acuity and brightness of the background is in general the same. It is seen that when the brightness of the background is low the visual acuity increases very rapidly with increasing

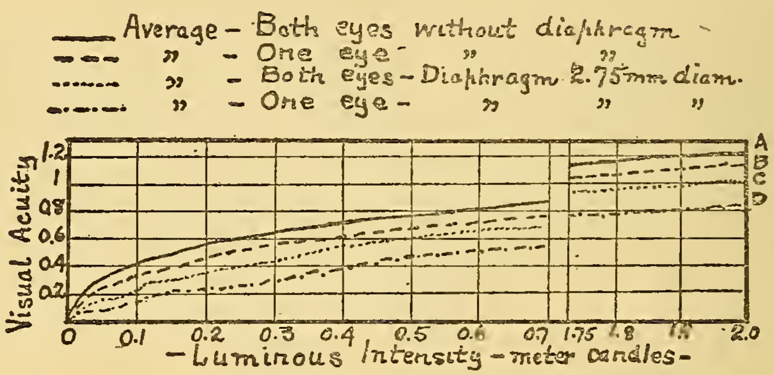


Fig. 156.—Variations of the Visual Acuity with the Luminous Intensity.

brightness of the background; after the brightness has reached a certain value, e. g., when the illumination has reached an intensity of one foot-candle, the visual acuity increases only very slowly with increasing brightness of the background. The relation for ordinary intensities may be thus expressed:

$$\text{Visual acuity} = k \log. I$$

in which k is a constant and I is the intensity of illumination. Experimentation shows that the value of k is not the same in the low and medium regions of illumination.

302. V. *Visual sensitiveness and acuity.* By the term visual efficiency is meant the ratio of the service rendered to the eye to the expenditure of nervous energy. The proper maintenance of visual efficiency demands attention to two important factors which we have discussed, viz., the sensitiveness of the eye to differences of luminosity

and the acuity of the eye. Fig. 157 indicates the variation of the sensitiveness of the eye to differences of luminosity and the variation of its acuity with the luminosity of the field when white light is used. It will be noted that the two curves become asymptotic after passing a luminosity of one lumen per square foot. This may be summarized into a working principle by which the degree of illumination required for any surface to meet the ordinary optical requirements may be ascertained, to wit:—the illumination of any surface requiring the continued application of the eyes shall be such that the light reflected or transmitted by it shall be equivalent to a luminosity of approximately one lumen per square foot. Inadequate luminosity and exces-

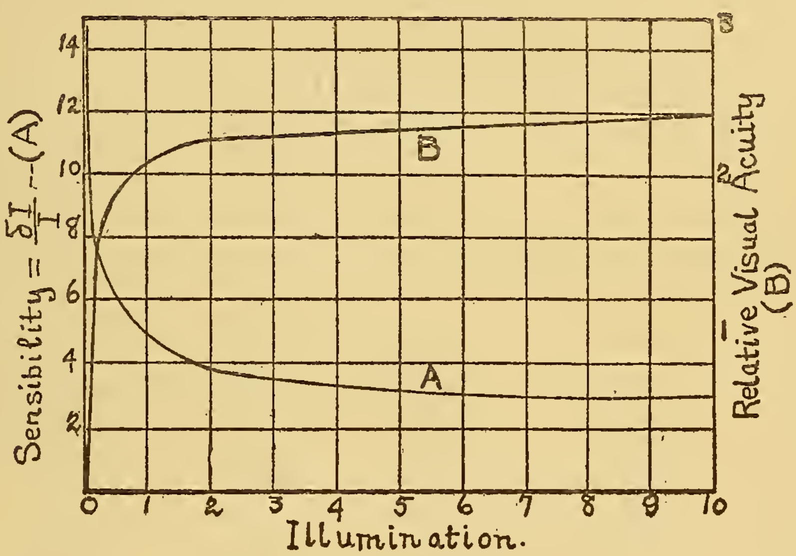


Fig. 157.—Sensitiveness and Acuity of the Eye for White Light.

sive luminosity of the direct field of view are about equally undesirable. In the former case the retinal images are not distinct and the eye grows fatigued in its efforts to sharpen their perception. Excessive brightness produces images which are intensely bright causing injury to the retina. Optical and engineering specialists differ in their estimates of the safe maximum of intrinsic brilliancy for direct vision; the values range from 1.75 to 5 candle-power per square inch of apparent light source.

303. VI. *Visual acuity in lights of different colors.* In his experiments upon the influence of the intensity of illumination upon the visual acuity Cohn made some observations using monochromatic light sources and found that the visual acuity attained a maximum value

under yellow light. Under an illumination of 36 meter-candles he obtained the following results:

Light	*Visual Acuity*
White	2
Yellow	2.15
Red	2.00
Green	0.66
Blue	0.37

Macé de Lépinay and Nicati as collaborators and Uhthoff came to the conclusion that it is possible to attain the same visual acuities with any color of illumination provided sufficiently high intensities are employed. The first two of the above named experimenters studied the visual acuities in the different colors as a function of the objective luminosity; they concluded from their curves that the acuity is the same for all radiations of wave-length greater than 5,070 Angstroms when taken in conjunction with the Purkinje phenomena. Taking as the unit for each color that luminous intensity which gives a visual acuity of 0.33 they obtained the following results:—

Visual	Intensity of Illumination				
Acuity	$\lambda = 5070$	$\lambda = 4970$	$\lambda = 4580$	$\lambda = 4420$	$\lambda = 4280$
0.47	5.00	8.18	....	....	....
0.42	2.67	3.71	5.48	6.51	....
0.33	1.00	1.00	1.00	1.00	1.00
0.26	0.48	0.38	0.22	0.21	0.18
0.22	0.33	0.23	0.13	0.12	0.10

It will be seen, therefore, that the visual acuity increases and decreases more slowly for the colors having short wave-lengths than for the less refrangible radiations for any given variation in the objective luminous intensity and that this difference is all the more accentuated when one has under consideration radiations which are more refrangible than $\lambda = 5{,}070$ Angstroms. The luminous intensity can vary considerably in the blue without appreciably influencing the visual acuity. And, again, if a test object is lighted in turn by different portions of the same spectrum it will be found that the clear recognition of the object is due solely to the illumination furnished by the less refrangible portion of the spectrum.

304. It has already been pointed out that the eye is not achromatic, with the result that the image of an object illuminated by light of

rather wide spectral limits is not sharply defined upon the retina. Bell has compared the acuity of the eye in tungsten and mercury arc lights and has obtained results indicating an advantage for this latter illuminant; this is probably to be attributed to the more nearly monochromatic character of the light from the mercury arc. Luckiesh has confirmed these results and extended them using lights of the same color but differing in spectral character; this is possible by employing proper absorbing solutions. Some of the results of Luckiesh are given in the subjoined table; it is to be remarked that "His data, except in case 4, were not obtained by the method of using fine detail at the limit of discrimination but instead in terms of equal readability of a page of type, which proved after some practice to be a rather definite criterion."

Relative Illumination for Equal Readability

Case	Source	Color	Approximate Foot-candles	Relative Illumination
1	Mercury Arc	Green line	2.0	1.00
	Tungsten	Green		1.75
2	Tungsten lamp	Yellow	4.0	1.00
	Tungsten lamp	Yellow (diff. shade)		1.33
3	Sodium line	Yellow lines	0.5	1.00
	Tungsten lamp	Yellow		1.66
4	Mercury Arc	Green line	0.5	1.00
	Tungsten lamp	Green		5.10

No stress can be laid upon the accuracy of the absolute values given above but these and similar results from other experimenters make it conclusively evident that monochromatic light is superior for discriminating fine detail. It has been found that monochromatic light is superior to daylight for such discrimination. In this case the Ives' acuity object was viewed against a white magnesium oxide surface which was illuminated to an intensity of approximately one foot-candle. The acuity on the Snellen scale was found to be 1.11 and 1.28 respectively for daylight and monochromatic green light of equal intensities; another experiment showed that for a visual acuity of 1.28 on the Snellen scale the intensity of illumination with daylight or tungsten lamps was nearly three times that required for the same visual acuity with monochromatic green light. As the brightness of the background was increased it was found that the difference in visual acuity under a given illumination of tungsten light and monochromatic light decreased. Luckiesh has recently conducted a very thorough-going series of experiments for determining the visual acuity in monochromatic lights. His results are platted in Fig. 158. Curves *a* and *e* represent extreme series of observations made by the experimenter showing the fluctuation in the ability of the eye to distinguish fine details; curve *b* is the average of a great many observations. These investigations indicate that monochromatic lights differ in their defining power and that yellow monochromatic light is superior to others in this respect. It was also found that for any given change in brightness of the test-object the change in visual acuity was least for yellow monochromatic light.

305. *Relations between the refractive condition of an eye and its visual acuity.* The relative size of the retinal image depends upon the visual angle under which the object appears and upon the distance of the second nodal point from the retina of the eye, the size of the

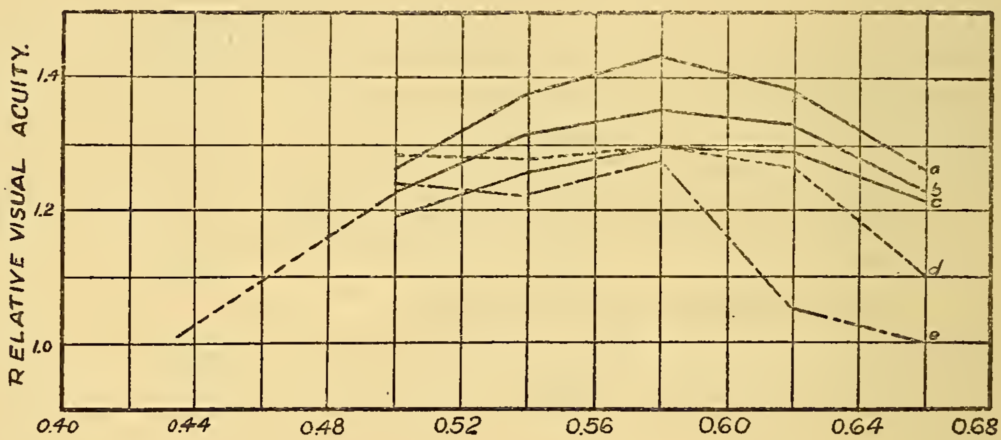


Fig. 158.—Visual Acuity in Monochromatic Light of Equal Brightness.

image being proportional to this distance. The distance of the nodal point from the retina and, therefore, the sizes of the retinal images varies due to the influence of accommodation and of the ametropia. In order to eliminate both sources of error Donders laid down as fundamental that the visual acuity be measured at a distance sufficiently large to exclude the influence of accommodation and that the ametropia be corrected. When the ametropic error is axial, the employment of correcting lenses gives to the retinal images the same relative sizes which they would have in an emmetropic eye in a state of repose on the condition that the correcting lens is in the plane of the anterior focus of the eye. In cases of ametropia due to curvature, however, the correcting lens still permits of the existence of an inequality in the size of the retinal image as compared with the emmetropic eye, but this inequality can be in general neglected under the methods in current use for determining visual acuity. In the state of accommodation, which is a condition of temporary myopia of curvature or, in certain cases where convex spheres are employed to replace accommodative action, the two nodal points will be displaced forward towards the cornea. This displacement increases the distance of the nodal points from the retina and, therefore, the size of the retinal images. For the accommodation necessary at a quarter of a meter it can be shown that the relative increase of the retinal image is approximately one-fifth of the size which it possesses when in a state of repose.

306. In Fig. 159 are diagrammed simple axial ametropic conditions in which the angle subtended at the nodal point is the same, but the retinal images of the object AB vary in size. E_1E_2 represents the image of the object formed by the emmetropic eye, while H_1H_2 and M_1M_2 indicate these images in cases of axial hyperopia and myopia respectively. We need, then, only to imagine three eyes which are

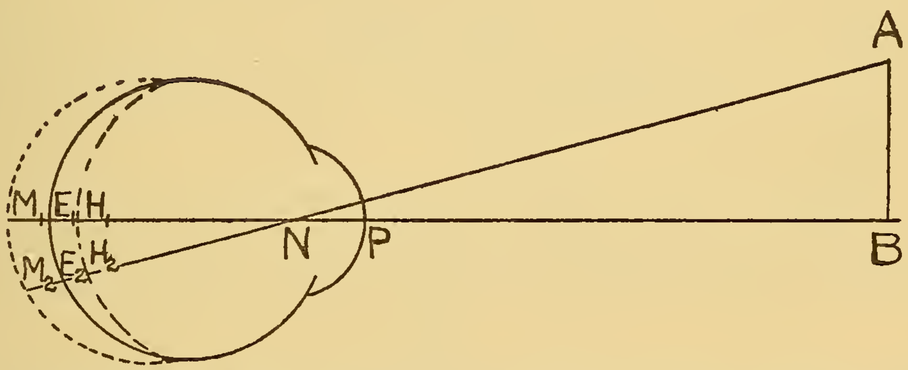


Fig. 159.—Relations Between Refraction and Visual Acuity—Axial Ametropias.

respectively emmetropic, myopic and hyperopic, the ametropia in the latter two being due to differences in axial length. With the accommodation at rest, each will form images equal in size by virtue of the equality of their dioptric powers, but the images of a distant object will not be clearly defined in the hyperopic and myopic eyes since they do not fall on the retina. The hyperope, with the aid of his accommodation, increases his refraction sufficiently to bring the image forward to the retina but at the same time he shortens his anterior focus and hence obtains a smaller image than the emmetrope. The myope is, of course, unable to obtain a sharp retinal image of a distant object. If, on the other hand, the three eyes view the same object at such a distance that all three can receive clear, sharp images upon their retinæ, i. e., let the object be at the punctum remotum of the myopic eye, then the latter, in which no accommodation is exerted, has the largest retinal image, the emmetropic image being next and that under the hyperopic conditions being the smallest.

307. In Fig. 160 we have considered curvature ametropias, which are produced by a variation in the curvature of the cornea or the crystalline lens. The object *AB* gives rise to the retinal images *RH*, *RE* and *RM* due respectively to hyperopia, emmetropia and myopia. N_2, N and N_1 represent the single reduced nodal points for hyperopic, emmetropic and myopic conditions. These three points, it will be remembered, coincide with the centers of curvature of the cornea of the reduced eye in each refractive state. The sizes of the images, therefore, increase from the hyperopic to the myopic states when due to curvature variations.

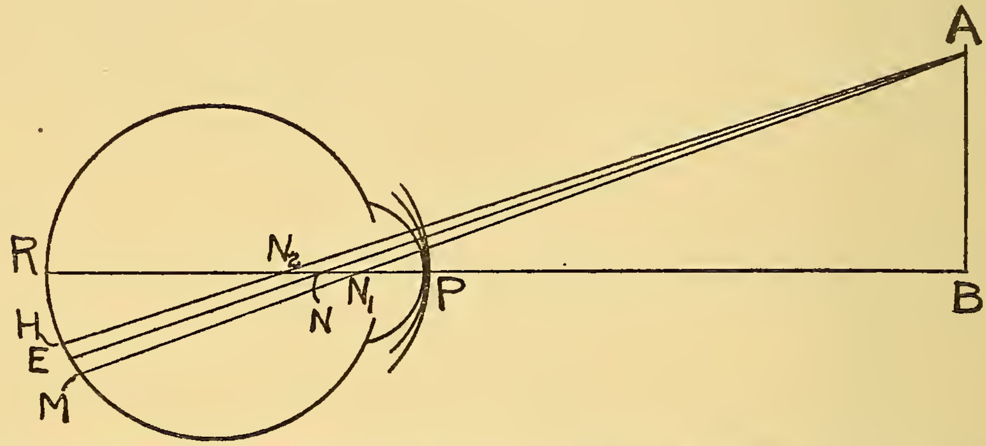


Fig. 160.—Relations Between Refraction and Visual Acuity—Curvature Ametropias.

308. If the ametropia is *refractive* due to abnormal indicial conditions, however, the axial length of the globe being the same in each case, the conditions described in connection with axial ametropia are reversed. When the accommodation is relaxed in the three conditions, the image formed in the vitreous of the myopic eye is the smallest of the three, the emmetropic being next and the hyperopic being the largest if the retina were to be conceived of as removed in order to permit of the formation of the image. Hence, with the object placed at the far-point of the myopic eye, there will be the same sized image in the hyperopic, emmetropic and myopic states since the hyperope and emmetrope, by means of accommodation, shorten the focal length of the eye sufficiently to enable the image of an object at a distance corresponding to the myopic punctum remotum to be formed on the retina.

309. It is not difficult to show that the following is correct in cases of the correction of axial ametropia, namely: *a lens placed in the anterior focal plane of the eye has no effect on the size of the image formed, the latter being merely moved forwards or backwards as the*

case may be; the image is, then, of the same size as in emmetropia. If, therefore, it were possible in cases of axial ametropia to place the correcting lens exactly at the anterior focus of each eye the retinal

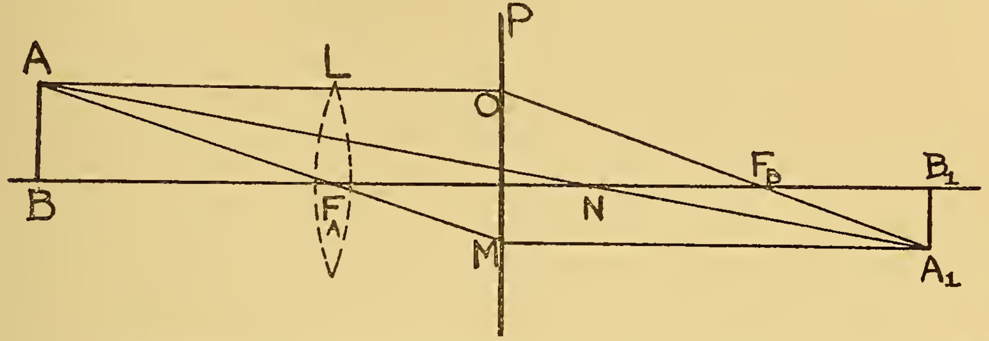


Fig. 161.—Image Formation not Changed in Size but in Position when a Lens is Inserted at the Anterior Focus of the Eye.

images would be identical in size neglecting the influence of aberration and distortion produced by the lenses. In Fig. 161, let *P* be the single refracting plane of the eye, *N* the single or united nodal point

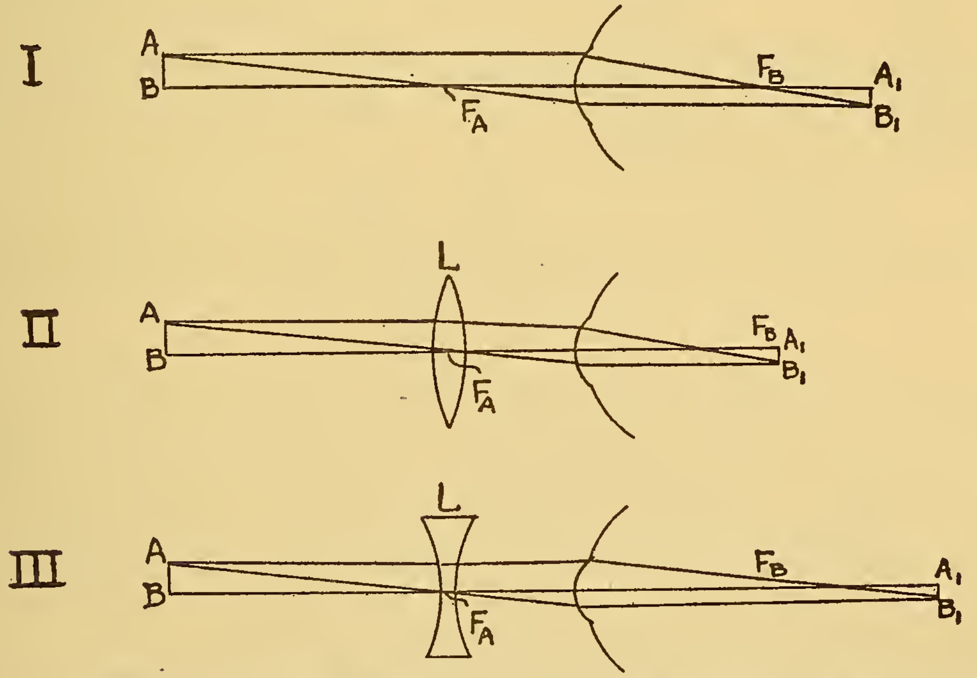


Fig. 162.—Effects of Lenses Inserted at the Anterior Focus of the Eye upon the Position of the Image.

and F_A and F_B the anterior and posterior focal points. The image of an object *AB* is easily constructed by tracing the courses of the three known rays *AO*, *AN* and *AM*. The ray, *AM*, passing through the anterior focus F_A, is refracted parallel to the axis BB_1; *AO*, parallel to the axis, passes, after refraction, through the posterior focus F_B and AN, through the center of curvature or single nodal point, proceeds without deviation. If a thin lens of any power or sign is placed at F_A, as in Fig. 161, the image will not be altered in size since the direction of the ray AM is unaltered by the lens because it passes through the optical center of this lens and the ray BF_AN passes through without refraction. To be exact, therefore, one should say that the optical center of the lens is to lie at the anterior focus of the eye. The image A_1B_1 is merely brought forward by a convex lens or carried backward by a concave lens. The three conditions are diagrammed in Fig. 162. Diagram I represents an object AB and A_1B_1 its image without an auxiliary lens before the eye. Diagram II is the same having a convex lens in the anterior focal plane, the image A_1B_1 being drawn forward but not changed in size. In Diagram III, the effect of the concave lens is to throw the image farther back of the cornea but to leave it of the same size.

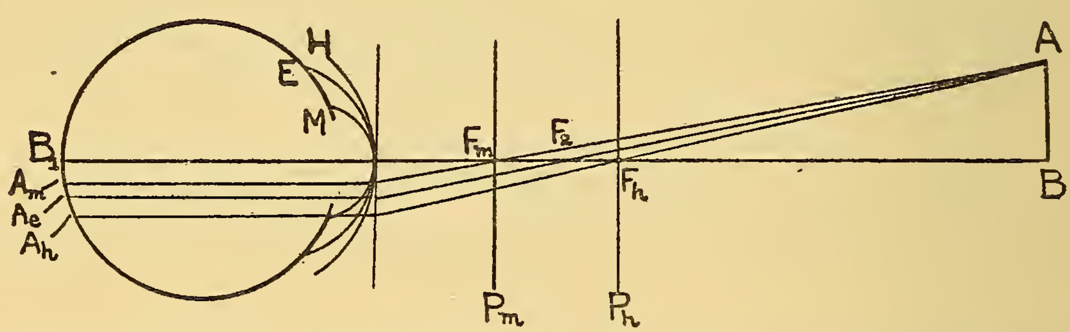


Fig. 163.—Illustrative of the Variation of the Retinal Image when the Position of the Anterior Focus Changes.

310. It is likewise easily seen from a consideration of these diagrams that the size of this image A_1B_1 will vary, however, when the position of the anterior focus F_A changes as in cases of curvature ametropia. The condition of affairs will then be somewhat as pictured in Fig. 163, in which F_e represents the anterior focus corresponding to the emmetropic refraction. The anterior focus, F_m, of the myopic eye will be nearer and the point, F_h, of the hyperopic eye will be farther from the cornea than for the emmetropic eye in *curvature* ametropias. P_h and P_m represent the correcting lenses for the curvature hyperopia and myopia placed at the respective anterior focal points of an eye possessed of these curvature variations. By construction, using the rays from A passing through these various anterior focal points, we find that the retinal image formed by the eye which is myopic due to curvature is smaller than that formed by the emmetropic eye which, in its turn, is smaller than the image formed under the curvature

hyperopia. If the lens which corrects the curvature ametropia could be placed in contact with the cornea it would then be possible to make the anterior focal distances of the myopic and hyperopic eyes equal to that of the emmetropic eye; in that case the retinal images would be of the same size in all three cases. Since the visual acuity is proportional to the size of the retinal image it follows that it will be increased by a lens correcting the hyperopia due to curvature placed at the anterior focus of the eye and diminished by a lens correcting the myopia caused by curvature when placed at the ocular anterior focus. Since the correcting lenses cannot be placed in contact with the cornea but are always placed at distances from the eye ranging from eight to fifteen millimeters, the retinal images of a series of test-objects thereby obtained do not indicate the linear distances corresponding to the visual acuity which they are presumed to measure.

311. Widmark measured the visual acuity of the myopic students in the schools of Stockholm and found that the visual acuity of corrected myopes, comprised between zero and eight diopters, showed a gradual diminution. The average values taken from his tables show the following:—

Myopia (Diopters Correction)	*Visual Acuity*
0.5	1
1.5	0.85
3.0	0.80
4.0	0.72
...	....
8.0	0.60

Seggel, after examining some sixteen hundred soldiers from twenty to twenty-five years of age, concluded that the apparent visual acuity, i. e., the acuity of the eye when wearing its correction, was less than for emmetropes. Nimier found an inferiority in the acuity of hyperopes. In cases of compound hyperopic astigmatism in which the irregularities of the cornea produce a diminution of the visual acuity it is found that the wearing of the correcting lenses gives larger retinal images than in emmetropia.

312. It is possible to distinguish between the visual acuity which the ametropic eye possesses when it re-unites upon its retina the luminous rays without the aid of correcting lenses and that acuity procured with external lens assistance. The first of these constitutes the *true* visual acuity of the ametropic eye and the second its *apparent*

visual acuity. Bordier * has defined the true visual acuity in the following manner: The true visual acuity of an ametropic eye is that which is obtained without changing the visual angle by correcting lenses. The acuity of an ametropic eye determined under the condition of constancy of angle is that which this eye possesses without the aid of correcting lenses when sharp, clear retinal images are obtained of test-objects presented in such a manner that the myopic eye receives divergent rays and the hyperopic eye convergent rays (Badal's optometer). Bordier proposed as a definition of the true visual acuity that which is obtained when the visual acuity of the corrected ametropic eye is measured by placing the test-objects in such a manner that an eye receives parallel light.

313. In Fig. 159 are shown the retinal images in emmetropic and axial ametropic conditions under the constancy of visual angle. In Fig. 164 is sketched a diagram of an emmetropic and of an hyperopic

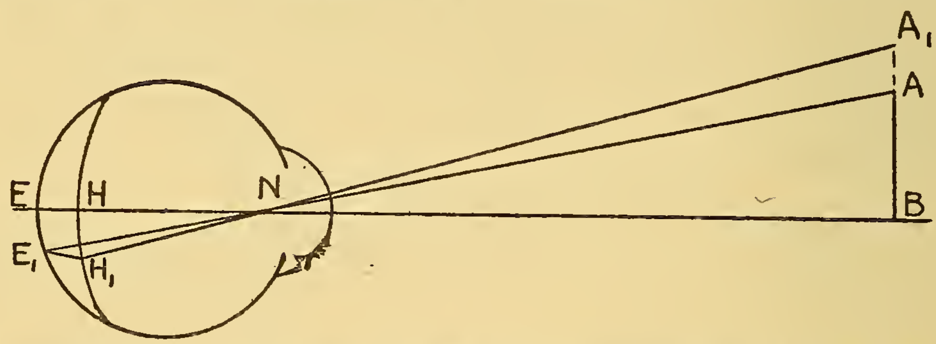


Fig. 164.—Equality of Retinal Images of the Emmetropic Eye and the Axial Hyperopic Eye Corrected: Inequality of Visual Angles.

eye in which the retinal images are made equal, the axial hyperopia being corrected by a proper convex lens placed at the anterior focus of the eye. The inequality of the visual angles in the two cases is apparent. We therefore conclude that the placing of a lens correcting an axial ametropia in the anterior focal plane of the eye produces an equality in retinal images by comparison with the emmetropic eye but that the object appears under a different visual angle from that of the emmetrope. For the hyperopic eye this angle is greater and for the myopic eye it is less than in emmetropia.

314. If, then, V represents the *true* visual acuity and V_a signifies the *apparent* visual acuity it can be shown from Fig. 164 or similar constructions that

$$V = V_a(1 \pm R \cdot f)$$

* D. E. Sulzer, writing upon the "Determination Qualitative et Quantitative des Fonctions de la Rétine" in the third volume of the *Encyclopédie française d'Ophthalmologie* says:—"Il (Bordier) propose de l'appeler acuité *vraie* de l'oeil amétrope, par opposition à l'acuité *apparente*, qu' on obtient quand on mesure l'acuité visuelle de l'oeil amétrope corrigé en disposant les optotypes (optometer of Badal — — inserted by the writer of this manual) de façon que d'oeil amétrope corrigé reçoit des rayons parallèles."

where R represents the degree of refractive error in diopters and f the length of the reduced emmetropic eye which we have taken as 20 mms. This formula shows that the true visual acuity of an axially myopic eye is greater than the apparent acuity, while in the case of hyperopia it is less. In the case of curvature ametropias the differences are in the same direction but considerably different in amount. The four formulæ can be written as:

$$(1)\ V_a = \frac{V}{1 + 0.02R} \text{ for axial myopia}$$

$$(2)\ V_a = \frac{V}{1 - 0.02R} \text{ for axial hyperopia}$$

$$(3)\ V_a = \frac{V(15R + 1)}{20R + 1} \text{ for curvature myopia}$$

$$(4)\ V_a = \frac{V(15R - 1)}{10R - 1} \text{ for curvature hyperopia}$$

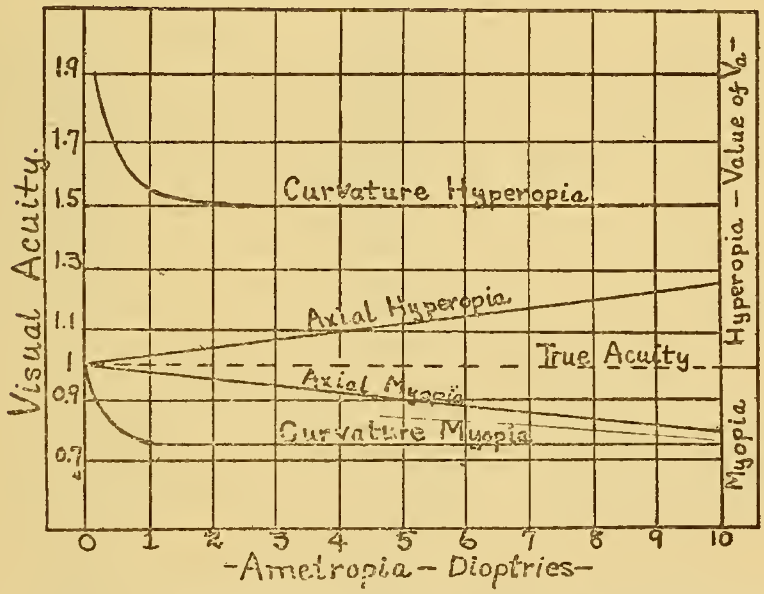


Fig. 165.—Graphic Representation of the Relations which Exist Between the True Visual Acuity and the Apparent Acuity in Different Conditions of Ametropia. (After Bordier.)

Fig. 165 gives a graphical representation of the relations which exist between the *true* and *apparent* visual acuities in different ametropic conditions from Bordier's work. These are the values of the apparent acuities, the basis of the true acuity being taken as unity.

By substituting in the above formulæ for V (the true acuity) the value of unity and in place of R (the degree of error) in turn the values from 1 to 10 diopters, one obtains the following tabular results:—

I. Axial Myopia

Degree of Myopia (R) in Diopters	For $V = 1$ Value of V_a is	For $V_a = 1$ Value of V is
1	0.98	1.02
2	0.961	1.04
3	0.943	1.06
4	0.926	1.08
5	0.91	1.1
6	0.89	1.12
7	0.87	1.14
8	0.86	1.16
9	0.84	1.18
10	0.833	1.20

II. Curvature Myopia

Degree of Myopia (R) in Diopters	For $V = 1$ Value of V_a is	For $V_a = 1$ Value of V is
0.2	0.80	1.25
1	0.762	1.31
2	0.756	1.3226
3	0.754	1.3261
4	0.753	1.3279
5	0.7525	1.3289
6	0.7520	1.3297
7	0.751	1.3304
8	0.75	1.3305

III. Axial Hypermetropia

Degree of Hypermetropia (diopters)	For $V = 1$ Value of V_a is	For $V_a = 1$ Value of V is
1	1.02	0.98
2	1.04	0.96
3	1.06	0.94
4	1.08	0.92
5	1.11	0.90
6	1.13	0.88
7	1.16	0.86
8	1.19	0.84
9	1.22	0.82
10	1.25	0.80

IV. Curvature Hyperopia

Degree of Hypermetropia (diopters)	For $V = 1$ Value of V_a is	For $V_a = 1$ Value of V is
0.5	1.625	0.616
1	1.55	0.643
2	1.526	0.655
3	1.517	0.659
4	1.512	0.661
5	1.510	0.662
6	1.508	0.663
7	1.507	0.6635
8	1.506	0.6639
9	1.505	0.6642
10	1.50	0.6644

PART FOUR

BINOCULAR VISION AND OCULAR MOVEMENTS

XIX. FUNDAMENTAL PRINCIPLES OF OCULAR ROTATIONS AND MOVEMENTS

315. *Center of rotation of the eyeball.* It is important that the position of the center about which ocular movements occur should be understood as definitely as possible before proceeding to a discussion of these movements. Different competent observers, including Mueller, Volkmann, Donders, Valentin and Barrow, have given considerable attention to this subject: their results are not entirely uniform but approximate each other. The earlier experimenters placed the center of rotation very nearly at the center of the optic axis. Volkmann, as a result of his researches upon the crossing of the lines of direction (center of similitude) believed that this point was situated at the center of the axis of the eye or 12.5 mms. behind the apex of the cornea. Helmholtz and Barrow obtained results in accord with those of Volkmann. Donders concluded that the distance at which the center of motion lies behind the cornea must undergo modifications depending upon the degree and kind of ametropia, recognizing as he did the fact that ametropia depends principally upon a difference in length of the visual axis. In conjunction with Doyer, Donders instituted researches from which the conclusion was drawn that in the emmetropic eye the center of motion is situated at a distance of 1.77 mm. behind the middle of the visual axis. Assuming the length of such an eye to be 23.53 mms., the distance of the center of motion

behind the apex of the cornea averaged 13.54 mms. and in front of the posterior surface of the sclera 9.99 mms. The subjoined table gives the average of results obtained by Donders for emmetropic, myopic and hyperopic subjects.

	Length of Visual Axis (mms.)	Position of the Center of Motion			
		Behind Cornea	In Front of Posterior Surface of Sclera	Behind Middle of Visual Axis	Angle Between Visual and Optic Axes
Emmetropia	23.53	13.54	9.99	1.77	5.1°
Hyperopia	22.10	13.22	8.88	2.17	7.55°
Myopia	25.55	14.52	11.03	1.55	2°

Donders' method consisted essentially in finding out how great the angles of motion must be, with equal excursions on both sides, in order to make the two extremities of the measured horizontal diameter of the cornea coincide alternately with the same point in space. This means that he determined the center of rotation by procuring the elements of a triangle of which one side, the diameter of the cornea, was known. The diameter of the cornea was measured by the aid of the ophthalmometer of Helmholtz, the flame of the lamp being placed perpendicularly above the instrument, and its image as reflected from the cornea viewed through the ophthalmometer. The cornea was illuminated by a separate lamp screened from the instrument. The eye under investigation was given a definite direction by fixing a sight or mire which was movable: thus it was possible to bring the eye into such a position that the reflected image of the lamp appeared exactly at the center of the cornea. The ophthalmometric images being doubled, it was possible to make the images of the flame fall upon the extreme border of the two images of the cornea. The number of degrees required to bring the double image into this position gave one-half of the chord subtending the cornea. It was then necessary to ascertain the arc which the cornea must describe in traversing this known distance, i. e., its own transverse diameter. In order to make this measurement a ring was suspended before the examined eye. It carried a fine hair placed perpendicularly. The number of degrees the eye must be moved in order that the hair should appear first at one and then at the other margin of the cornea was found; this number of degrees corresponded to the angle which the eye had described from its center of rotation. This angle in normal eyes was found to be about 50°. The knowledge of the half-diameter of the cornea and of the above range of motion then permitted of a determination of the center of motion; for if we call "p" the semi-corneal chord, "y" the half angle of motion and "x" the distance from the center of motion to the corneal chord we know that $p = x \tan y$. Giraud-Toulon criticized these measurements of Donders, saying that he assumed the very point in question. Mauthner later obtained results in fair agreement with those of Donders. *The eye, therefore, rotates about a point approximately at its geometrical center or the mid-point of its optic axis.* It possesses no translation, however, properly speaking: for if we regard the head as fixed and confine our attention to a study of the voluntary movements of the eyeball we find it approximately true that translation of the globe is prohibited by virtue of its attachments to the orbit. Since, however, the evidence is reasonably conclusive that the center of rotation of an eye is a trifle back of the geometrical center of the eyeball, it follows that the globe is slightly translated in whatever direction the eye is made to turn. Maddox says that in maximum excursions of the eye this translation is probably not less than one or greater than two millimeters. (For detailed mathematical and experimental proofs see Howe's *Muscles of the Eye,* Vol. I, pages 123-126, and an article by Ferree and Sheard on "Ocular Movements and the Center of Ocular Rotation," *Journal of Ophthalmology, Otology and Laryngology,* 1916.)

316. *Individual ocular muscles and planes.* All the rotations of the eye occur, then, about a fixed point known as the center of rotation. The six muscles which contribute to these rotations are divided into three pairs and each pair acting by itself causes the eye to rotate upon a definite axis which in every instance cuts the center of rotation. The plane of action of the *rectus internus* and *rectus externus* may be regarded as practically identical although from observations by Fuchs and Stevens we may conclude that there are variations from the ideal insertions and therefore variations from the rotations according to the rule. Hence the axis of turning for the eye under the influence of the externus and internus is vertical and by the action of the lateral muscles the eye will be turned exactly upon this axis and the axis will not be forced from its original vertical position.

317. As in the case of the lateral muscles, so also is the plane of action of the *superior* and *inferior* recti practically common to both. Unlike the lateral recti, however, the course of the superior and inferior recti is not parallel with the antero-posterior axis of the eye. Inasmuch as the center of traction from the insertion, both for the superior and for the inferior, is nearly at the point cut by a sagittal plane through the center of the eye and since the line of action in each case

is inward, it follows that this line of traction will not fall through the center of rotation but to the inner side of it. Due to this arrangement, therefore, it will be seen that if all the other muscles were at rest while the vertically acting muscles were in active and equal contraction, the eye would be rotated upon the vertical axis inward. The horizontal rotation axis for these two muscles cuts the optic axis obliquely. This rotation axis is at an angle of 70° in relation to the line of regard as determined by Ruete. Mauthner makes the angle with the transverse axis about 30°. This axis of rotation then points outward and backward and inward and upward and lies in the horizontal plane when the eye is in its primary position. But the action of these two muscles upon the eyeball cannot be uniform under all conditions: for in the primary position the axis of rotation for these muscles is at an angle of about 30° with the transverse diameter of the eye while as the eye leaves the primary position through, for example, the influence of the lateral muscles, the relations between the axis of rotation and the transverse axis of the eye must change. Hence a change of the direction of the optic axis outward 30° would bring this axis and the axis of rotation into coincidence so that in this position the action of the two muscles together would neutralize each other or, separately, one would roll the ball directly upward and the other directly downward without rotation on the optic axis. If, on the other hand, the eye could be turned *in* about 60°, the muscles would exercise their traction directly around the antero-posterior axis and acting together would rotate it inward, while acting separately they roll it upon this axis without modifying its direction. Hence at all points intermediate to these extreme positions the eye not only experiences a change in direction of its optic axis due to the separate action of these muscles but it must also be revolved upon the optic axis as a wheel upon an axle. The extent of this revolution must be dependent upon the angle between the axis of action of the two muscles and the transverse axis of the eyeball. If, then, as Stevens remarks, "There were to be found upon the cornea a vertical white line, this line, when the eye would be turned from the primary position inward (through the action of the internus) and upward (through the influence of the superior rectus) would be observed not only to move inward and upward with the general movement of the eye, but to tilt with its upper end inward and the farther the eye were to turn inward the more would the originally vertical line lean toward the median line of the face." The turning of the eye under the externus and superior would produce like results but with a tilting of the vertical line outward. These rotations of the eyeball about its own fixation line under the influence of contractions of the muscles are known as *torsions:* we shall discuss them further under *Listing's law.*

318. The third pair of muscles, *the obliques,* have approximately a common plane of action and like the other pairs of muscles they are mutually nearly antagonistic. Their axis of rotation is, like that of the superior and inferior recti, horizontal and this axis forms with the line of regard, according to Ruete, an angle of about 35°, according to Volkmann about 39° and according to Mauthner about 42°. These muscles, acting together against the center of rotation, rotate the eye inward; singly the action changes. Stevens says that it is questionable whether the first part of the preceding statement is correct; their combined action does, however, sometimes force the eye forward. Each of these muscles, acting individually, swings the anterior pole of the eye outward; the contraction of the superior oblique, when the head and eyes are in the primary position, causes the anterior pole to describe a curve downward and outward, while the contraction of the inferior oblique gives an outward and upward movement. The most notable result of the action of these oblique muscles is the rolling of the eye upon its antero-posterior axis. This action is similar to that of the superior and inferior recti but more pronounced; the action of the superior rectus, for example, draws the eye upward and inward and tilts the upper end of the vertical meridian of the cornea inward while the superior oblique, acting singly, turns the eye downward and outward and gives the vertical corneal meridian an inward turning of its upper end. A similar comparison can be made of the actions of the inferior rectus and inferior oblique.

319. As in the case of the vertical muscles, we find that the effective action of the obliques varies with the position of the line of regard with respect to the primary position. As the line of regard is carried to the temporal side their influence becomes less upon the rotations laterally and vertically while the torsion becomes greater, while as the line of regard is shifted to the medial side the influence of these muscles becomes greater in vertical movements as the degree of turning inward increases, while the torsion is proportionately reduced.

320. The muscular planes and axes of rotation for the vertical recti and the obliques are diagrammed in Fig. 166. The letters *R,R* refer to the muscular planes and *r,r′* to the axes of the recti, while the letters *O,O* and *o,o′* refer to corresponding terms for the obliques.

As to the primal muscular functions, then, we find that each eye possesses one muscle pre-eminent for abduction, namely the external rectus; another for adduction, the internal rectus; for elevation or supraduction, the superior rectus; for depression, infraduction or subduction, the inferior rectus; for intorsion, the superior oblique and for extorsion, the inferior oblique. In addition to these prime actions each muscle has secondary actions; this secondary action is a minimum with the lateral muscles which are purely adductors and abductors respectively except possibly when the eyes are elevated or depressed. We may say in brief summary that the superior muscles cause intorsion and the inferior muscles extorsion; the obliques abduction and the recti (superior and inferior) adduction as *secondary* or *subsidiary* actions.

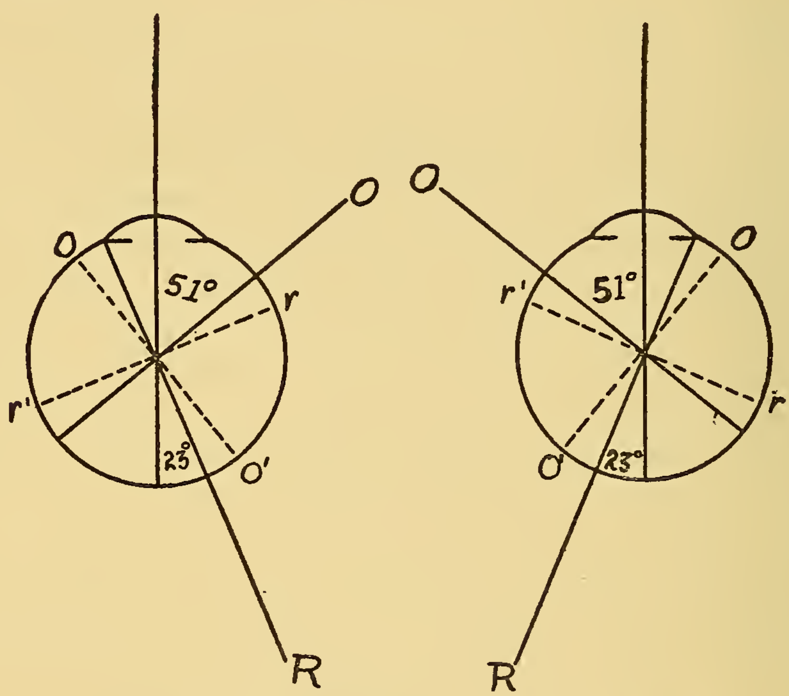


Fig. 166.—The Muscular Planes and Axes.

O—The muscular planes and oo′ the axes of the obliques. R, R—The muscular planes and r r′ the axes of the recti: O, O and o, o refer to the corresponding planes and axes for the oblique muscles.

321. *False torsion. Listing's law. Donders' law.* Donders observed that each time the visual regard returns to the same point, no matter in what way, the eye always reassumes the same position. If, for example, by fixing a colored ribbon stretched horizontally there is produced thereby an after-image which is then projected onto a wall, then, keeping the head motionless, it is found that the image assumes a position which is not in general horizontal but which is always the same every time that the visual regard returns to a given point. In other words, Donders observed that whatever position the eyeball may take there belongs to that position a definite amount of torsion which remains the same no matter how often the eye may return to that position and however many motions it may make in arriving at

it. He says: "For any determinate position of the line of fixation with respect to the head, thereto corresponds a determinate and invariable angle of torsion, a value independent of the volition of the observer, independent also of the manner in which the line of fixation has been brought into the desired position." Helmholtz has stated the same law more concisely as follows: "The wheel-movement of each eye is, with parallel fixation lines, a function only of the elevation angle and of the lateral deflection angle."

322. Experiment has shown that one degree of freedom is lost in all voluntary movements of the eyes which start from the straightforward position; latent torsion is not voluntary however. The degree of freedom which is lost is that of rotating about the antero-posterior axis considered as fixed in the head, the two degrees of freedom remaining being those of rotation about vertical and transverse axes. Simultaneous rotations about the vertical and transverse axes can be variously compounded into rotations about any intermediate axis. This amounts to saying that they are limited to rotations about all conceivable diameters in one plane, which is that plane containing the vertical and transverse axes and which is called or known as Listing's plane. Listing's plane passes through the center of motion of the eyes and is a vertical transverse plane fixed in the head and perpendicular to the fore-and-aft axis, about which no rotation can take place. When the head is erect and the eyes look straight forward at a very distant object on the horizon they are generally said to be in their *"primary position."* No matter, then, how many or complex the motions of an eye may be in glancing from point to point, the ultimate result is equivalent to a single rotation of the globe about some axis in Listing's plane provided the eye started from the primary position. The primary position of the eye is practically identical with the equatorial plane of the eye.

323. To revert again to Listing's law, it is to be said that this law goes a step further than that of Donders' and is as follows: "When the line of fixation passes from its primary to any other position, the angle of torsion of the eye in this second position is *the same as if* the eye had arrived at this position by turning about a fixed axis perpendicular to the first and second positions of the line of fixation." According to this law, therefore, an eye may be brought from the primary position to any secondary position by a rotation around an axis perpendicular to the two successive directions of the visual line. Listing's law means that when the eye starts from the primary position and glances toward an object situated obliquely, the line of fixation takes the shortest possible cut to its new position and must,

therefore, move along a plane common to its original and its new position. In order that this may be accomplished, the eyeball must rotate around an axis perpendicular to this plane and hence perpendicular to the line of fixation throughout the whole of its motion. This law is likewise essential to the rapidity of the ocular movements. "The exquisiteness of this design is apparent," says Maddox, "when one considers that no fewer than three muscles are concerned in every oblique motion of the globe, not one of which, acting individually, would rotate the eye about the required diameter." Certainly the arrangement on which Listing's law is based entails an absolute minimum of motion, hence it results that the momentum of the ocular contents is the least possible, the time is the shortest, the work done is the least and the lowest kinetic energy is developed.

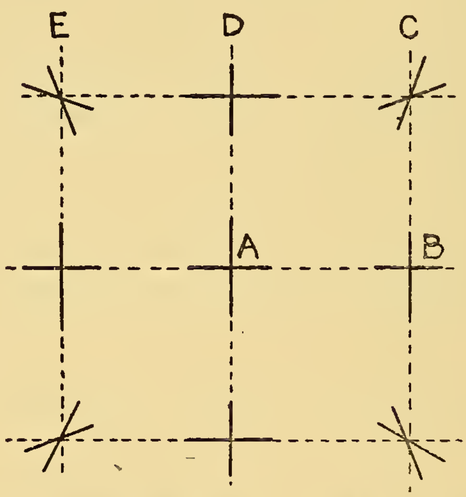


Fig. 167.—Demonstration of Law of Listing.

324. *Ruete's demonstration of Listing's law.* In the method, due to Ruete, the observer places himself at a distance of one or two meters from a wall on which is placed a fixation point *A* (Fig. 167) on a level with the eyes. A head-rest similar to that used on the ordinary ophthalmometer or better yet the planchette of Helmholtz enables the subsequently taken measurements to be obtained with fair accuracy. At the fixation point *A* (Fig. 167) is placed a cross with its arms horizontal and vertical. This cross ought to contrast boldly with the background; red on a gray is very satisfactory. With the head properly fixed it is possible to find a position such that, on moving the gaze along the prolongation of each of the arms of the cross, the after-image of this arm glides upon itself as indicated by the dotted lines. When this position is found the planchette is rigidly fixed in order that the observer may be able at all times to return to this condition in which, when the point *A* is fixed, the eye is in its

primary position. In fixing, then, a point *B,* situated on the prolongation of the horizontal arm, the eye has moved in its primary plane and hence it is apparent that the gaze may be swung from *A* to *B* by a motion around the vertical axis, which is an axis perpendicular to the two positions of the visual line. The same is true for displacements in the vertical direction. But if, in these experiments, the gaze is directed along the prolongation of the arms of the cross we observe phenomena which are apparently at variance with Listing's law. For if the observer, after raising his gaze, were then to turn his eyes to the right, as at *C* in the accompanying figure, the after-image would no longer remain vertical but would slope to the right; and again on looking to the left, as at *E,* it would slope to the left. There is then a rotation of the after-image. Hence it appears that this is a simple consequence of the law of Listing and that the meridian which was vertical when fixing *A* cannot remain vertical when the eye turns around an axis perpendicular to the direction *AC.* It can be experimentally demonstrated that the law of Listing is also verified in cases of oblique displacement by tilting the cross represented in Fig. 167 so that its arms are not vertical and horizontal but make an angle of 45 degrees with these directions. It can then be shown that the after-image of one of the arms of the cross glides all the time on its prolongation, hence the eye turns around an axis perpendicular to this meridian. In the experiments recorded above relative to the sloping of the after-images when gazing obliquely it might be concluded that when the eyes occupy oblique positions they experience torsion equal to that of the after-image. But since those portions of the wall upon which the after-images fall are not perpendicular to the line of sight, these slopes are exaggerated. As a variation of the experiment, let the head be rotated considerably to the left and kept temporarily fixed in that position. Then, after gazing at the cross or ribbon, turn the eyes up the wall immediately above the fixation object and the image will appear to become more twisted to the right the higher the gaze is raised. This proves that torsion does take place on looking up and to the right. But this experiment, though it correctly indicates the presence of torsion and the true sense in which it occurs, does not measure it correctly since there is not perpendicularity of plane and visual line in the secondary position. Le Conte remedied these defects by placing an experimental plane in such a way that the line of sight is at right angles to this plane when the gaze is turned up and out, down and out and so on, and thus obtained results uniform with the laws of torsion in all positions. To obtain correct results it is necessary either that the *accidental* *images* be projected upon a surface which is in every case at right angles to the line of regard, that is, upon the inner surface of a sphere at the center of which lies the eye, or that the plane upon which the observations are made shall be marked with lines representing a series of spherical coordinates. The diagram in Fig. 168, due to Helmholtz, shows the inclinations for the horizontal and retinal images for different positions of the line of regard when the secondary image is projected upon a plane vertical surface. Hence, when the line of regard is elevated and directed to the right the accidental image tilts to the left; when directed to the left, the image tilts to the right;

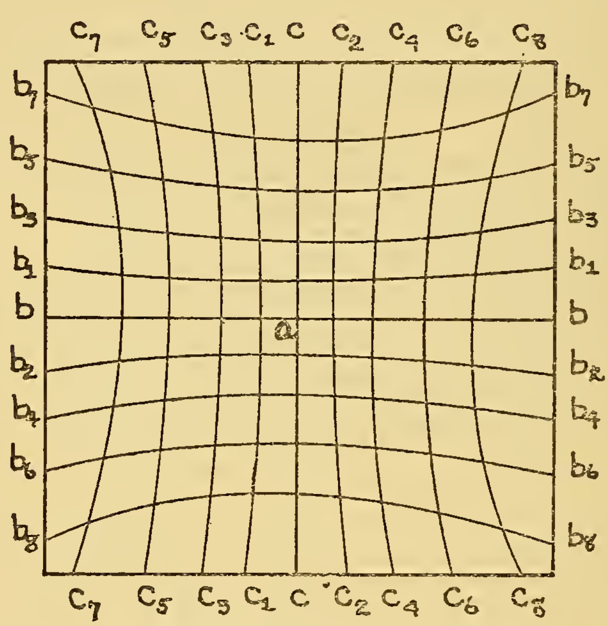


Fig. 168.—Inclination of the Horizontal and Vertical Images when Projected upon a Plane Wall.

If *a* is the first point fixed, the accidental image, as the regard passes from *a* to some other point, the position of the accidental image will conform to the direction of the line on which the regards rest. (Diagram from Helmholtz: description taken from Stevens.)

when the line of regard is depressed and directed to the right the accidental image tilts to the left and when directed to the left the image tilts to the right.

325. If the gaze then passes from one secondary direction to another the position of the eye is determined by the law of Listing since, having reached its new secondary position, it must have the same position as if it had arrived there starting from the primary position. For if the gaze passes from *B* to *C*, Fig. 167, following the prolongation of the vertical arm, it is observed that the after-image of this arm starts from the prolongation and rotates more and more so as to attain the position which it should have when the gaze has arrived at *C*. In

making this movement the eye does not rotate around an axis perpendicular to the visual line. If the line of regard is so displaced that the after-image moves at all times upon itself, then the point of fixation describes a curve the convexity of which is turned towards the point *A*. The same is true for the horizontal arm; if the point of regard moves from *C* to *E*, Fig. 167, so that its after-image moves on itself there is obtained a curve with its convexity downward. These statements are in agreement with the curves shown in Fig. 168. The following illusion, described by Helmholtz, is explained by the foregoing observations. If, after having fixed the point *A* in the primary position, the eyes are raised and survey quickly a horizontal line situated in a higher plane it will appear concave toward the floor. This may be explicable on the basis that oblique directions of the gaze are uncommon; when we desire to look at any object we ordinarily turn our heads in such a manner that the eyes are nearly in their primary position so that horizontal lines lie on the horizontal meridian. On account of this habit there is a tendency to consider the horizontal retinal meridian as horizontal even when it is not so.

Hence, in any of these movements which we have been considering, what we call the vertical axis at one instant ceases to be the vertical axis as soon as the globe has changed its position; the result is an apparent rolling of the globe although not a true wheel motion. This movement has been called by Maddox and other writers "false torsion" to distinguish it from the true wheel motion.

326. In order to get a clearer notion of the motions of this group it is desirable to make use of some form of ophthalmotrope. A simple rubber ball pierced by three needles at right angles to each other answers admirably. This should be modified or added to as follows: (1) Attach a thread to the anterior polar axis at its point of emergence from the globe. Make the thread a little longer than the radius of the circle representing the cornea and to the loose end of the thread attach a small weight so that this may act as a plumb-line. (2) Mark off about thirty degrees at the lower edge of the circle which indicates the edge of the cornea or any other circle on the eye concentric with it. (3) Transfix the ball with another needle which is to constitute the axis upon which the globe revolves into the oblique position. When the visual axis (the antero-posterior axis) is made to pass upward and outward, for example, the plumb-line shows that the originally vertical axis in the primary position no longer remains so but that another axis which marks another vertical plane has taken its place. The number of degrees between the two positions occupied by the plumb line indicates the amount of false torsion. These effects can

also be illustrated in a very simple manner by a circular disc of cardboard as suggested by LeConte and elaborated upon by Maddox. LeConte says:—"A simple experiment will show the kind of rotation which takes place in bringing the eye to an oblique position. Take a circular card, Fig. 169, and make on it a rectangular cross which shall represent the vertical (*VV*) and horizontal (*HH*) meridians of the retina. A small circle at the center represents the pupil. Now take hold of the disc with the thumb and finger of the right hand at points *VV* and place this line in a vertical plane. Then tip the disc up so that the pupil shall look upward 45° or more but the line *VV* still remaining in the vertical plane. Finally, with the finger of the left hand turn the disc on the axis *VV* to the left. It will be seen that *VV*

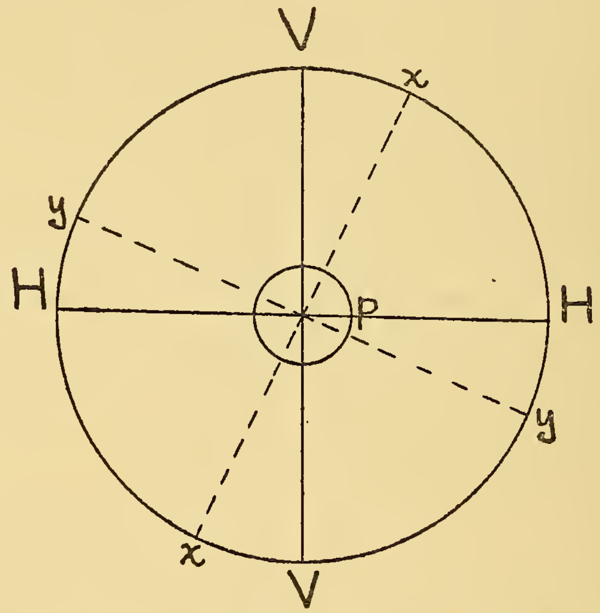


Fig. 169.—Cardboard Model to Illustrate False Torsion.

is no longer vertical, nor *HH* horizontal, but some other line *xx* is vertical and *yy* horizontal. In other words, the whole disc seems to have rotated to the left. But there is evidently no true rotation on a polar axis but only an apparent rotation consequent upon reference to a new vertical meridian of space."

327. We may, therefore, conclude:—

(1) When the eye moves from its primary position up, down, in or out no torsion occurs.

(2) When the eye moves in an oblique position the axis which is vertical in the primary position is replaced by another vertical axis, the former vertical axis now being oblique.

(3) The after-images which are projected on a flat surface are not in the same position when projected on a concave surface to which the visual axis is perpendicular.

(4) The so-called "false torsion" is not a true wheel motion of the globe.

(5) The exact amount of this torsion with parallel visual axes can be calculated for any given position of the visual axis. This was done originally by Helmholtz; his formula is

$$\tan\left(\frac{\Upsilon}{2}\right) = \tan\left(\frac{\alpha}{2}\right) \cdot \tan\left(\frac{\beta}{2}\right)$$

in which α is the vertical movement, β the lateral movement and Υ is the size of the angle of rotation. The proof of this is given in Helmholtz *Handbuch der Physiologischen Optik;* further amplifications are to be found in Howe's *Muscles of the Eyes,* Volume 1.

(6) The relation of this form of torsion to the positions of the double images seen in certain cases of paralysis is sometimes important. Suppose, for example, a paralysis of the abducens (sixth nerve) on the right side. The axis of vision of the right eye then turns towards the left or nasalward; a case of homonymous diplopia will arise. The image with the left eye is in its normal position. To a patient viewing a candle held vertically and directly in front, no torsion or tipping of the two candles as seen by virtue of the diplopia will occur. But if the candle is moved upward and to the right the image which is seen with the right eye is no longer vertical, the upper end now being tipped more or less away from the median plane. The tendency of the eye is, of course, to undergo the kind of torsion which it would if one of the muscles involved were not paralyzed. This eye in this condition acts practically independently of its mate; the degree of the paralysis together with the factors stated in Donders' law determine the amount of inclination given to the false image.

328. *The directions of the apparent vertical and horizontal meridians of the eye.* If the point of regard of the two eyes is fixed in the median and horizontal plane and at an infinite distance so that the head of the observer is in the primary position it might be assumed that the vertical meridians of each retina would coincide with the plane perpendicular to the plane of regard and that the horizontal meridians would coincide with the plane of the gaze. But the views of some of the ablest experimenters are at variance upon this proposition. Helmholtz concluded that the horizontal retinal meridians so nearly coincide with the plane of regard that they may be considered as identical, but characterized the vertical meridians as "apparent" only, for he reasoned from his own experiments and those of Volkmann that these apparently vertical meridians actually converged

downward to the extent of 1.25° for each eye in normal eyes, thus making an angle between the two vertical meridians about 2.5° with the lower extremities of the meridians approaching each other. Meissner and Hering found tiltings of the apparent vertical meridian, while LeConte found the vertical meridians to be vertical in his own case but his proof is not conclusive.

329. Tscherning has described a phenomenon, first observed by Meissner, which we encounter when we wish to judge whether a line is vertical or not. A plumb-line is held in front of a uniformly painted wall and a point situated a little in front of this line is fixed; one must take care not to approach too closely to the line else the influence of convergence will enter into this experiment. Under these conditions of fixation at a point slightly in front of the line, the line will be seen double. One would expect to see two vertical and parallel lines; the two lines, however, appear to converge upwards; seen with the right eye the upper extremity of the line seems to lean to the left according to Tscherning. If a point behind the line is fixed the images are crossed and appear to converge downwards. A vertical line seen with one eye only does not then appear vertical but its upper extremity leans either to the left or the right depending upon which eye looks at it. A rectangular cross, carrying vertical and horizontal arms, will therefore appear differently to each of the eyes, for the two angles at the upper right and lower left will appear to the right eye larger than the other two angles, while the reverse is true for the left eye. Since a vertical line appears to lean to the left for the right eye, there should exist then a line actually leaning to the right which appears vertical. That this is the case can be demonstrated by taking a white circular disc which can be rotated about its center and drawing thereon a diameter. Along the border is a scale graduated in degrees and so arranged that the zero corresponds to the position of the line when vertical. This scale must be placed so as not to be visible to the observer. The observer then states when the diameter appears vertical; it is found experimentally that in the majority of cases, using the right eye, the upper extremity will be placed some degrees too far to the right and with the left eye some degrees too far to the left. It is, of course, necessary that the experiment be arranged in such a manner that the observer cannot correlate the line with surrounding objects. In the case of the horizontal meridian the phenomenon is less apparent.

330. Volkmann has given a method of determining the apparent vertical meridians of the two eyes in which two small revolving discs are placed on a vertical support so that the distance separating their

centers is equal to the distance between the eyes. A radius was drawn on each disc and the discs placed as shown in Fig. 170. One of the discs was placed so that its radius was vertical; the discs were then observed as with the stereoscope, the right eye fixing the right object and the left eye the left object. The attempt was then made to cause the two radii to form a single straight line; it was found that they must form an angle of about two degrees before this was possible. Or, if a stereoscope is used which carries small discs like those of Volkmann and upon which are drawn radii exactly parallel, it will be found that on fusion the two discs will form but one, but the diameter appears broken.

331. It is probable that the downward direction of regard which is demanded in nearly all of our everyday pursuits may be the cause of these phenomena; certainly we are accustomed to some convergence and downward gazing in reading and near work and even in walking,

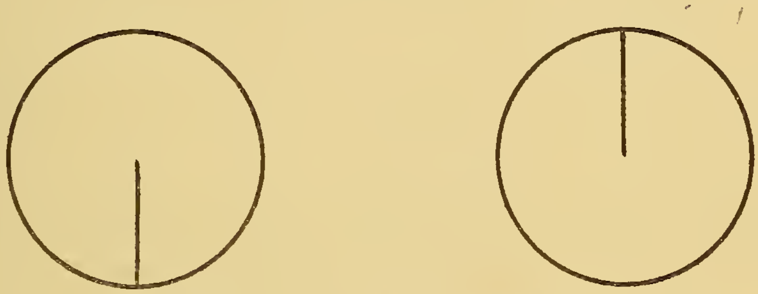


Fig. 170.—Discs of Volkmann.

when the gaze most frequently follows the ground. If the experiments of Meissner are repeated in such a manner that the lower extremity of the plumb-line is approached toward the observer until, in relation to the line of regard, this line occupies about the same position as would the page of a book when held in the customary position for reading, the two images of the line will appear parallel.

332. All of these phenomena can be readily observed by using a pencil or a hat-pin placed ten to thirteen inches from the nose and in the median plane. If the head is held in its approximately primary position and a point on a distant wall is fixed, the pencil being held in a vertical plane and placed at right angles to the direction of regard, two images of the near object will be seen and as these are allowed to slowly fuse by the voluntary control of the fusion on the part of the observer it will be found that the lower extremities of the two images will coalesce first, giving a V-shaped single image. In other words the image for the left eye will lean to the right and for the right eye toward the left. But if the pencil is held in the position in which a book is ordinarily placed with respect to the line of regard

and a distant point first fixed and the two images obtained, it will be found that, as the fusion is allowed to take place by degrees, the two images will remain parallel and that both the upper and lower extremities of the images will coalesce at the same time, as a general rule.

333. These conclusions as drawn from these experiments of Helmholtz, Volkmann, Meissner, Hering and others are not accepted by Stevens, who has done a considerable amount of careful work upon this subject and who has given us the clinoscope as one of the products of his investigations. Stevens points out as the prerequisites for determining the actual positions of the meridians, the following:—(1) A knowledge on the part of the observer of the adjustments of his own eyes with respect to heterophoria, anaphoria or kataphoria; (2) a means by which the exact position of the head may be maintained: the position described by Volkmann, Helmholtz and others is inexact, uncertain and irregular; (3) examinations in this field of inquiry are of little if of any value when the observer can see surrounding objects, and (4) when it is desirable to blend or compare test objects in the field of regard of the two eyes, as for the distant point, the blending or comparison should be made with the lines of regard of the two eyes parallel. All of these conditions for physiological research in respect to the meridians are met, according to Stevens, by his clinoscope. A modified form of instrument with short tubes, thus permitting of convergence within a few inches of the eyes, has replaced the older form of instrument. The clinoscope objectives are the Volkmann's discs shown in Fig. 170. When the observer looks into the tubes the discs blend and the two pins become one long pin with the common head in the middle. When each pin is brought into a position such that it appears to the observer to be exactly vertical and remains so it marks the position of the vertical meridian of the observer's eye as indicated by the pointer and scale above the tube for that eye.

334. Stevens is not in agreement with the views and the results of the experiments of Helmholtz and others but says: "All my experiments, which have now been continued during several years, lead to the conclusion that the typical normal position for the vertical meridians is the exactly vertical position and that the typical position of the horizontal meridians corresponds with the external horizon." Deviations from these positions when the regard is directed in the primary position are anomalies—so-called cyclophoria, or the tendency of the vertical axes to be turned inwards or outwards from the true vertical meridian. Whether or not, however, we are to assume that under normal conditions what we call the normally vertical axes are

not in reality quite vertical but that the upper end of each axis tips outward at an angle of from one and a half to two degrees from the median plane is a mooted question in regard to which we have presented some of the experimental evidence.

335. *Monocular rotation.* Monocular rotation differs in some essentials from binocular rotation: the same muscles are concerned in each but the innervations are not identical throughout. The volitional brain centers, with one exception, are alike concerned in both classes of rotation but in monocular rotations there is no *fusion* demand. To effect all possible monocular rotations the four straight muscles and the two obliques are needed. A straight muscle can effect a given rotation only when that muscle is bisected by the rotation plane from its origin to its insertion. Two muscles are required to effect a cardinal rotation and three muscles are required for any oblique rotation. While only six muscles are required to effect monocular rotation in any direction, eight voluntary centers are requisite, one center for each straight muscle and two for each oblique muscle. Only one muscle and one innervation are needed for a cardinal motion either toward the temple or the nose if the lateral recti are bisected by the plane of the horizontal retinal meridian. Two muscles and two innervation centers are necessary if the visual axis is to be rotated in the plane of the vertical meridian. If the visual axis is to be rotated in the plane of any oblique meridian this must be done under the simultaneous and harmonious action of three muscles under impulses from three volitional centers. One of these muscles, the oblique, will prevent any rotation on the visual axis while it is being rotated in the rotation plane by the other two muscles around two moving axes, these axes being always the transverse and vertical axes of the eye. These three forces combined (not converted into one force, however, in the sense of creating a fixed axis) rotate the eye in an oblique plane without torsioning just as if the eye had been rotated first on the vertical axis to a point in the horizontal plane directly beneath the secondary point and thence directly up around the transverse axis of the eye to that point. These preceding statements constitute the essentials of the views of Savage as expressed in the opening chapter on the fundamental principles of ocular rotations in his *Ophthalmic Myology.* He gives the following as his formulation of the law of monocular motion. "(1) The visual axis, which is the line of intersection of all the planes of all meridians, must be rotated in the plane of that meridian on which lie the first and second points of view and their retinal images. (2) In the plane of the horizontal, or that of the vertical, meridian, the rotation must be effected around a single fixed axis at right angles to

the rotation plane and cutting it at the center of rotation—if in the horizontal plane, around the vertical axis of the eye; if in the vertical plane, around the transverse axis of the eye. (3) In the plane of an oblique rotation, whatever the degree of obliquity, the rotation must be accomplished around two moving axes by two forces acting simultaneously, these axes being the transverse and vertical axes, both at right angles to the visual axis, but neither one at right angles to any oblique rotation plane; while a third force prevents any rotation around the visual axis.'' As Savage remarks, the third portion of this law is open to controversy, since it denies the possibility of a resultant fixed axis. The purport of this statement is that, while the visual axis is being rotated in a given plane, the vertical axis of the eye shall never lose parallelism with the median plane of the head. Hence, Savage says, ''Listing's plane cannot be a plane of reference, nor can it be a plane containing the axes of all rotations starting from, or returning to, the primary point of view. The equatorial plane of the eye contains both the vertical and transverse axes of the eye and it is around one or the other or both of these that all rotations, cardinal and oblique, occur. Listing's plane, like Listing's law, should be forgotten in the interest of truth.''

336. These opinions of Savage are presented ahead of the more commonly accepted notions of the actions of the associated muscles in a single eye because of their simplicity and because they are at variance with a considerable number of orthodox views. The ordinarily accepted ideas upon the composition of rotations treat them in a manner analogous to the composition *of forces* in physics. The amount of rotation imparted to a rotating body can be represented in linear measure by laying off along the axis a distance proportionate to the rotation. Since a body can rotate in two directions about any axis it is necessary to choose one diameter to represent rotation in one sense and the other direction to indicate rotation in the opposite direction. By a single measured line it is, therefore, possible to represent three quantities: (1) the axis of rotation, by the direction of the line; (2) the amount of rotation, by the length of the line, and (3) the sense of the rotation by the direction from the center in which the line is drawn. Any units may be chosen; millimeters may represent degrees. Since the forces acting upon an eye through the muscles in the interest of ocular rotation are tangential and since the lines of the forces may, with little error, be reckoned as equally distant from the center of rotation it follows that the moments of the forces, which are a measure of the tendency to produce rotation about any point, may be taken proportionate to the forces. Doubtless the resistances

to rotations of the eyeball are greater in some directions than in others, but this element of resistance cannot be calculated, hence the forces can be measured only by the rotations which they produce. Therefore, rotations and not forces must be compounded, since the forces are not known but the rotations can be investigated to a goodly degree of accuracy by the behavior of after-images. In the accompanying diagram (Fig. 171), let *C* represent the center of rotation. The arrowheads on the lines *Ca* and *Cb* represent the directions in which the lines are measured and therefore the direction of the rotation which takes place about each as an axis and which is the same as that of an ordinary screw turned right-handed in the direction of the arrow. These two forces, *Ca* and *Cb,* when compounded give the resultant *Cd* by the method of the parallelogram. The reason for the composition of the rotations is fairly obvious: for if the body were subjected to one of the rotations any point in it would move over a

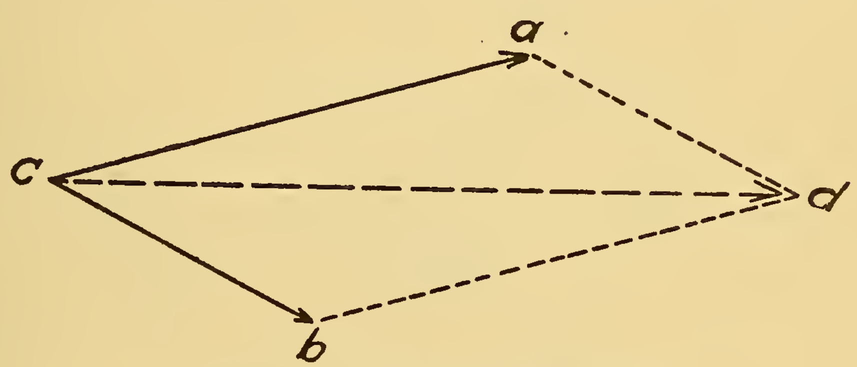


Fig. 171.—Composition of Rotations.

distance proportional to the amount of rotation and to the distance from the axis of rotation. When the rotations *Ca* and *Cb* take place simultaneously, points which lie between their axes will rise in consequence of one rotation and sink because of the second; the line *Cd* represents the locus of points such that the rising and falling exactly neutralize each other. The distance of each point in the line *Cd* is inversely proportional to the amount of rotation about the axes.

337. These principles are applied to the rotation of the eyeball as illustrated in Fig. 172. This represents a horizontal section of the eye with *A* and *P* the anterior and posterior poles of the eye, so that *AP* is the optic axis. The line *DE* is the transverse axis: *MN* is the axis of rotation of the superior and inferior recti and M^1N^1 is the axis of rotation for the obliques. A measured quantity, *Or,* along the line *ON* indicates a measured rotation of the globe in the sense of a screw proceeding from *O* to *N*. This rotation elevates the cornea and

would be effected by the superior rectus acting alone if this were possible. Similarly *Os* specifies a proportionate rotation by the inferior oblique which also elevates the cornea. These rotations (*Or* and *Os*), when they occur simultaneously, are compounded into a single rotation *OE* which takes place about an axis in Listing's plane.

338. We may likewise resolve rotations due to individual muscles. Taking the inferior rectus as an example, let the distance *OM* represent the maximum rotation it can produce. By dropping perpendiculars from *M* upon the transverse and optic axes we find that these perpendiculars cut off distances from *O* along these axes which give the component depression and torsion respectively. *Om* represents

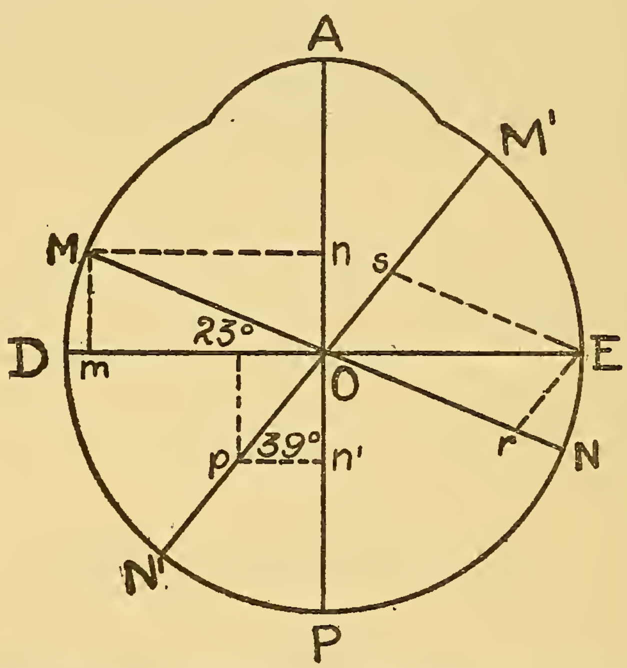


Fig. 172.—Horizontal Section of an Eye to Illustrate Composition of Rotations.

the depression of the cornea and *On* its torsion or extorsion. The lengths of these two lines are readily found trigonometrically; if we take the obliquity of the axis of the superior and inferior recti as 27° from the transverse axis, the component *Om* will be 0.89 or about nine-tenths of the whole rotation, *OM,* and the component *On* will be 0.45 or about nine-twentieths of the whole rotation about *OM*. Hence the elevating action is double that of the torsional effect.

339. We are, finally, desirous of finding out how much rotation the superior oblique must effect in order to be a perfect associate of the inferior rectus. If, then, subduction is to be unaccompanied by torsion, the extorsion *On* must be offset by an intorsion On_1. Marking off, therefore, On_1 equal to *On* but in the opposite direction from *O,* erect a perpendicular to the axis *AP* at the point n_1: where this

line cuts the axis of rotation of the obliques (ON_1) gives the direction Op indicating the exact proportion of intervention needed on the part of the superior oblique, since the torsional component On_1 balances the torsional component On of the superior rectus while the subducting component n_1p supplements the subduction of the rectus. As a matter of fact the lengths Om and n_1p represent the relative proportion of subduction due respectively to the inferior rectus and the superior oblique; the latter is about two-fifths of the former.

340. As Maddox, from whom the essentials of the above treatment of monocular motion is taken, remarks: ''These calculations are at best only approximate but we can by their aid determine with more or less approach to truth the provinces of the motor field over which different muscles hold sway.''

341. *Binocular rotation. Conjugation of the eyes. Conjugate innervations.* The relative movements of the two eyes are governed by the desire for and necessity of seeing the object single; the two eyes are so placed and so adjusted as to make binocular vision possible in obedience to the law of corresponding or identical points. It is necessary, therefore, that an image of the object fixed be formed on each fovea. When the point of regard is changed the two eyes make associated movements: both turn to the right or to the left, upwards or downwards, and so on. If the objects are in the median plane but at different distances it is necessary that the eyes make a movement of convergence in order that the point of regard be changed from the more remote to the nearer object; both eyes normally turn inwardly to the same extent. If, however, the two objects are in different directions one being, for example, farther to the right than the other, but at different distances from the eyes, then the eyes execute a combined movement of association and of convergence; if the second object is more remote than the first object viewed there is a movement of divergence, i. e., a relaxation of convergence.

342. A movement of one eye can be made, however, without an apparent conjugate movement of its mate: in the last analysis, however, it can be shown that it is impossible to cause a movement of one eye without a motion of the other also or at least a tendency on its part to move. Hering has described a simple experiment which is of importance in understanding the relation between the movements of the eyes. Suppose the two eyes to fix a point P and that there is placed in the visual line of the right eye an object O. If the party under observation fixes the object O by changing his point of regard from P to O, the left eye will be moved toward the point O while the right eye remains motionless. But upon close observation it is found that

this eye (the right eye) in reality makes two slight changes of position, for instead of receiving no innervation, as is apparently the case, its muscles receive two innervations one of which would cause it to make an associated movement to the right and the second of which would cause it to execute a movement of convergence to the left: the net result of these innervations neutralize each other so that the eye remains stationary.

Binocular rotation is the rotation of the two eyes in the interest of binocular single vision and correct orientation; binocular single vision is possible in obedience to the supreme law of corresponding retinal points. We shall pause at this juncture to ask the question, "What is the fundamental fact of corresponding retinal points?" and to indicate various opinions and conclusions which have been formulated in answer thereto.

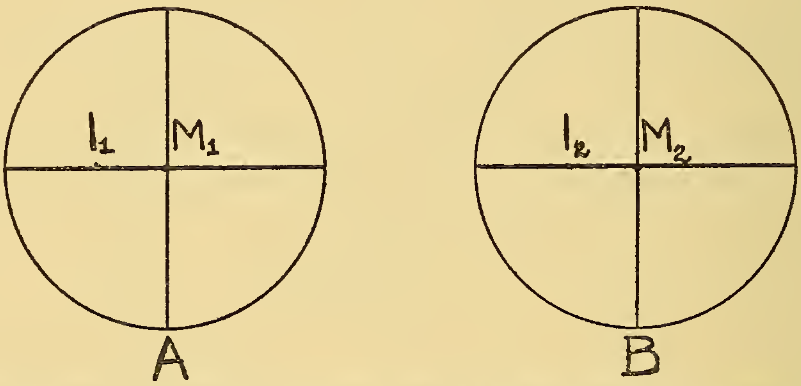


Fig. 173.—Mercator Projection of the Two Retinæ, Showing Corresponding Retinal Points According to the Accepted Doctrine.

343. *Corresponding retinal points.* This doctrine as it is commonly accepted and expressed in treatises on physiologic optics may be stated somewhat as follows. If the image of a given point is located at the temporal side of the macula of the right eye the impression will also be located at the nasal side of the macula of the left eye and at a distance from it equal to that of the impression of the right eye from the macula. Likewise, if the image is impressed at an horizon above that which passes through the maculæ or below that horizon, then the impression for each eye will be equally above or below this horizon. These points are, therefore, not anatomically but rather geometrically similar. This is diagrammatically represented in Fig. 173. Let A and B represent the two retinæ and M_1 and M_2 the maculæ. If the selected image is located on the retina A at I_1 (i. e., the temporal side of the macula) it will be located at the point I_2 of the retina B (at the nasal side of the macula) and at the same distance from M_2 as I_1 is

from M_1. It is evident, of course, that the two retinal images are in widely different localities anatomically. They are, however, not only equally removed from the maculæ but they lie on corresponding horizontal meridians of the two retinæ. Also, if the point I_1 is situated on a horizontal meridian above or below the meridian of M_1, the point I_2 will be on a horizontal meridian equally above or below the meridian of M_2. Hence we may say that a point on one retina *corresponds* or is *identical* with a point of the other one when the images of the same exterior point falling on the two retinal points are seen as a single

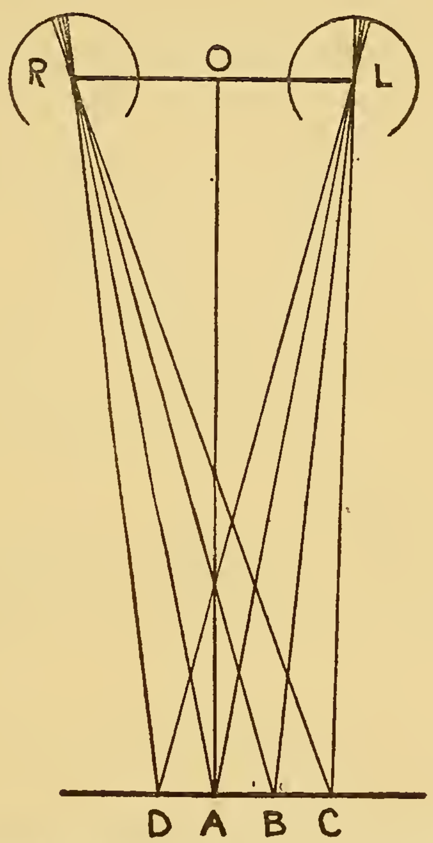


Fig. 174.—Diagram Illustrating Contention that Retinal Corresponding Points are not at Geometrically Equal Distances in the Two Retinæ. (After Stevens.)

image. If, on the other hand, the image is formed on any other point it is not blended with that of the first eye and the point is seen double. Helmholtz stated the law of corresponding points as follows:—"Upon the apparently vertical concordant lines of the two eyes, points which are at equal distances from the horizontal meridians are corresponding points," and "Points which in the retinal horizons are at equal distances from the point of fixation are corresponding points." With respect to corresponding points in the field of vision he says: "Corresponding points in the two visual fields are those which are at equal distances and equal in direction from the corresponding horizontal and apparently vertical meridians.'' Stevens has criticized these statements of Helmholtz and holds that the propositions are inconsistent with the illustrations given and states that one of the most conclusive proofs that retinal corresponding points are not at geometrically equal distances in the two retinæ is that a straight line drawn in the vertical or horizontal direction does not appear curved as it would positively do were the accepted doctrine correct. Fig. 174 taken from Stevens' work, is offered by him in support of the contention ''that the accepted view that the points of retinal correspondence are, by superficial measurement, equal is incorrect.'' In this diagram, *R* and *L* are the nodal points of the two eyes and *A* the point of fixation. The points *B*, *C*, and so forth are outside the point of fixation. If, therefore, $RO = 1.25$ inch and $AO = 15$ inches, then the angle $RAO = 4° \ 45' \ 49''$ and we find that the angle $ARB = 1° \ 53' \ 26''$, $ALB = 1° \ 54' \ 5''$, $ARC = 3° \ 46' \ 1''$ and $ALC = 4° \ 39' \ 58''$. The points corresponding to the incidence of the lines *CR* and *CL* are not thus equally removed from the maculæ.

344. There is a mental cognizance of relations of distances in space and distances on the retinal surfaces, but they are recognitions of *angular* rather than of equally removed spatial values on the two retinæ. Stevens, then, sums up his conclusions respecting this subject in the following statement:

''Corresponding points are those points in the retinas which answer to proportionate degrees of rotations of the eyes about their centers of rotation and which, from given points in the plane of the point of fixation, receive incident rays which must pass through the nodal points.''

345. The question as to *why two points are corresponding* and two others are not has been considerably discussed. Most of the advocates of the theory of identity suppose that there exists an anatomical relation between the two corresponding points. They suppose that the nerves carrying the impressions of two corresponding points unite on their way to the chiasma into one which conducts the impression to the brain. Savage believes that the secret of corresponding retinal points is common brain-cell connection: that one macula corresponds point for point with the other macula only because these corresponding points have, going from them, two fibers which meet in the optic tract and which go, side by side, into the same cuneus to terminate in one common cell in the visual center. This explains double impressions yet a single sensation—two images, yet a single object. The theory of projections, in which impressions on the nasal side of the macula are referred in space to the temporal side and so forth, has

been advanced as an explanation by others. In this theory a point on the left retina, for example, situated 10 degrees to the left of the fovea, localizes its impression at 10 degrees to the right of the point of fixation: the point situated at 10 degrees to the left of the right fovea localizes its impression in the same direction and as the two impressions are localized in the same direction they are blended into one. This bases the whole theory of corresponding points upon experience: in fact, the identity of the two foveas might be a result acquired by experience. But according to Savage the law of visible direction does not explain corresponding retinal points: for this law, he says, is violated in the interest of binocular single vision whenever a prism is placed before either of the two eyes and that duction is possible only

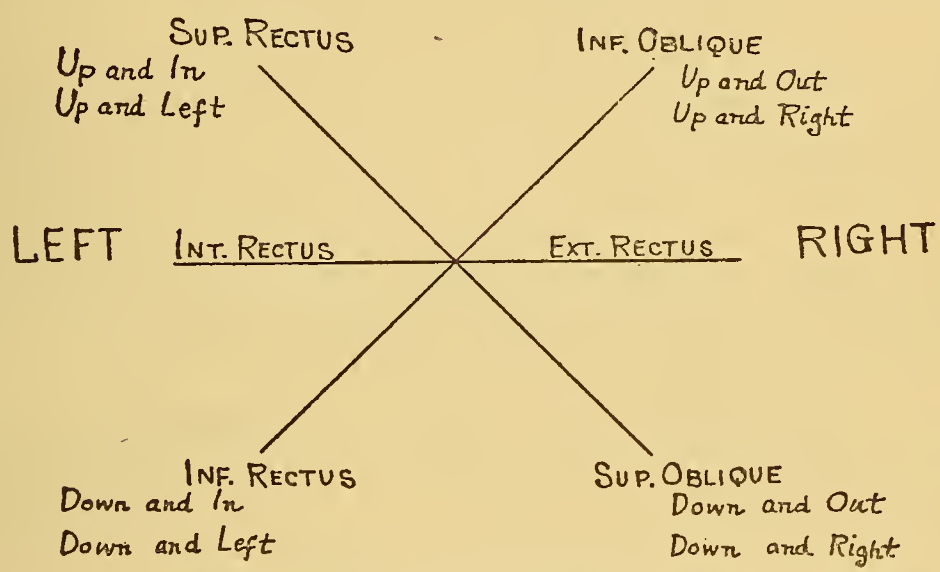


Fig. 175.—Showing Dominant Action of the Muscles of the Right Eye.

in violation of the law of direction. This objection does not appear to the writer as valid however, since in duction tests it is the function of the extra-ocular muscles to keep the eyes in such positions that the impressions of objects in space are received upon their foveas; when this is the case there follows a mental interpretation of singleness of object irrespective of the directions of the rays of light within the eyes.

346. *Conjugate innervations.* The presentation of the essential facts relative to the motions of one eye under the caption *Monocular rotations* has, of necessity, included much which relates to the motions of both eyes. Any study of the motions of both eyes is, then, rather a study of their action together in associated movements. It is known that the externi and interni have purely lateral action; that the superior rectus moves the eyeball upward and medianward; that

the inferior rectus draws the eyeball downward and medianward; that the superior oblique moves the cornea downward and temporal-

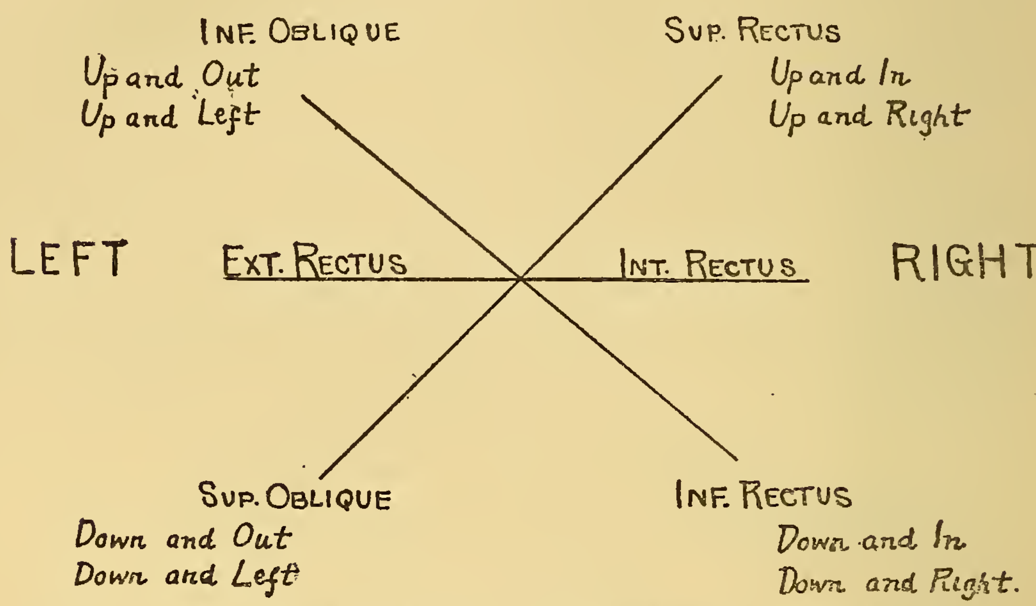


Fig. 176.—Dominant Action of the Muscles of the Left Eye.

ward, while the inferior oblique moves the eyeball upward and temporalward. Figs. 175 and 176 diagram the dominant action of

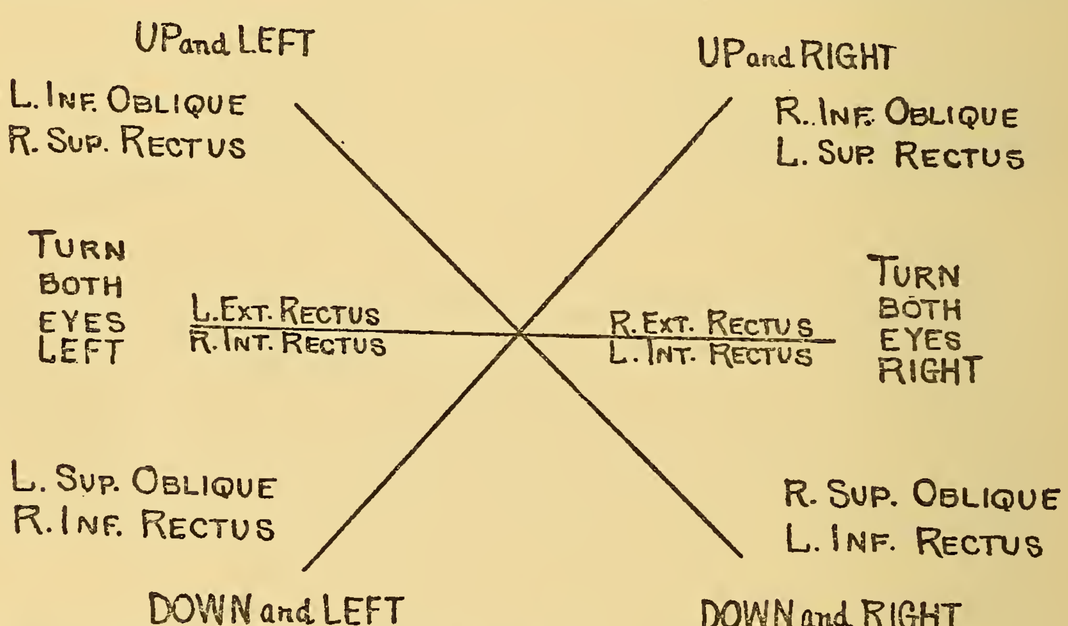


Fig. 177.—Superposition of Figures 175 and 176. The associated muscles in the two eyes are thus schematically shown.

the muscles of the right and left eyes respectively; these diagrams are based on the assumption that the eyes are in the primary position when the movement in the various directions begins. A superposition of these two diagrams gives us Fig. 177 in which the associated muscles in the two eyes are shown. It will be seen at once, for example, that in the movement of both eyes directly to the right the muscles which predominate in this action are the right external rectus and the left internal rectus; in movements of the eyes up and to the right the two muscles principally concerned are the right inferior oblique and the left superior rectus, and so on through the six principal movements of the eyes. It follows, therefore, that the two eyes work together as one; as Hering says in introduction to his monograph on binocular vision, the two eyes may be regarded as halves of a single organ. It is impossible for one eye to move in one direction without the other paralleling its action in every particular when normally acting. A nervous impulse from the cortex must necessarily be divided between the two eyes. The number of conjugate innervations is at present unknown. Five have long been recognized: (1) binocular elevation, (2) binocular depression, (3) binocular dextroduction, (4) binocular lævoduction and (5) that for the totally distinct act of convergence. These five innervations are more or less under voluntary control. (See Hansell and Reber, *Ocular Muscles,* pages 26-31.)

347. To the above conjugate innervations Maddox adds the following: (1) binocular dextrotorsion and (2) binocular lævotorsion, the existence of which can be deduced from physiological experiments, phenomena of rotational nystagmus and changes in cylindrical corrections necessary when the head is sloped toward either shoulder; (3) binocular intorsion and (4) binocular extorsion for regulating the parallelism of the vertical meridians of the retinas with each other; (5) divergence, (6) one for raising the right visual axis above the left and (7) another for raising the left visual axis above the right.

348. Savage has given a most elaborate discussion of the conjugate innervations and their relations to muscular defects in his books on *Ophthalmic Myology* and *Neuro-myology.* According to this writer there are nine conjugate brain-centers all under the control of the will and each connected with two muscles, one belonging to each eye; there are twelve centers controlled by the fusion faculty of the brain, each center being connected with only a single muscle. These fusion centers exist in the interest of binocular single vision; these centers must act independently of and co-ordinately with the conjugate centers. To accomplish their work the twelve muscles belonging to the two eyes have *nine conjugate innervations:*—(1) the one to elevate both eyes, the two superior recti, (2) the one to depress both eyes, the two inferior recti, (3) the one to converge both eyes, the interni, (4) the one to move both eyes to the right, the right externus and the left

internus, (5) the one to move both eyes to the left, the left externus and the right internus, (6) the one to keep the vertical axes from diverging above, the two superior obliques, (7) the one to prevent their converging above, the two inferior obliques, (8) the one to maintain parallelism of the vertical axes and the median plane of the head when the point of view is obliquely up and to the right or down and to the left, or the right superior oblique and left inferior oblique and (9) the one to maintain parallelism of the vertical axes of the eyes and the median plane of the head when the point of view is obliquely up and to the left or down and to the right, thus involving the left superior oblique and the right inferior oblique. Hence the internal recti and the four obliques are each connected with two conjugate innervation centers, while the remaining muscles are each under the control of only one conjugate innervation center. There are in addition *twelve fusion brain-centers* not under the control of the will; when one of these basal centers discharges neuricity only a single muscle responds; when a conjugate center discharges neuricity both muscles of a pair respond (see Savage *Ophthalmic Neuro-myology* and the section on "Ocular Muscles," Volumes X and XI of *The American Encyclopedia of Ophthalmology.)*

349. We may with propriety ask the question: By what mechanism can we explain the motor impulses which rotate both eyes in the same direction at the same time as we look from right to left or again in opposite directions as in convergence? Innumerable theories have been offered and discarded in turn as our knowledge of the functions of the cells in the nuclei and in different portions of the brain have grown more exact. The third, fourth and sixth pairs of cranial nerves and the carotid plexus of the sympathetic system innervate the muscles governing the movements of the eyeball, the accommodation and the iris. From these cortical centers are derived nerve fibers which run indirectly to the nuclei and undoubtedly have connections with other centers in the brain the functions of which are associated. The fibers have not been dissected or strictly outlined. Their presence must be assumed in order to explain mental processes, a portion of the evidence being the voluntary although not always conscious ocular movements. The nuclei have been studied and their locations, their relations to each other and their functions to a large extent determined. From the nuclei large numbers of nerve fibers are given off which unite to become distinct nerve trunks easily seen at the base of the brain and followed to their exit through the sphenoidal fissure to be distributed to their respective terminations in the muscular tissues. Russell's experiments led him to conclude that the cerebellar cortex plays no

little part in the ocular movements and that it is associated with the cerebrum in these functions. The areas which are supposed to preside over the different eye movements, according to Russell, and which are five in number, are above the center of the fissure of Sylvius just anterior to the large motor area.

350. The mass of cells composing the nucleus of the third nerve lies on both sides of the median line next to the corpora quadrigemina and under the aqueduct of Sylvius. The nucleus is from six to ten milli-

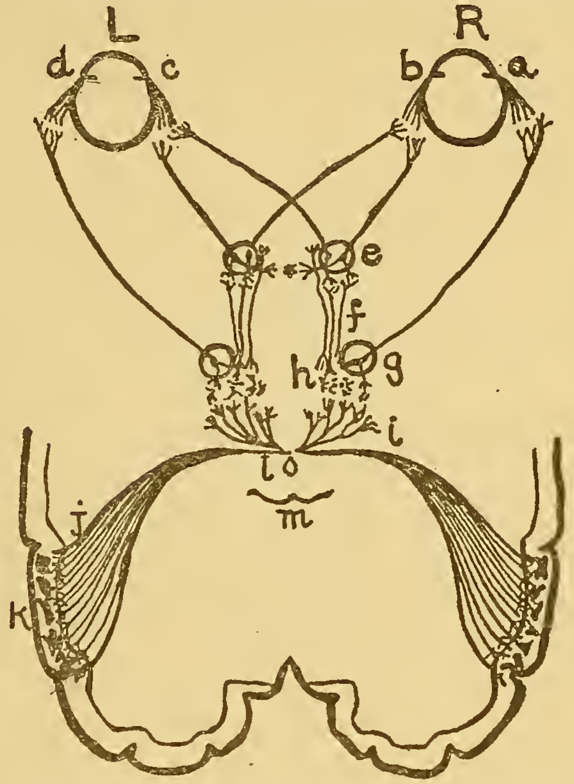


Fig. 178.—Bernheimer's Scheme to Illustrate the Action of the Associated Muscles in Lateral Movements and in Convergence.

a, Right external rectus; *b*, right internal rectus; *c*, left internal rectus; *d*, left external rectus; *e*, third nerve nucleus; *f*, communicating fibres; *g*, sixth nerve nucleus; *h*, fimbriated cells; *i*, arborizing ends of the fibres to the opposite side of the cerebral cortex; *k*, cortical centers; *l*, aqueduct of Sylvius; *m*, roof of the corpora quadrigemina. (Graefe-Saemisch *Handbuch*, second edition.)

meters in length and of varying breadth, mingling with adjacent cells. Posteriorly they encroach upon the cells of the fourth nucleus without a distinct demarcation between them. The mass may be divided into nucleoli, each with its separate function and muscle control. Fig. 178 gives Bernheimer's scheme to illustrate the action of the associated muscles in lateral movements (lateral conjugations) and in convergence. In the diagram the letters have the following significances: *a*, right externus rectus; *b*, right external rectus; *c*, left internal rectus; *d*, left external rectus; *e*, third nerve nucleus; *f*, communicating fibers; *g*, sixth nerve center; *h*, fimbriated cells; *i*, arborizing ends of the fibers to the opposite sides of the cerebral cortex; *k*, cortical centers; *l*, aqueduct of Sylvius; *m*, roof of the corpora quadrigemina. It will be seen that the fibers from some of the nuclei run directly to the muscles on the same side, while others are crossed to stimulate those on the other side, the crossing taking place chiefly in the anterior half of the nucleus. Strictly, the inner or median part of the nucleus belongs to the intraocular muscles and the outer part to the extraocular muscles. The anterior and principal part of the nuclear masses belongs to the third nerve.

351. Bernheimer conducted a large number of experiments on apes to determine the site of the centers for eye movements and reached the following conclusions:—(1) The gyrus angularis and especially its middle part of both hemispheres is the only cortical center for synergic eye movements. (2) The right gyrus angularis controls movements toward the left, up and left and down, while the left gyrus controls movements toward the right, up and right and down. (3) The anterior corpora quadrigemina are neither a reflex center for eye movements nor the passage for the neurons. (4) The connection-neurons between the nuclei and the cortex are all crossed in the angular gyrus in the median line under the plane of the aqueduct of Sylvius between it and the nuclei. (5) The end filaments of the connecting fibers communicate probably by other cells (*schalzellen*) with the roots of the motor ganglion cells of the nuclei. (6) The schalzellen lie probably imbedded and scattered in the central gray matter and form no cell mass. (7) Since there is a partial crossing of the third, the complete crossing of the fourth and the connection of all the oculomotor nuclei with each other, it may be asked whether the crossing connecting fibers of one angular gyrus equally influence the muscles of both eyes.

352. *Convergence.* When normally balanced motor muscles are at rest a single distant object can be fixed by the two eyes and the images, which fall upon the maculæ, are fused into a single mental impression. In the binocular single vision of a near object the eyes must be converged or turned toward each other so that the images may still be formed on the maculæ. Convergence is independent of and can be associated with any other motor muscular action, such as lateral rotation, elevation or depression of the eyes; in fact, depression is a usual accompaniment of convergence since reading, writing and other near work are generally done below the level of the eyes. Since convergence is an angular movement of the eyes effected around the centers of rotation it is measured in degrees: the farther the two

eyes are apart the greater must be the angular movement for fixation and *vice versa.* Hence for any given distance of the object viewed the actual convergence depends upon the interpupillary distance. It is common custom (although not correct) to disregard the interpupillary distance and to measure and express this function in meter-angles. The *meter-angle* is that angular displacement of the one visual axis from its primary position of parallelism when a point on the median line one meter from the eyes is fixed. It is, therefore, equal to the angle *c,* Fig. 179, between the median line *DF* and the visual axis *RF.*

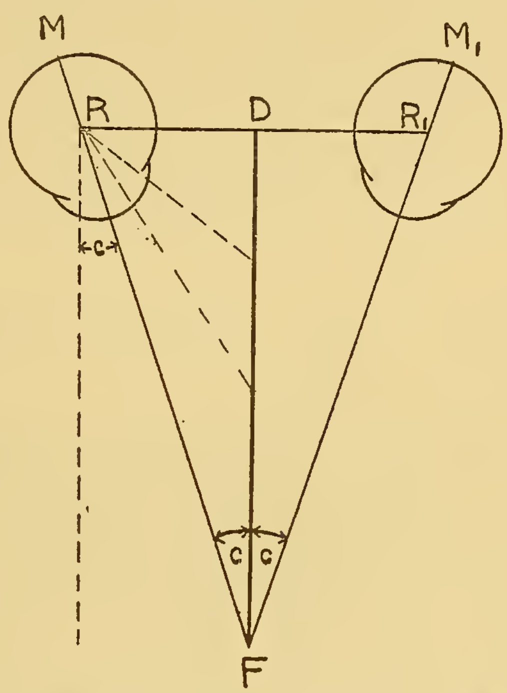


Fig. 179.—Illustrating Convergence and the Principles of the Calculation Trigonometrically of the Value of the Meter Angle.

The meter-angle is then that angle whose sine is half the interpupillary distance, i. e.,

$$\sin c = \frac{RD}{RF}.$$

Since the distance between the rotational centers of the eyes is considered fixed and the sines increase less rapidly than the angles, the angular value of two meter-angles is slightly more than twice the value of one meter-angle and so on.

353. The value of the meter-angle varies with the interpupillary distance. If the latter is 60 mms. the value of one meter-angle is 1° 45′. If the P. D. (interpupillary distance) is 64 mms. the M. A. (meter-angle) is 1° 50′. If we neglect the difference between sines

and tangents, the M. A. can be expressed in terms of the prism diopter, due to Prentice, as equal to half the P. D. in centimeters. Thus, if the P. D. is 64 mms., the M. A. equals 3.2△. In other words, a 3.2△ prism, base out, before each eye in the primary position will cause a convergence of 1 M. A. in order that binocular vision at infinity may be retained. Prentice gives the following simple rule: Read the patient's interpupillary distance in centimeters, when half of it will indicate the prism-dioptries required to substitute one meter-angle for each eye. This rule likewise enables us to quickly calculate the amount of convergence demanded for any intraocular distance with fixation at any specified points. For instance, if the interpupillary width is 70 mms., then 1 M. A. = 3.5△ and if such eyes are fixing a point at 33.3 cms. (equals 3 M. A.), each eye must converge 10.5△ or the total convergence demanded is 21△. The number of M. A. of convergence demanded at any point can be calculated from the relation that

$$\frac{100}{\text{Distance in cms.}} = \text{M. A. of convergence.}$$

This meter-angle system was invented by Javal and Nagel to measure the convergence in a manner analogous to the measurement in diopters which is used for refraction and accommodation.

354. The *range of convergence* is the actual distance between the near and the far points of convergence. It usually extends from infinity or (possibly) more often from a negative position to within a few inches of the eyes. The *range of convergence,* or the space over which convergence can be exerted is expressed by $r_c - p_c$ in linear measure. The near point of convergence is found by approaching a pencil or a card carrying a black vertical line or row of printed letters towards the eyes to the nearest point at which it is seen single, or by the use of an ophthalmoscopic lamp as advocated by Hansell and Reber, in which the observer watches the corneal reflections of the light as it is approached toward the eyes and in which failure of one or both eyes to fix is noted by the shift of position of the corneal image. The far point of convergence is found by using a Maddox rod or similar device before one eye and noting the position of rest and measuring its amount by prisms bases in or out; the position of rest may be one of parallelism, divergence or convergence. The *amplitude of convergence* is the total amount of convergence force that can be exerted and is, therefore, the distance between the punctum remotum and punctum proximum expressed in meter-angles or prism-diopters:

i. e., $A_c = P_c - R_c$. Age has a much less effect upon the amplitude of convergence than upon the amplitude of accommodation; it is stated by some writers and investigators that age has very little effect and this statement seems very likely correct. The *relative convergence* is the amount, either positive or negative, that can be exerted when the accommodation is fixed for a given distance. This *relative amplitude of convergence* is measured by the difference between the strongest prisms, base in and base out, through which vision is single. These measurements are made preferably with the rotary prism, an instrument composed of two superimposed prisms of the same strength and operated by a special mechanism which allows them to be turned in opposite directions. This rotary prism is turned in such a manner as to throw the apices outward or bases inward while the observed party looks at a distant small luminous source; the strength of the prism is increased until the subject sees two images. We thus obtain the *abduction*, or the negative relative convergence: for normal eyes this is about 5 to 7 prism degrees. The positive relative convergence, or the *adduction*, can be found by turning the apices of the prisms inward or bases out and increasing their strengths until diplopia results. Adduction is normally much stronger than adduction; it may easily reach 20° to 30° of prism power or even more. The difference between—in reality the arithmetical sum of—the two, i. e., the adduction and the abduction, gives the *total relative amplitude of convergence,* the accommodation remaining passive if the eyes under test are emmetropic or else put in this condition due to the correction of refractive errors. It is worthy of note, in passing, that these determinations frequently vary amongst themselves because the observer does not attempt with equal effort to fuse at all times, because there is fatigue under repeated trials and also because the rotary or mobile prism and the insertion of and replacing of prisms one by one from the trial case apparently give different results. Furthermore, it should be pointed out that the amount of deviation or turning of the visual line in any case is approximately one-half of the prism value in degrees inserted before the eye. The deviation produced by a prism corresponds exactly to half its angle if the index of refraction is 1.50. If one can overcome a prism of eight degrees, apex outwards, it is equivalent to saying that he can make the visual line diverge four degrees.

There are two general classes of convergence which demand differentiation, the so-called *static* and *dynamic* convergences. The term "*static*" is applied to convergence when the eyes are at rest and it may, therefore, be positive, negative or zero. "*Dynamic*" convergence is always positive; it involves the muscular process by which the visual axes are so turned that the point of fixation may have an image on both maculæ.

355. Maddox divides convergence into three portions:—

(1) *Tonic* convergence, which is exerted in order to fix an object at infinity, i. e., to render the visual axes parallel. It may be positive, negative or nil according to the evidence furnished by the muscle balance tests at twenty or more feet.

(2) *Accommodative convergence.* This accompanies convergence and is always positive.

(3) *Fusional or supplementary* convergence, exerted in order to fuse the images.

The sum of these three causes the visual axes to meet at a near point so that single and simultaneous binocular vision results. In distant vision only the tonic or initial convergence is demanded. When accommodation is exerted it is always accompanied by convergence or conversely convergence is normally accompanied by accommodation. The two are simultaneously coexistent and so naturally associated that it is difficult for ordinary eyes to exert the one function without the other. It is commonly taken that for one diopter of accommodation one meter-angle of convergence is also normally exerted.

356. *Accommodation and convergence.* Because of a common or associated innervation or harmony of action there is an intimate connection between these two functions. If it be taken that neither comes into action for clear binocular vision at infinity, then for a near point or object both are equally required. At 50 cms., for example, 2 diopters of accommodation and 2 M. A. of convergence would be required. It is almost impossible to judge which is brought into action first or whether their action is simultaneous, but the weight of evidence is that fixation first occurs to be followed simultaneously by the accommodative act.

357. It has been taught by many that when the eyes are accommodated for any given distance the initial (tonic) and accommodative convergences then in force should be just that quantity required for fusion of the images and that there is normally no call for supplementary or fusional convergence. But if the initial and accommodative convergences are insufficient for the distance under test some positive fusional convergence must be exerted to obtain single vision, while if the sum of these two quantities exceeds the amount of convergence demanded then some negative supplementary convergence, or divergence, must be brought into play. Positive supplementary convergence may be demanded if the initial position is that of diver-

gence or if less than one M. A. of convergence comes into operation with each diopter of accommodation exerted. Negative fusional convergence may, in turn, be demanded because of the initial convergence or because more than one M. A. of convergence results with each diopter of accommodation. These statements must be, in their import, correct; the chief point upon which differences of opinion exist lies in the question of the accommodative convergence. If the accommodation and convergence are dissociated and investigations are carried out at the normal reading point, while the patient accommodates, by means of the insertion of a 4△ base up and down before each eye respectively, the patient viewing a line of type and subjoined arrow or if the Stevenson's muscle testing device is used, experimentation by such authorities as Maddox, Howe and Worth has led to the conclusion that there is a normal or physiologic exophoria at 12 to 13 inches of approximately 4°. It does not appear valid, therefore, to attribute the whole of the innervation necessary to binocular vision at a near point to the tonic and accommodative convergences only but rather to the tonic, accommodative and fusional convergences. If the hypothesis of physiologic exophoria is correct we should *not* then expect to find orthophoric conditions at distant and near points (as indicated by such tests as the Maddox rod or von Graefe's dissociation test) to be in agreement in any case. We should have such a status of affairs as the following:—at 20 feet no vertical or horizontal imbalances; at 13 inches some 4 to 6△ base *in* to bring the images into a line with each other or 6△ of exophoria at near. If such a basis is assumed to indicate orthophoria for distance and near it must follow that there is an association of convergence with accommodation but that there is not the usually stated 3 to 1 ratio between these two functions *per se* but a lesser ratio of approximately 2 to 1, the remaining portion of the convergence demand being met by the fusional convergence and ultimately, therefore, giving a relation such as that commonly stated of 1 M. A. of convergence to 1 diopter of accommodation. An emmetropic pair of eyes evidencing orthophoria at infinity (20 feet) would, therefore, possess binocular single vision at near points through the functions of both accommodative and fusion convergences. This topic is discussed in detail by Sheard in his volume on *Dynamic Ocular Tests.* (Note:—Simple calculation by the Prentice rule or by trigonometry shows that approximately 18△ to 20△ of convergence is demanded when a pair of eyes, about 64 millimeters from center to center, fix a point at 13 inches.)

358. While the accommodation and convergence are thus intimately related and normally co-existent, yet the one can be made to exceed the

other. For at any given distance we can reduce accommodation with convex lenses and increase its operation with concave lenses without producing double vision, thus proving that convergence does not conform to the altered accommodation. With prisms base *in* we can decrease and with prisms base *out* we can increase the convergence without disturbing the clearness of vision. This disturbance to clear vision, it is commonly stated, would occur if accommodation was proportional to convergence. The whole of these phenomena are fairly readily and logically explained if due regard is paid to fusional convergence as associated with, but entirely separated from, the accommodative convergence.

359. And again, since there is normally such intimate correlation between these actions of accommodation and convergence, it might appear possible that one single innervation would serve the purpose of

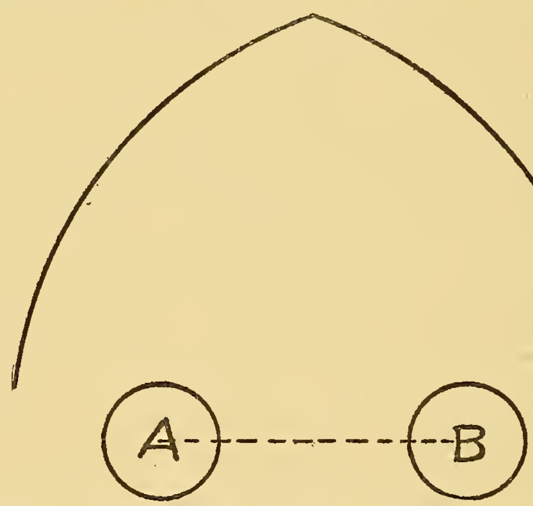


Fig. 180.—Line of Equal Accommodation. (After Maddox.)

the two. Maddox has raised this query and discussed it from geometrical and clinical viewpoints. Whenever the eyes are turned to the right or the left a differing proportion between convergence and accommodation is needed: for slight lateral movements of the eyes accommodation needs to be relatively increased but as soon as the motion exceeds a certain limit, the demand is reversed and the greatest demand is for convergence. In Fig. 180 we have a representation of the line of equal accommodation for near vision. It is made up of two curves: if an object is placed at any point thereof accommodation remains the same. It is composed of two arcs of equal radius described from the centers of their opposite eyes. It is assumed that the centers of accommodation are so intimately connected that one eye does not normally accommodate more than the other when looking at any point outside of the median plane. Since accommodation with normal refraction implies positive effort, the eye which is farthest from the object and can see it with least effort determines the accommodation for both.

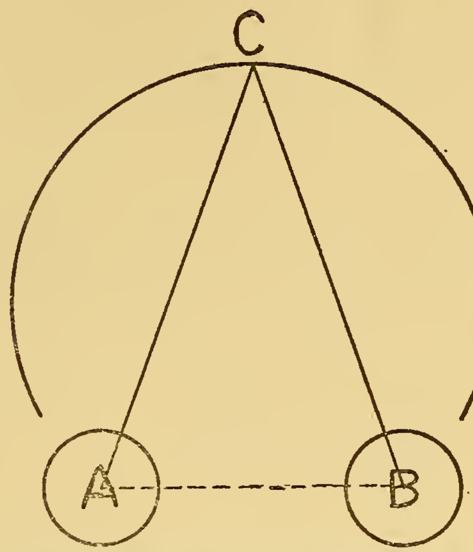


Fig. 181.—Line of Equal Convergence. (After Maddox.)

360. The equal-convergence curve in Fig. 181 is a portion of a curve which passes through the centers of rotation of the two eyes and possesses these attributes:—

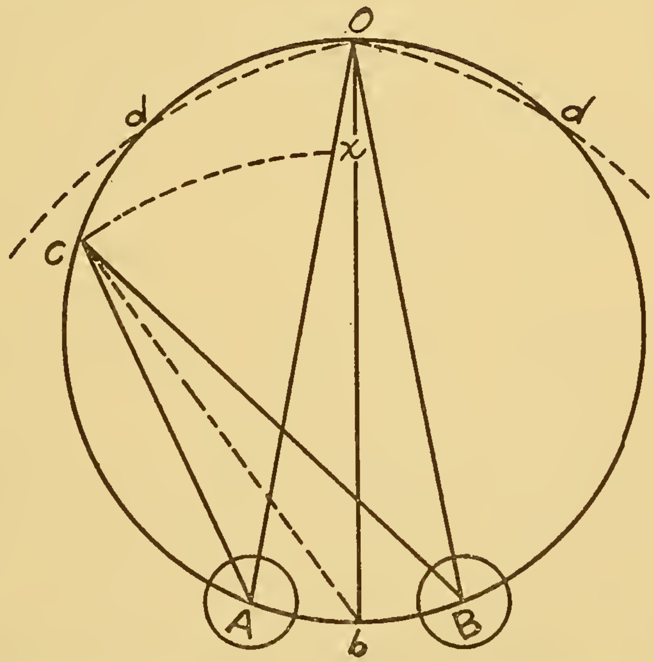


Fig. 182.—To Illustrate the Relation Between Convergence and Accommodation in Lateral Fixation. (After Maddox.)

(1) The angle of convergence is the same whatever point in it is made the point of binocular fixation.

(2) In glancing from any one point in it to any other, both visual

axes traverse equal angles. Thus in Fig. 182 the angles *OBc* and *OAc* are equal. In contrast to this we have the condition in which the eyes fix a point in a plane surface, such as a wall, when the point of regard is shifted to the right or to the left. If the point of fixation is to the left of the median plane the left axis passes through a greater angle than the right.

(3) The line which bisects the angle of convergence is the one to which, hypothetically, objects upon the maculæ should be mentally referred. This line is represented as *cb* in Fig. 182. It is inclined to the median plane by an angle which measures the obliquity of vision: it is equal to the angle through which each visual axis has turned in shifting the fixation from the median plane point *O* to any other point in the circle of equi-convergence.

361. If the two curves of Figs. 180 and 181 are applied to each other as in Fig. 182, the dotted arcs indicating the line of equal accommodation, we see demonstrated the fact that within a certain degree of obliquity of vision the proportion of convergence to accommodation is greater than in the median plane, while for increased obliquity the proportion is less. At the points *d,d* the relation between accommodation and convergence is the same as at *O*. The distance of *d* from *O* is equal to the inter-central distance, *AB,* of the observer. This diagram also shows how the accommodation and convergence required in looking at any point obliquely may be compared with those needed in the median plane. In the case of any given point, *O,* we need only to describe a circle through it and the centers of rotation of the eyes, and from the center of the farthest eye to draw the arc *cx* from *c* to the median line. The arc is then part of the line of equal accommodation. In binocular vision at the point *c,* therefore, convergence must act as if for *O* and accommodation as if for *x*. Were the relation between accommodation and convergence inflexibly that, for instance, demanded for objects in the median line there would be a diplopia for any object outside of this median plane except possibly at one point on each side within which diplopia would be heteronymous from relative divergence and without which it would be homonymous from relative convergence. Furthermore, the farther the point of fixation is moved from the median plane the greater becomes the physiological difficulty of converging the eyes so that the excess of convergence effort required above the demands made upon the accommodative efforts increases in proportion to the lateral deviation of the line of regard. Bolton found the following deficiencies in

convergence with accommodation for 10 inches at the various angles indicated. (See Maddox, *Ocular Muscles.*)

For an Object Distant	Exophoria
On looking straight forward (median plane).....	— 6°
Looking 10° to the right........................	— 7° 10′
Looking 20° to the right........................	— 8° 54′
Looking 30° to the right........................	—10° 45′
Looking 35° to the right........................	—12° 36′

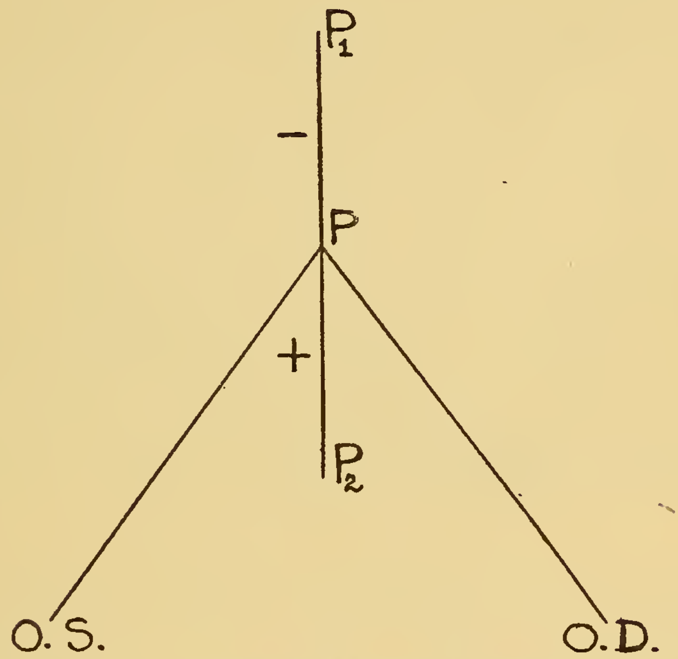


Fig. 183.—Illustrating Relative Accommodation.

The last figures show the visual axes to be actually divergent so that if they were prolonged they would meet at a point behind the head. "It is," says Maddox, "just as much as we can do to overcome this tendency to diplopia in the lateral limits of the field of fixation."

362. *Relation of accommodation to convergence—Relative accommodation.* The amount of accommodation which it is possible for an individual to exert or relax with respect to a given degree of convergence is called the *relative accommodation.*

Suppose that the eyes are accommodated and also converged to a point *P*. Then if concave glasses of gradually increasing strength are placed before the eyes, the patient will retain the same degree of convergence but will be forced to increase his accommodation up to a certain limit. This degree will be represented by the strongest concave lenses which can be overcome and is equivalent to accommodation at a point nearer than that to which the eyes are converged, as for example, P_2 in Fig. 183. The distance PP_2 will then represent the *positive* portion of the relative accommodation. In a similar manner, if the person continues to converge to the point P and convex lenses of gradually increasing strength are placed before the eyes, there will be relaxation up to a certain point or limit, for example, P_1. The distance PP_1 will then represent the *negative* part of the relative accommodation; hence the total range of relative accommodation will be equal to the sum of the positive and negative portions. A simple illustration will not be inappropriate; the reader is referred for further information to the writings of Donders and to the more recent and excellent treatises of Howe on *The Muscles of the Eye.* Assuming an emmetropic condition, we find the positive part of the relative accommodation by inserting before the patient's eye the strongest concave lens which does not blur the line which should be seen at that distance: these lenses represent approximately the degree of extra accommodation made by the ciliary muscles. Let us suppose that they are — 3.25 D. S. Since this represents the apparent rather than the real amount of accommodation exerted, the real amount must be calculated. If this lens is placed about 30 mms. from the nodal point of the crystalline lens, then

$$\frac{1}{R_1 - 0.03} - 3.25 = 0$$

or the real value, R_1, of the positive accommodation is 2.95 diopters. The negative portion of the relative accommodation should next be measured by using convex lenses; inasmuch as we have assumed emmetropia we have an actual negative accommodation of zero value however.

363. Let us next take convergence at one meter. In this case the patient fixes test-letters properly constructed for the distance at which they are to be used as in Howe's optometer. Care must be exercised that the pupillary distance of the frames carrying the lenses is changed so that the optical centers of any lenses which may be inserted are in the lines of the visual axes when the individual under examination is viewing the test-type at the distance specified. An emmetrope at one meter's distance exerts naturally an accommodation of one diopter. We then find the strongest concave glasses that can be overcome in the manner previously described. Suppose this to be — 3.00 D. S. The actual amount of positive relative accommodation can be shown

to be about 2.6 diopters, since calculation (see footnote) indicates that the total accommodation is 3.6 diopters of which the emmetropic eye without a lens, converging at one meter, will exert one diopter. The negative part of the relative accommodation with 1 M. A. of converg-

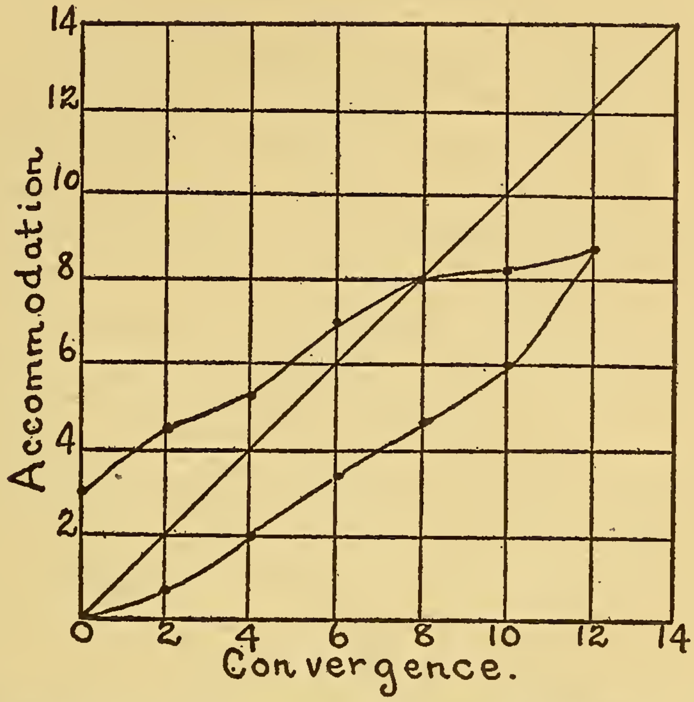


Fig. 184.—Lines Showing the Relative Accommodation as Plotted in a Given Case. (Howe's *Muscles of the Eye.*)

ence must next be obtained. This is done by the employment of convex lenses until the type normally readable at the distance under which the tests are made begins to be blurred. Suppose this is + 0.75 D. S. The total range of the relative accommodation at one meter is,

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Let *P* represent the distance from the nodal point of the eye to the point *p* and P_1 the distance from the nodal point to the point p_1 and let *d* be the distance of the lens from the nodal point. In these formulae *p* represents the point looked at while p_1 is the point for which the eye is adjusted. Then if a *convex* lens is placed before the eye, its dioptric value, $\frac{1}{F}$, is correctly determined from the equation

$$\frac{1}{F} = \frac{1}{P - d} - \frac{1}{P_1 - d}$$

$$\text{or } \frac{1}{P - d} = \frac{1}{P_1 - d} + \frac{1}{F}$$

When a *concave* lens is placed before an eye the formula becomes

$$\frac{1}{P - d} = \frac{1}{P_1 - d} - \frac{1}{F}$$

P_1 is, of course, the term whose value is sought in these experiments upon the relative accommodation.

therefore, 3.32 diopters. Similar measurements on the negative and positive portions of the relative accommodation should then be made with convergences of two, three and four meter-angles and so forth. In Fig. 184 there is plotted a set of curves showing the relation between the positive and negative portions of the relative accommodation and convergence. The diagonal running across from the lower left-hand to the upper right-hand corner represents the relation which would exist if there was found to be one meter-angle of convergence associated with each diopter of accommodation. Hence the positive part of the relative accommodation is recorded above the diagonal and the negative part below it: the accommodation values are usually plotted vertically and the convergences horizontally. In Fig. 184, for example, with fixation at infinity in a specified case of emmetropia, the positive relative accommodation is plotted as 3 diopters while the negative relative accommodation is zero. Passing on to 1 M. A. of convergence, we proceed to plot our positive and negative relative accommodations from the diagonal line at the point opposite 1 diopter of accommodation and 1 M. A. of convergence and not from the horizontal axis. In the figure as plotted we see that the positive relative convergence is equal to 3.5 — 1 = 2.5 diopters and the negative accommodation to 1 — 0.25 = 0.75 diopter, and so on. When there is no longer any positive portion of the relative accommodation the curve crosses the diagonal. Beyond this point there is only a range in the negative portion of the relative accommodation.

364. If the patient is ametropic the statement that at a distance of one meter there is exerted one diopter of accommodation does not hold. Thus, in cases of myopia, convergence occurs while accommodation is impossible: in a case of myopia of 4 diopters there will be a convergence of six meter-angles but an accommodation of two diopters only. In hyperopia more accommodative than convergence effort is required: a hyperope of 3 diopters will exert 5 diopters of accommodation when using 2 M. A. of convergence. These facts must be taken into account in the plotting of accommodation—convergence curves. This is done by starting the diagonal either above or below the zero mark taken for emmetropia.

And again, the *negative* part of the relative accommodation for any point of convergence may be found objectively by the methods of *dynamic skiametry.* The mechanical arrangement necessary for such tests consists simply in the attachment of a small card, carrying printed characters, to the side of the retinoscope. Observation and fixation points are then one and the same. To insure the presence of both convergence and accommodation the subject under test is required to read or decipher (i. e. to at least make the effort) the material on the fixation card. The operator proceeds to add plus lenses binocularly, viewing the reflexes in each eye in turn, until *reversal* of shadow is obtained. This gives the total lens quantity demanded, from which there must be algebraically deducted the static corrections found if we are desirous of knowing the true relative negative accommodation when the eyes have been optically re-established as nearly perfect as seems possible. Such tests can, of course, be made at any fixation points desired. Furthermore, such tests as made by dynamic skiametry in which such additional lens quantities are added as will afford neutrality of shadows at the point fixed, often furnish valuable information as to the optical assistance needed in order to adequately supply accommodative demands, correlate and harmonize economically accommodation and convergence or furnish an objective method of determining the proper reading correction in presbyopia. (See *Dynamic Skiametry* by A. J. Cross and *Dynamic Ocular Tests* by C. Sheard.)

365. The importance of such investigations may be questioned by the reader. Donders laid down the very important principle that "The accommodation can be maintained only for a distance at which, in reference to the negative part, the positive part of the relative range of the accommodation is tolerably great." Howe says:—"It is desirable to determine whether the action of that (ciliary) muscle is normal or excessive or insufficient. At least a general idea as to the power of the ciliary muscle is shown, as already stated, simply by placing a minus three (diopter) glass before each eye and asking the patient to read again the distant test-type. I have learned to regard this as *one of our most important tests.*"

366. *Relation of accommodation to convergence—Relative convergence.* The procedure for measuring relative convergence is similar to that for measuring the relative accommodation. It is very similar in theory also. With a given accommodation, then, the strongest adductive (bases out) prisms show the relative near point of fusion while the strongest abductive prisms show the relative far point of fusion. If a person whose ocular base line is 58 mms. can, when fixing the test object at 6 meters, overcome adducting prisms amounting to 14 degrees, we can calculate (or take from tables already worked out) the amount of this positive relative convergence as 2.2 M. A. The same method is followed in obtaining convergence powers at other fixation points.

367. Fig. 185 gives curves showing the positive and negative portions of the relative convergence. It is plotted in the same manner as an accommodation-convergence curve, except that, instead of reckon-

ing vertically from a certain point of the diagonal, we count horizontally from that same point of the diagonal. So many squares to the right show the positive part of the relative convergence and so many squares to the left show the negative part. We find that the lines representing positive and negative convergences are often almost parallel to the diagonal in the system of co-ordinates employed. To plot the relative convergence with parallel axes we should represent the positive part on the right of the first horizontal line, i. e., about two squares from the zero point in the illustrative case taken in the preceding paragraph, whereas the negative portion would be represented on a continuation of that line to the left by a distance of 1.8

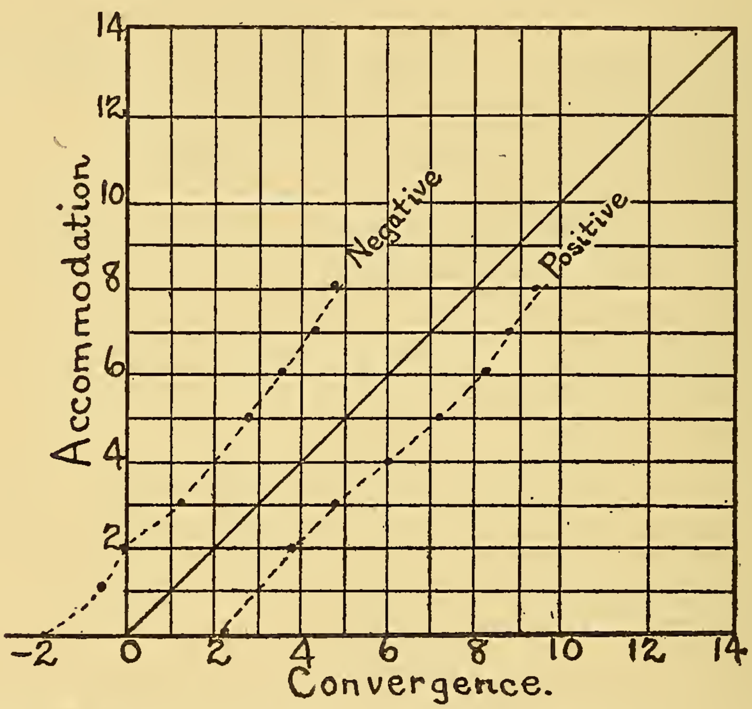

Fig. 185.—Lines Showing the Relative Convergence as Plotted in a Given Case.

squares. With one diopter of accommodation the positive part of the convergence would be at a distance of two squares from the right of the diagonal and the negative portion at a distance of 1.8 squares to the left of the diagonal; these figures are assumed to have been clinically found in a particular case. The remaining points of the positive and negative relative convergence curves are then determined in a similar manner for different amounts of accommodation demanded.

XX. THE PROJECTION OF VISUAL IMPRESSIONS

368. *The general law of projection.* An impression at any point of the retina is projected outward into the visual field following the line of direction. This line of direction is one which passes through

the retinal point involved and the nodal point of the eye. It is important to recognize that projection is not a faculty of the retina but is a mental act. The more perfectly the projection is performed the more nearly do the projected images or pictures of external objects coincide with the objects themselves. Projection is a congenital faculty but is perfected during the exercises of childhood when the real position of objects is constantly being discovered by other senses. Conversely it may be said that an exterior point upon which the eye is focused has its image formed at the point of intersection of the line of direction with the retina. When referring, therefore, to objects seen distinctly the law of projection is equivalent to saying that we see objects in the direction in which they really are. Since the image of an external object formed upon the retina is perverted and inverted it follows that a ray proceeding from the temporal side of an object will meet the retina at the nasal side and a ray proceeding from the upper portion of the object will meet the retina in its infra or lower portion, the macula being regarded as the visual center of the retina, and so on. It follows that the field of projection is re-inverted so that its right half corresponds to the left half of the retina and its upper half to the lower half of the retina. The law of projections applies not only to the ordinary phenomena of vision but also to all the retinal impressions, the phosphenes, after-images, entoptic phenomena and circles of diffusion. The deformities of objects seen indirectly, which appear to show that this law is not followed very exactly for very peripheral parts of the retina, may be cited as exceptions.

In order to be able to form a correct idea of the position of an exterior object it is necessary that we know its direction and its distance. Judgment of direction is formed better monocularly than binocularly: the superiority of binocular vision is apparent in the judgment of distance.

369. *Projection of the visual field.* The law of projection regulates the manner in which objects are localized in the visual field but does not regulate the projection of the visual field in its entirety. The latter depends upon the manner in which one judges of the position of his own eye or the direction of the visual line. One of the most important factors in regard to the judgment of form, size and direction of objects is that these judgments are largely based upon what is known as the *muscular sense.* Helmholtz says that there should be distinguished under the term "muscular consciousness" a number of sensations essentially different. They are: (1) the intensity of the effort of the will by which we endeavor to cause the muscles to act, (2) the tension of the muscles, i. e., the force by which these muscles

strive to act, and (3) the result of the effort which is indicated externally by an effective shortening of the muscle. To these Stevens would add a fourth which he calls "the consciousness of the intensity of the will effort to accomplish the muscular change": in other words, an element of the muscular sense is the knowledge gained by experience of the individual, or inherited from the experience of others, of the intensity of the will impulse demanded for the execution of a muscular act. We are, for example, fixing a point *A* and desire to fix another point *B*. As long as *A* is fixed, *B* is seen by indirect vision and the distance between the images permits us to judge of the amount of innervation required to bring the line of regard to *B*. Generally and normally this judgment is quite exact and instantaneous so that we bring the look toward *B* almost without hesitation. Due to the innervation there results a contraction of the muscles, a change in the position of the eye and a change of the position of the retinal image until the image of *B* is formed on the fovea. As Tscherning remarks, it might appear plausible that the sensation of the more or less considerable contraction of the muscles, the gliding of the eyes between the lids and other correlated phenomena should furnish us with information on the direction of the visual line, but this is not true. This direction is judged solely by the degree of innervation which has been used to bring the line of regard into this direction. Observations of patients affected with ocular paralysis establish this statement. If, for illustration, a patient is affected with paralysis of the right external rectus and he is told to close his left eye and to look to the right, he may supply the necessary innervation. But the eye remains practically motionless on account of the paralysis while the patient, on the other hand, believes that he has moved his eye to the right so that there results a false projection; if the patient under observation is told to move his finger rapidly towards an abject situated to the right, not having sufficient time to guide himself by the sight of his finger, he consistently moves it too far to the right. A person with normal ocular innervations and muscular responses can perform this experiment satisfactorily by looking to one side while a traction is exerted in the opposite direction on a fold of the skin near the external canthus. This traction is communicated by the conjunctiva to the eyeball. Because of the resistance thus offered one is obliged to use a stronger innervation to bring the "visual regard" to the opposite side; from this it may be concluded that the look is carried farther in this direction than it actually is and this causes projection of the visual field in a false manner. The judgment of the degree of innervation demanded and used is quite exact because it is always corrected by the results obtained. Suppose one places a ten-degree prism, base out before one eye and base in before the other, and then looks directly in front. As seen through the prisms an object situated at ten degrees to the right appears five degrees from the visual line and an innervation corresponding to five degrees only is demanded in order to fix it. One believes, therefore, that it is situated at five degrees to the right and if one desires to grasp the object the hand is not brought far enough to the right. A few repetitions of this experiment, however, suffice to remove the deception: one can learn very quickly how to allow for and reckon with prisms. When the direction of the visual line is correctly judged there is in monocular vision no possible illusion as to the direction in which objects lie. In monocular vision the center of co-ordinates is represented by the nodal point of the eye and the law of projections gives the direction of any radius vector. The position of any point in space is determined by the direction and length of the radius vector from the center of co-ordinates to the point in question: uniocular vision gives the direction but the length of the radius vector is lacking.

370. *Projection in binocular vision. Physiologic binocular diplopia.* The two laws—the law of external projection and the law of direction—are two of the most fundamental principles underlying vision. What has been said thus far treats only of monocular vision. Most individuals possess two eyes which act as practically equal visual machines: these are not, however, to be considered as mere duplicates one of the other or that, if one is lost, the other is still left. On the contrary, the two ordinarily act together as one instrument and there are many visual phenomena and many judgments based upon these phenomena which result entirely from the use of two eyes as one instrument. The phenomena of binocular vision are less physical and more psychical than those of monocular vision. They are more obscure, illusory and much more difficult to analyze since they are more subjective and more closely allied to psychical phenomena. When the two eyes perform their functions correctly both of them fix the same object; hence the *impressions of the two maculæ are projected to the same place.* Images formed upon the two foveæ are projected under all conditions to the same spot in space. In recent paralytic squint two candles held in line with the two visual axes appear as one. If a sheet of paper be punched with two small holes separated by the interocular distance and this is held up close to the eyes so that distant objects can be seen through them the two holes will be seen as one and will lie in the median line. And again, a bright point of light may be fixed with one eye so as to produce a small foveal after-image.

It will then be found that no matter what object is fixed with the other eye, or however much the eye having the after-image be displaced or made to squint, the after-image will be seen at whatever point is fixed by the other eye.

371. When a pair of eyes is in the primary position (i. e., looking straight ahead into distance) the fields of vision of the two eyes overlap except in a sector of about thirty degrees toward the temporal periphery of each field. A normal-sighted person, therefore, sees objects with both eyes simultaneously, the exception being for objects which lie on the extreme right or left which are seen with one eye only. When a distant object is under regard the two eyes assume such a position that a picture of the object is formed simultaneously on the central part of each retina. All other distant objects are focussed on functionally corresponding points of the retina. These impressions are then blended in the brain and one is conscious of only one picture. But binocular vision of near objects is a much more complex act inasmuch as it is necessary that there be ultimately blended images which do not fall, as we have previously discussed, upon corresponding points of the retinæ. In every ordinary act of vision, then, a large number of objects do not have their images formed on corresponding points of the two retinæ. For every position of the point of fixation there is an *horopter* in which all objects are seen single, while all other objects are seen double if close analysis is made. We have, then, the phenomena of *physiologic binocular diplopia*. The following is a simple but practical illustration. If two fingers are held before the face in the median line or plane, one being somewhat in advance of the other, and the nearer finger is fixed, two images of the more remote finger will then appear. The right image will disappear on closing the right eye and the left on closing the left eye, which shows that the *distal* diplopia, as Maddox calls it, is *homonymous* or uncrossed diplopia. If, on the other hand, the more remote of the two fingers is fixed, the nearer one will be seen double, the left image corresponding to the right eye and the right image to the left eye. The *proximal* diplopia is then crossed. These experiments teach us that objects nearer than the point of fixation and the horopteric surface connected with it have crossed images while all objects beyond the point fixed have homonymous images. This physiologic diplopia must, therefore, be constantly present yet no one is ordinarily conscious of seeing double.

372. It is evident, then, that when an object is seen double there is at least one of the images which does not coincide with the object. When one eye is closed the corresponding image disappears while

the other image does not change position. The information which the eyes under these conditions furnish us gives rise to a false interpretation; a false judgment must, therefore, persist at least for one of the eyes. The sight of normal persons is not necessarily similar to that arising in monocular vision. Physiologic diplopia is due to the fact that the different positions of the two eyes are not taken into consideration. One cannot, without a special investigation, tell whether an image belongs to one eye or the other. Every visual impression, from whichever eye it may come, is referred to a common and single center. In projecting the retinal fields into space the mind must have some "point or origin" or "line of direction." Reference has already been made in a preceding paragraph to the fact that the nodal point of the eye is the so-called center of co-ordinates in monocular projection. If, then, we take into account the different positions of the two eyes we shall have two centers of co-ordinates and the notion of the direction of an object would suffice to fully determine its position. The line of reasoning would then be somewhat as follows:—Since, when we fix a remote object *A* and see two images of a nearer object *B*, we see with the right eye only the object *B* to the left of *A* and with the left eye the same object to the right of *A*, the object must then lie in the middle plane and nearer than *A*; the object *B* would then be seen single and in its correct position. Instead of this, however, the impressions are referred as in monocular vision to a *single* center and we are informed that the object must be double since it is seen at once to the right and to the left of the fixed object. Hering places "this origin of co-ordinates" for binocular vision at a point midway between the two eyes exactly as if they were united into one *cyclopic eye*. This is, without doubt, the true type of binocular vision in which neither of the eyes plays a dominating or directing part. Some, however, and possibly the majority even of those who have equal visual acuity in the two eyes, apparently use one eye chiefly as the auxiliary to its mate rather than as an equal in the processes of projection. Under these conditions one eye is called the "directing eye" and the origin of projections appears to coincide with the origin of co-ordinates of this directing eye. Tscherning gives some experimental proofs to show that the center of projections in his own case coincides with his right eye. In fact it is commonly found in the physiologic diplopia of persons who are right-handed that the image which belongs to the right eye is more "substantial-looking" than the other. Tscherning describes the following simple experiment and draws from it a rather interesting conclusion as to the inferiority of binocular vision with its origin of projections midway between the eyes. He says: "We fix binocularly an object placed at some distance in the median plane and we try, by quick experiment, to place a stick quite near the face in the direction in which we see the object: it is better to conceal the movement of the hand with a screen. Making this experiment I bring the stick pretty exactly on the visual line of the right eye. * * * Most persons examined show a tendency to prefer one eye or the other, which seems to indicate a tendency to a development of a uniocular vision in addition to the binocular vision like that which I have described for my eyes. Persons enjoying pure binocular vision must place the stick in the median plane; as the center of projection does not coincide with either of the eyes, these people cannot project correctly objects seen indirectly. This type of vision, therefore, seems inferior to the other as far as orientation is concerned.''

373. Since some exterior objects are seen double and some single, depending upon whether they lie within or without the horopteric circle on the one hand or upon it on the other hand, one might think that there would thereby result considerable confusion. But there is not: most persons have never observed double physiologic images before making the experiments involving this phenomenon. Under ordinary circumstances one's *attention* is brought to bear upon the object fixed and, furthermore, the gaze never remains for any length of time on the same object so that there is little time in which to observe double images. We know also that objects not fixed form their images on the peripheral parts of the retina where the sensitiveness and resultant perception are less distinct than at the macula. It is scarcely possible to suppose a serviceable binocular vision if the entire retina had an acuity equal to that of the fovea (Tscherning). Why this physiologic diplopia, which must be constantly present, does not ordinarily make one conscious of seeing double has been explained in two distinct ways which we may, for brevity, designate as (1) *the elasticity of the fusion faculty theory* and (2) *the image suppression theory.* This customary freedom from diplopia is possible, says Worth, not by the mental suppression of one of the images but by the marvellous elasticity of the fusion faculty. Both sets of impressions are received by the brain and by their combination assist in the perception and appreciation of depth. Some such view is evidently held by Stevens, who believes that the images of those points or objects which are seen doubly are not mentally suppressed but that they constitute an important, if not essential, part of the physical impressions which unite to constitute the basis for a complete mental conception of the field of view, and that the beauty of perspective and the harmony of objects in the field of view would be absent although

the skeleton of the field of regard might remain. Stevens cites in support of these statements the experiences of persons subject to gradual atrophy of the optic nerve, for when the field of view is much reduced they are unable to see their way to walk though they may read letters directly in front of them at the standard distances. "The images which fall upon spatially non-corresponding points therefore serve an essential purpose. They serve as finders in the field of space. By means of these peripheral images not only are the eyes enabled to turn from one object to another and from one part of the same object to another part, but there is a mental estimate of their relative positions on the retinas from which conclusions respecting the positions of objects in space are drawn. * * * What has been said of single images when the impressions are located at corresponding points and of double images when they are located at non-corresponding points is, then, in an important sense, a physical law, but there are circumstances which indicate that a higher law governs all these phenomena. It is the law of *unconscious conclusions.*"

374. We now turn to briefly discuss the *suppression* theory. As has been previously remarked, it is commonly found that in the physiological diplopia of persons who are right-handed the image which belongs to the right eye is more "substantial-looking" than the other. While attention is, then, diverted from the diplopia and concentrated upon the point of fixation, the less substantial image of objects outside of the horopter is in most persons so entirely ignored by the mind as to be "suppressed." Tscherning writes:—"It seems that this suppression of the images of one eye plays a great part in binocular vision and that it is this which generally causes us not to observe double physiologic images. It is not easy to know which of the two images is suppressed, for as soon as we pay attention to this question both appear. Generally it is the more eccentric image, or, in other cases, the image which, on account of the perspective, occupies the smallest retinal surface (Javal) which disappears. But in most persons there seems to be developed a certain superiority of the eye which is most frequently used separately and then it is always the image of the other eye which is suppressed."

375. *The horopter—The isogonal circle.* No subject in physiologic optics is so replete with conflicting views of different investigators and none shrouded in a greater mystery or confusion of contradictory ideas than that of the *horopter.* Helmholtz devoted considerable space in his *Handbuch* to the experimental and mathematical developments of the horopter: he worked out a single horopter of the infinite number which may exist and even that one, it is claimed by many, is

faulty because it is based on false premises. Some have, in the language of Giraud-Teulon, characterized the horopter as a "transcendental fancy": "when," he says, "all the labor of determining the surface curve (fulfilling the geodescial condition of the horopter) was ended it was found that this surface assumed the form of a torus. * * * It was not noticed that a table with four legs, a chair placed before us, was seen singly, although they certainly had not the attributes of a torus."

376. The *horopter* may be defined as consisting collectively of all the points in space whose images, with a given alignment of the eyes, fall upon corresponding points of the retinæ. All points outside of the point fixed are not seen double as in the case of physiologic diplopia in which the second object is inside or outside of the point fixed. If we fix a point *A*, for example, then a point *C* some ten degrees to the right or left of *A* is seen as well with the right eye as with the left eye: it is therefore always normally seen single. The entirety of the points seen single when a given point is fixed is the horopter. The definition of the horopter is apparently the only point upon which various investigators agree. By some it has been described as a line, by others as a surface and by Helmholtz as a complex combination of curves, planes and straight lines. "In a single case only," says Helmholtz, "is the horopter a surface; it is when the point of regard is situated in the horizontal and median planes and at infinite distance. The plane of the horopter is then parallel to the plane of regard. In the case of normal eyes thus directed toward the horizon the horopter coincides approximately with the ground on which the observer walks." This has been objected to somewhat strenuously by Stevens who believes that if it were correct much ocular inconvenience would result and that "according to this proposition, if the eyes should be directed to the ground at a few feet in advance of the pedestrian he would bring his horopter beneath the soil and all the objects on his pathway would appear, so far as an horopter is concerned, confused and indistinct."

377. Two fundamental concepts or tenets constitute the essential foundation for the doctrine of the horopter. They are: (1) the theory of the position and direction of the meridians of the retinas and the law which regulates the position of the eyes (law of Listing), and (2) the theory of corresponding points. In respect to both these principles there is considerable diversity of opinion. Helmholtz, Volkmann, Hering and others came to the conclusion that the horizontal meridians were all parallel with the external horizon but that the vertical meridians were only apparently vertical and that they

leaned out above and approached each other below. That Helmholtz included in his mathematical calculations his own individual defects, which he assumed were physiological features common to all, is the claim of those investigators who hold that the actual and proper position for a vertical meridian is the vertical position. And, again, as previously pointed out, there is no universal agreement upon the notion of corresponding points. For, to quote from Helmholtz, "Corresponding points in the two visual fields are those which are at equal distances and equal in direction from the corresponding horizontal and *apparently* vertical meridians." Stevens sums up his conclusions respecting this subject in the following statement:—"Corresponding points are those points in the retinas which answer to *proportional degrees of rotation* and which, for given points in the plane of the point of fixation, receive incident rays which must pass through the nodal points." They represent, on this view, the relation between the muscular and the retinal senses. It is, then, but little wonder that the horopter and its calculation and significance appear so confused in the literature, for various investigators have not been able to agree upon the status and interpretation of the two fundamental tenets underlying the horopter.

378. Assuming the viewpoint of Stevens, we may say that when the point of fixation is at infinite distance and in the median plane all horizontal meridians are horizontal and all vertical meridians are vertical: so again, if in the plane of the horizon the point of fixation is brought nearer, the meridians maintain their original relations and these relations will continue if the eyes are directed upward or downward, provided the visual lines remain parallel. If the point of fixation is such as to demand convergence of the lines of regard and if it is above or below the horizon, the head being in the primary position, then all horizontal and all vertical lines assume new directions; these torsional rotations are governed by the law of Listing. If the visual lines of the eyes converge and the plane of regard is depressed, the horizontal meridians of each eye will tilt downward toward the temporal side and upward toward the medial side; the vertical meridians will also tilt with the upper part outward and the lower part inward, the tilting being proportional to the depression and the lateral direction of the line of regard. Under these premises, three simple horopters can be found. First: the observer directs the gaze towards the horizon in the median plane at infinite distance, the head being in the primary position; the horopter will be a plane surface at right angles to the plane of regard. Second: if the gaze is directed downward and to a few feet in advance, the horopter will be very

nearly at right angles to the plane of regard, tipping forward slightly however, since, although there is depression of the plane of regard, the convergence is so slight as to induce small torsional action. As a third case, let us take the condition in which the eyes are directed to the page of a book in the ordinary reading position and that the gaze is directed so that the point of fixation is in the median plane and that the plane of regard is depressed 35 degrees. A mathematical calculation (see Stevens *Motor Apparatus of the Eyes,* page 184) gives the position of the page in relation to the plane of regard in which the horopter is most completely formed and it is found that the page should be tilted about 15 degrees beyond the right angle with the plane of regard or at about 105 degrees. It is thus possible to predicate the position of the horopter when the depression of the plane of regard and the convergence are known, if in addition the length of the base line between the nodal points is known. The above calculation can be verified by a simple experiment for those who can unite stereoscopic figures by convergence without the aid of a stereoscope. Two parallel vertical lines and at a distance of two and one-half inches are drawn on a card: this card is held so that in fixing the center of the lines the gaze is directed downward 35°. The card is to be held at about thirty inches from the eyes; one who is expert in such exercises will be able to unite the two lines at the normal reading point. If the two lines are not perfectly fused but are allowed to remain an eighth of an inch apart the angle at which the card must be held in order to render the two stereoscopic images parallel can be determined. In general it is found that the card must be tilted forward about fifteen degrees.

379. Savage, in his book entitled *Ophthalmic Myology,* has given a somewhat different method and mode of obtaining the horopter or *isogonal circle* as he terms it. The horopter, in the sense that it is the circle of binocular single vision, both direct and indirect, as shown in Fig. 186 (B), is based on three assertions:—(a) the macula of every eye is the posterior pole and the visual axis is the antero-posterior axis of the eye; (b) all indirect lines of vision cross the visual axis at the center of rotation; (c) corresponding retinal points have a common brain-cell connection and these points bear identical relationship in degrees to their respective maculas. In Fig. 186 (B) the circle is constructed through two fixed points and one changeable point. These fixed points are the centers of rotations of the eyes, *b* and *d,* and the changeable point that of direct fixation. The direct point of view, *c,* and its images *h* and *g* are connected by lines that cut the centers of rotation *b* and *d.* The secondary point of view, *a,* and its images, *j*

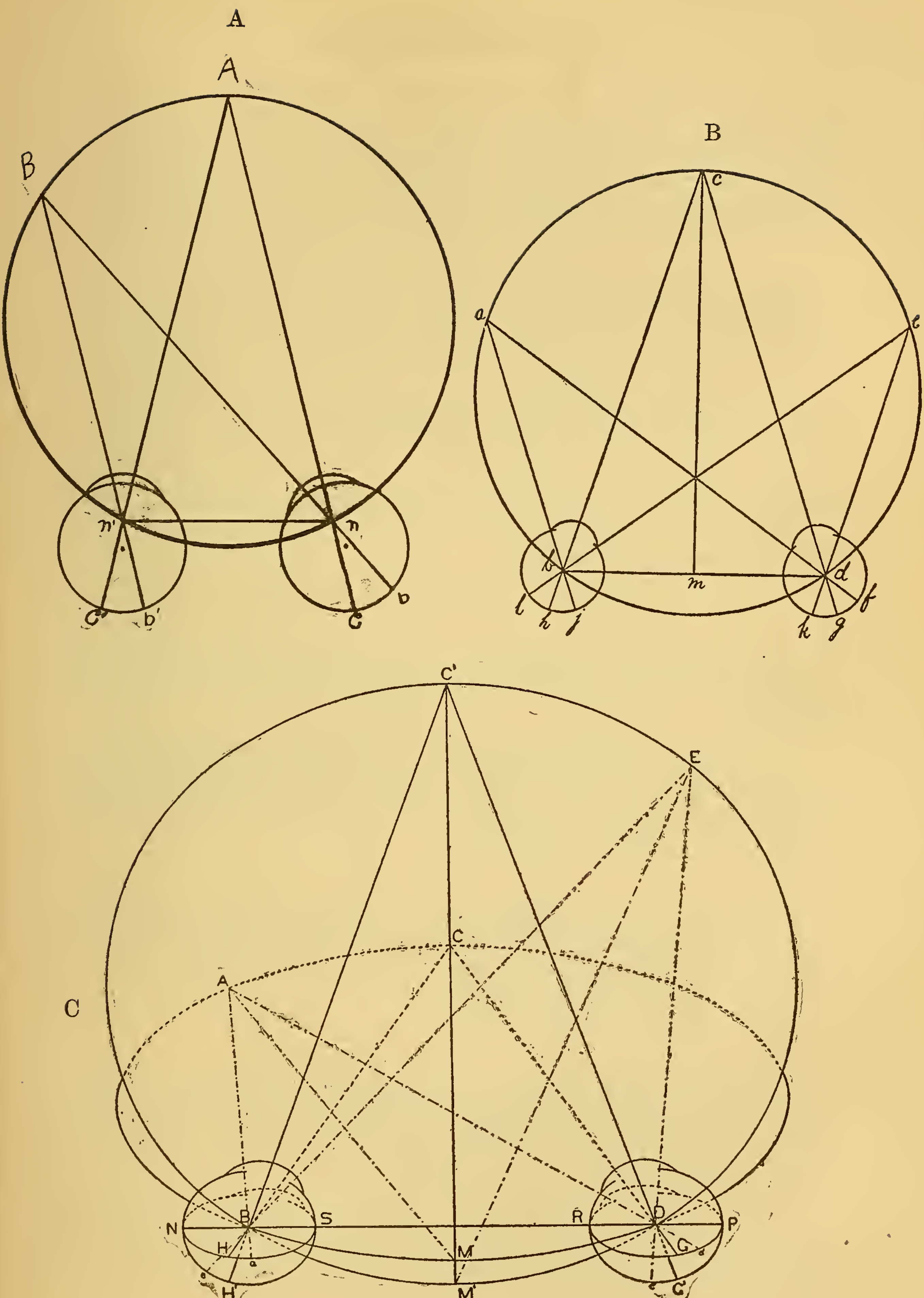

Fig. 186.—The Isogonal Circle. (After Müeller and Savage.)

and *f*, are connected by lines passing through the centers of rotation, *b* and *d*, and the secondary point *c* is connected with its images, *l* and $_{x}k$, by lines that cut the visual axes at the centers of rotation. All points on this circle, both direct and indirect, are seen under the same angle as is shown by the fact that each is measured by half of the arc *bd*. The figure also shows that the direct and the indirect points of view are related in degrees as are their respective images.

380. It is not inappropriate to pause at this point and to call attention to some of the teachings of Savage which are at variance with those of Helmholtz and others. Savage's first contention and assertion upon which he bases his isogonal circle is that the macula of the eye is the posterior pole and that the true optic axis is the visual axis. It is impossible to give, within the confines of this article, the reasoning upon which this statement is based. We can, then, but call attention to the main contentions of Savage in contradistinction to those of Helmholtz relative to ocular rotations. The following points are quoted from Savage's work.

Helmholtz	*Savage*
(1) The center of the cornea is always the anterior pole, and the center of the macula is the posterior pole only in ideal eyes.	(1) The center of the macula is always the posterior pole, and the center of the cornea is the anterior pole only in ideal eyes.
(2) The optic axis begins always at the central point of the cornea, passes backward through the center of rotation to the retina, rarely at the central point of the macula, but usually to a point between the macula and the optic disc.	(2) The optic axis always begins at the center of the macula, passes through the center of rotation and cuts the cornea, rarely at its center, but usually to the nasal side.
(3) The optic axis is the visual axis only in the ideal eye, and only then does the visual axis cut the center of rotation. Usually the visual axis misses the center of rotation by passing to the outer side of it, crossing the optic axis at the nodal point, and lying in only one meridional plane.	(3) The optic axis in all eyes is the visual axis and is the line of intersection of all meridional planes, hence it lies in the plane of every meridian.
(4) Visual lines are axial rays of cones of light, as is also the visual axis, and all these cross the optic axis at the nodal point. Even in ideal eyes the visual lines do not cross the visual axis at the center of rotation.	(4) Visual lines are not axial rays of light, but are radii of retinal curvature prolonged, all of them crossing the visual axis at the center of rotation, which is the center of retinal curvature.
(5) In passing from one point of view to any other, the visual axis of a non-ideal eye can not move in a plane of a meridian except when the rotation is directly to the right or left.	(5) In monocular motion every rotation plane is a meridional plane extended, and the visual axis always moves in this plane.

Helmholtz	*Savage*
(6) In cardinal and oblique rotation starting from the primary point of view or returning to it, the axis of any rotation lies in Listing's plane; the axis of rotation from one secondary point to another secondary point lies in a plane bisecting the angle between Listing's plane and the equatorial plane.	(6) The axis of every rotation, whether cardinal or oblique, whether from a primary to a secondary point of view, or vice versa, or whether from one secondary to another secondary point of view, lies in the equatorial plane.
(7) The object in space and its retinal image are connected by a straight line which crosses all similar lines at the nodal point, and never at the center of rotation.	(7) The object in space and its retinal image are always connected by a straight line which crosses all similar lines at the center of rotation and at no other point.
(8) The spacial pole, if on the same straight line with the two poles of the eye, can not be the direct point of view for non-ideal eyes.	(8) The spacial pole is on the same straight line with the two poles of the eye and is the direct point of view for all eyes.

And, again, the horopteric circle of Savage differs from those of Mueller and LeConte in that the indirect visual lines cross at the centers of rotation rather than at the nodal points.

381. As a result of his investigations, therefore, Savage defines an "isogonal surface" as follows: "The two visual lines, whether direct or indirect from corresponding retinal points, converging at any point on this surface, form the same angle as the two visual lines converging at any other point on this surface." All isogonal circles, whether primary or secondary, are alike in the following respects: (a) they are all constructed through two common points, the centers of rotation, of the two eyes; (b) they all have a common chord, the line connecting the centers of the two eyes; (c) they are all bisected by the extended median plane of the head; (d) all points on all the circles, belonging to one group, so located as to send light into the two eyes, will be seen as single points and (e) the two lines of vision connecting any secondary point, or any circle of a given group, with its two images, have the same angles as that formed by the convergence of the visual axes on the point of direct view.

XXI. MONOCULAR AND BINOCULAR PERCEPTION OF DEPTH

382. *The monocular perception of depth.* There are many ways in which a single eye can gain an idea of the third dimension: the eye, however, gives us no direct information as to the distance from which light comes to it. In the absence of direct information, however, a series of circumstances enables one to judge of the distance of an object, generally by an *unconscious judgment.* They are as follows:

(a) *Accommodation.* It is only in the judging of comparatively near objects that any assistance is derived from the conscious effort of accommodation. As a rule a greater effort of accommodation causes one to think objects are smaller. When the eye is accommodated for distant objects, near objects do not appear distinct, hence an experienced observer might use this phenomenon to judge of the distance of an object. The importance of accommodation in the judgment of distance is small, however, because generally such long distances are dealt with that the difference of accommodation is insignificant: we know, for example, that for all distances exceeding one meter the variation in accommodation does not reach one diopter.

(b) *Visual angle of known objects.* The knowledge of the nature of objects often furnishes the observer with a means of knowing their distances. If the size of an object is known its distance can be judged from its angular size. When a man is seen a long distance off, he does not appear to be small because we know the size he should be and conclude that he must be a considerable distance away since the angular size is small. This experience is characteristic of the manner in which unconscious judgments are formed and is a process of education.

(c) *Mathematical perspective.* The gradual decrease in the size of similar objects and the gradual approximation of parallel lines is well known. The number of intervening objects also influences the judgment; hence distances over water appear smaller than over land.

(d) *Shadows and overlappings.* Shadows are often important in the judgment of distance. If a surface is illuminated, the luminous source must be in front of it and if the object casts a shadow on this surface it must be nearer the observer than the surface. Shading added to a drawing gives a better idea of its reality.

(e) *Aerial perspective.* The more distant the object the greater the depth of the atmospheric veil and the greater the depth of the atmosphere the bluer is the veil. When there is considerable water-vapor in the atmosphere distant objects, such as forests and hills, appear more remote than they in reality are and consequently seem larger than they are. In mountainous districts, in which the air is usually very pure, distances are judged to be less than they actually are. It is, again, a matter of common knowledge that the sun and moon appear larger when they are near the horizon. Their angular sizes remain constant however. Since the moon, near the horizon, appears larger than at the zenith although it has the same angular size, we may say that we judge it to be farther away. The illusion is due to aerial perspective, for the moon is seen through a much thicker layer of the terrestrial atmosphere when it is near the horizon than when it is at the zenith. It is probable, also, that the comparison which is possible with terrestrial objects when in the horizon plays a part in the formation of the judgment.

(f) *Parallax.* This is by far the most important and valuable indicator in the formation of judgment as to the third dimension in uniocular vision. A single-eyed person who views an object not too far distant, while he cannot see the object as in binocular vision from two points of view simultaneously, yet can do so consecutively by moving his head from one position into another. An observer often sees, without being conscious that he does so, the relative movements of external objects and uses these to account for their relative positions. If, for example, an eye is displaced from *C* to *D*, Fig. 187, and the observer sees the object *A* displaced to the right relatively to the object *B, A* must be nearer than *B.* If, after having observed the

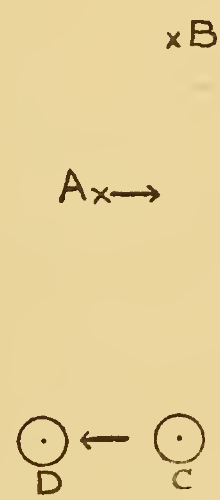

Fig. 187.—The Influence of Parallax.

objects *A* and *B* with the eye in the position *C,* the eye is closed and is again opened when in the position *D,* the observer realizes that *A* has changed position relative to *B.* This is sufficient to judge of its distance. The judgment is based upon the comparison of the successive retinal images. These images change position with each shift of the eye. But as all comparisons by memory are defective, one obtains a much clearer notion of the differences between the images and hence of the relief by a comparison of the images simultaneously with the two eyes. When binocular vision is enjoyed, each eye receives a perspective image of the objects situated before it; since the two eyes are not in the same place there are differences between the two images which are more pronounced the nearer the object is to the eyes. If, however, we look at a plane with both eyes the retinal images are identical. It is the perspective image of an object which permits us to distinguish an object of three dimensions from a plane. This difference in images exists only at near points: if the objects are at a larger distance the retinal images are alike. A landscape, therefore, presents nearly the same appearance whether we view it with one or both eyes.

XXII. THE BINOCULAR PERCEPTION OF DEPTH

383. *The influence of convergence.* While parallax is the most important and valuable indicator in the formation of judgment of the third dimension *monocularly,* the *degree of convergence* which it is necessary to use to fix an object gives us the most reliable information binocularly as to the distance of an object. The degree of innervation employed is the criterion for the judgment of the direction of the visual line in uniocular vision; it is the innervation and not the sensation of the position of the eyes which is the guide. The absolute amount of convergence actually used is not of any great aid in the binocular perception of depth; it is solely for differences of convergence that there is an exact sensation, for to any volitional increase or decrease of convergence the mind is very sensitive. We can judge with great exactness whether one object is nearer or farther away than another; the judgment of absolute distance is extremely difficult.

Many experiments have been made to determine the rôle played by the convergence in the perception of depth and of distance. Wundt (*Lectures on Human and Animal Psychology*) placed the face of an observer before a box open at that side and having a horizontal slit in the other side through which both eyes could look at a white screen, the surrounding objects being excluded from the field of view. A vertical thread, kept taut, hung between the slit and the screen. In the experiments which were made to determine to what degree of certainty the comparative distance of the thread, when it was made to approach or recede, could be determined, Wundt and his collaborators were careful, whenever the thread was moved, to close their eyes during the movement and upon opening them to look first at the screen and then at the thread. Such experimentation showed that, on the average, one could determine the approach or the recession of the thread to within $1/50$ of the distance and that the degree of accuracy increased with the degree of convergence of the visual lines. To illustrate: if the thread hung at a point fifty centimeters from the eyes it was found possible to determine the fact that it was nearer when the thread was moved up to the forty-nine centimeter point. Bourdon (*La Perception Visuelle de l'Espace*) arrived at conclusions which do not vary greatly from those of Wundt. He found that a convergence of eight minutes for each eye, the object fixed being 1.08 meter distant, was needed to recognize the fact of an approach or recession of a very small object. The fact that the nature of the change of adjustment of about one-quarter of a degree between the two visual lines could be constantly detected, after an interval of time during which the eyes had been variously moved, shows the sensitiveness or delicacy of the sense of movement of the visual apparatus.

384. *The stereoscope.* In 1833 Wheatstone enunciated the principle on which the stereoscope is constructed. His statement is as follows:—

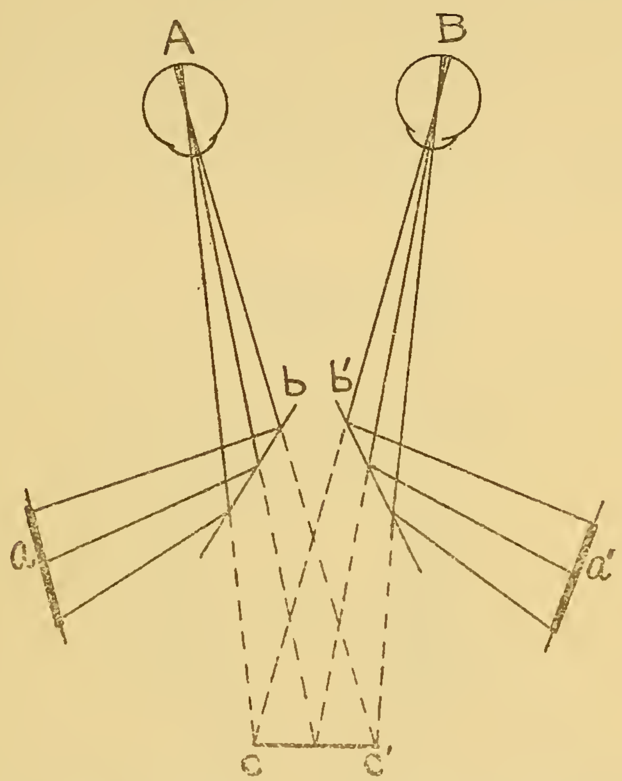

Fig. 188.—Diagram Representing the Principles of Wheatstone's Stereoscope.
a, a', The cards with the two pictures.
b, b', The mirrors.
c, c', The apparent position of the combined images.

"A solid object being so placed as to be regarded by both eyes projects a different perspective figure on each retina; now, if these two perspectives be actually copied on paper and presented, one to each eye so as to fall on corresponding parts, the original solid figure will be apparently represented in such a manner that no effort of the imagination can make it appear as a plane surface." Wheatstone's stereoscope consisted of two glass mirrors fixed in frames and adjusted to an angle of ninety degrees with each other. The drawings are placed in suitable holders at each side and make an angle of 45° with the mirror on the same side respectively. The image of each drawing is then seen by reflection; by means of mechanical devices the drawings may be moved to a greater or less distance from the mirrors and also their angles to the mirrors may be changed. The two pictures may thus be presented to the eyes when adjusted in parallelism or in convergence. The diagram, Fig. 188, represents the principles of the reflecting stereoscope: *a* and *a′* represent the cards with the two pictures; *b* and *b′* the mirrors and *cc′* the apparent position of the combined images. In order that the relief may not be reversed or pseudoscopic it is necessary to present to the left eye the image outlined for the right eye, since the mirrors reverse the images.

385. Each of the images of the stereoscopic picture is drawn in such a way as to form in the eye a retinal image like that which would be formed there by the object itself. Distant objects are represented by identical images, while the images of near objects are different. In

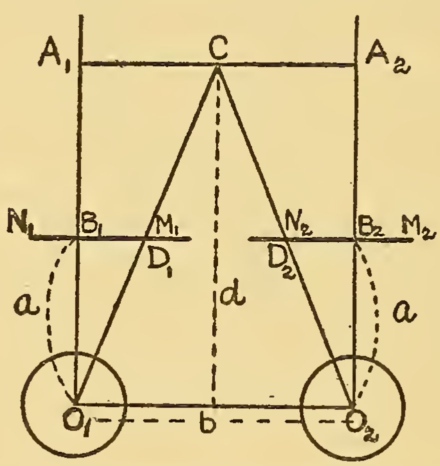

Fig. 189.—Illustrating Stereoscopic Parallax.

order to account for the way in which objects are represented on stereoscopic images, let us suppose two plates, N_1M_1 and N_2M_2, which are transparent, placed in front of the eyes at the position which the stereoscopic cards ordinarily occupy (Fig. 189). Straight lines directed toward the eyes are drawn from all exterior points: two such lines start from each point. The points at which these lines cut the corresponding plate is the reproduction of the exterior point. If this exterior point lies at infinity the two straight lines are parallel and the distance B_1B_2 between these points is equal to the base line O_1O_2. If we place the two transparent stereoscopic figures over each other so that the two reproductions of the same point situated at infinity overlap, then the reproductions of all the points situated at infinity coincide two by two. If, however, the exterior point C is not at infinity, the distance between the two reproductions is less than that of the eyes. The difference is known as *stereoscopic parallax*. The parallax of the point C is, as in the figure, $B_1D_1 + B_2D'_2 = E$. If we designate the distance between the two eyes by b, that of the object C from the plane of the two eyes by d and the distance of the plates from the eyes by a, we have

$$\frac{b - E}{d - a} = \frac{b}{d} = \frac{E}{a} \text{ or } E = \frac{ba}{d}$$

This shows that the parallax increases with the distance between the two eyes and that it is greater as the object is nearer the observer.

Fig. 190 gives the plan of Brewster's stereoscope. A and B are the

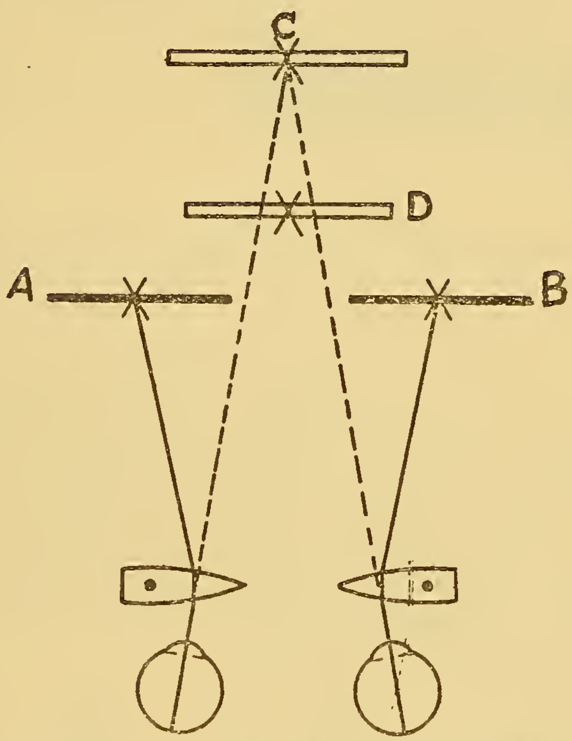

Fig. 190.—Plan of Brewster's Stereoscope.

pictures which are taken from slightly different points of view. Hence the distance between identical objects in the foreground of the two pictures is less than between identical objects in the background. To fuse the former more convergence is required than to fuse the latter. Foreground and background objects cannot be fused simultaneously. If this were possible the sensation of *relief* would disappear. When looking at the foreground of the landscape pictured there is a physiologic diplopia of the background and vice versa. By way of explanation at this point, it has been stated by Dove and other experimenters that objects appear solid when seen by the instantaneous illumination of an electric spark; if so, then the appearance of solidity must be due to the physiological diplopia of those portions which are not seen single. The quantitative perception of depth and the notions of relief require that the eyes should unite successively different parts of the object by consecutive increase and decrease of convergence. As a result of this, many persons discover with a stereoscope that the appearance of relief does not come until after a few movements, involving changes of convergence, have been made. If one takes a lead-pencil, held pointing forwards in the median plane and a little lower than the plane of the eyes, and fixes the far end of the pencil, the near end is seen double. By converging to a point at the center of the pencil both ends exhibit diplopia of half the magnitude which the far end at first showed. By converging still more so as to look at the near end of the pencil the far end exhibits homonymous diplopia. *Hence we conclude that if both foreground and background objects in stereoscopic images could be simultaneously fused there would be no sensation of relief.*

In the stereoscope proper, a decentering of the lenses outwards enables the eyes to converge somewhat, as for instance to the point *C*, Fig. 190. The convex lenses produce a certain magnification and their prismatic effect renders it unnecessary to make the visual lines parallel. As a general rule, however, the single united image of the stereograms is projected to a position *D* which is closer to the eyes and to the pictures *A* and *B* than is the point *C;* this occurs because of the knowledge that the observer possesses of the size of the stereoscope. The effect of the stereoscope is to give an idea of depth such as no other method of representation can. (See article by Isadore Franklin, M. D., on "Stereoscopic and Perspective Vision," *American Journal of Ophthalmology,* page 236, 1918.)

386. *Antagonism of the visual fields.* Under ordinary circumstances there are formed in one eye images of the same objects as in the other. As long as we place in the stereoscope images of real objects only we simply see the relief. When the images placed in the two fields are so different that they cannot be fused as, for example, if there is presented to one eye horizontal lines and to the other vertical lines, there will be observed the phenomenon known as antagonism of the visual fields. It is sometimes one field and sometimes the other which predominates; while one field is in the supremacy the other is suppressed and is not seen at all. The change from one field which has been dominant to the other takes place under external influences; a winking of the lids or a change in the direction of the gaze often brings about the change.

387. We shall now consider briefly two *haploscopic* fields containing diagrams which, upon being united, partially coincide with each

other; let us choose fields such as those diagrammed in Fig. 191 *A* and *B*. There is on the left in Fig. 191 a vertical black rectangular field perpendicular to the visual field and on the right a field similar in all respects to the first one except that it lies horizontally. In the center of these fields is a white cross. These crosses are made to coincide with each other: in the binocular field there will be formed large black

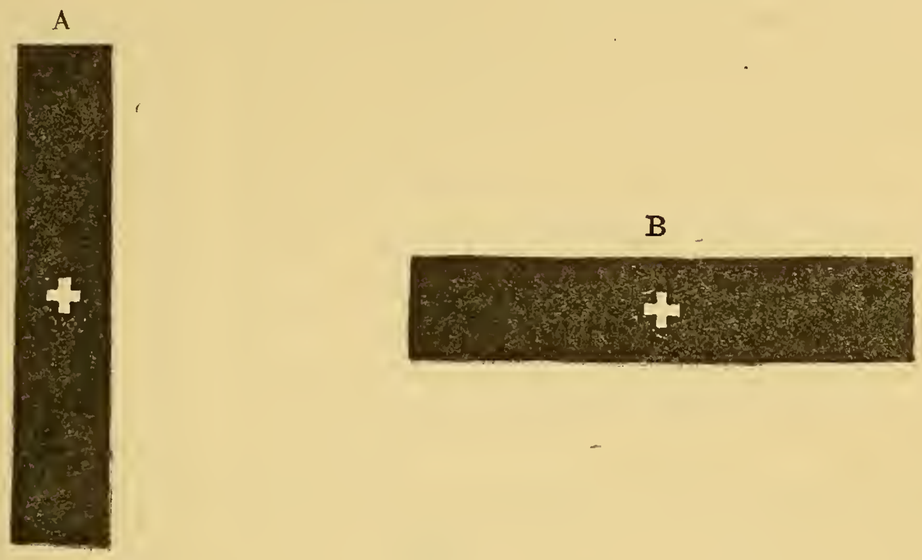

Fig. 191.—Diagrams for Obtaining Union of Two Haploscopic Fields.

crosses which are roughly represented in the three diagrams of Fig. 192, the fixation crosses being omitted from the diagrams. At those parts where the contours of one black band overlap those of the other there is a constantly wavering appearance as if there were a continuous rivalry between the two impressions which the object can convey. The impression is not that of an evenly black or evenly gray cross.

Fig. 192.—Illustrating the Effect of the Union of the Two Haploscopic Fields Shown in Figure 191.

In the portions where the black bands overlap each other one sees now the contour of one band and now that of the other and at times both at once as shown in the central diagram of Fig. 192. The condition in which the borders of the two objects make themselves particularly prominent is known as the *rivalry of contours*. Close to an edge there is always seen the brightness of the object whose border is at the time prevailing. The inner square of this figure is always black; just outside of each contour there is a white spot which gradually shades into black again. If the attention is fixed upon the right-hand (the horizontal) band, then the first of the diagrams in Fig. 192 is

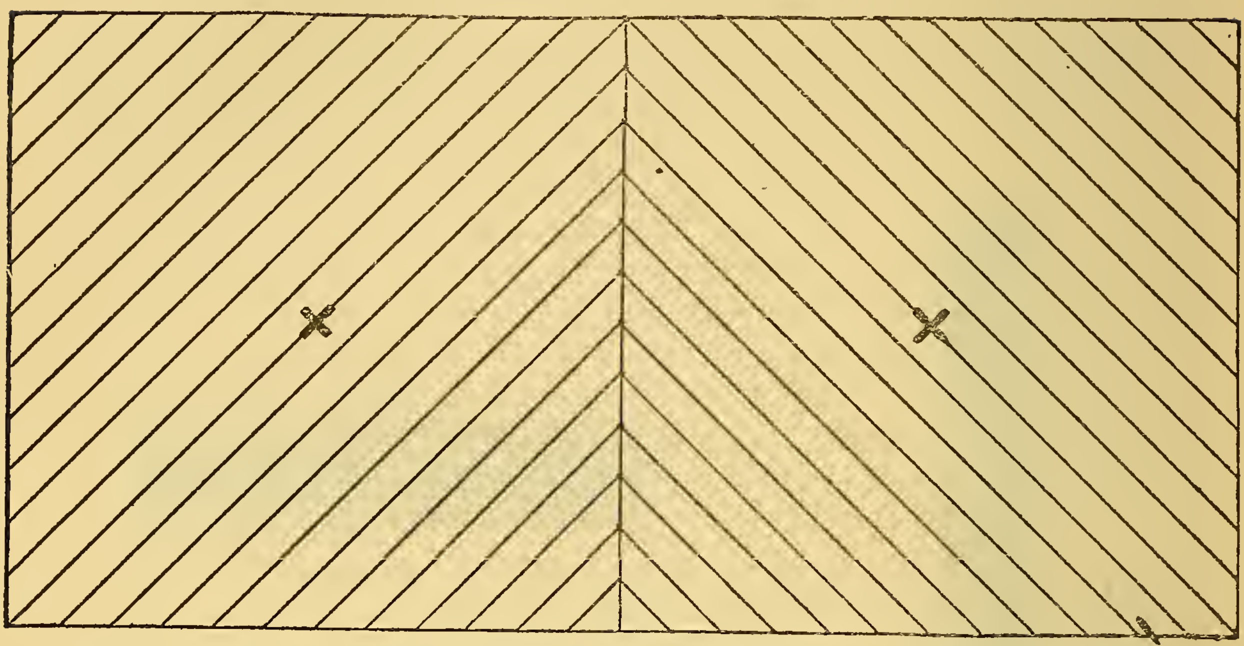

Fig. 193.—Another Type of Diagram for Obtaining Union of Two Haploscopic Fields.

obtained: if it is directed upon the left-hand band then the third of the diagrams of this figure is seen.

388. In the case of the diagrams represented in Fig. 193, each field

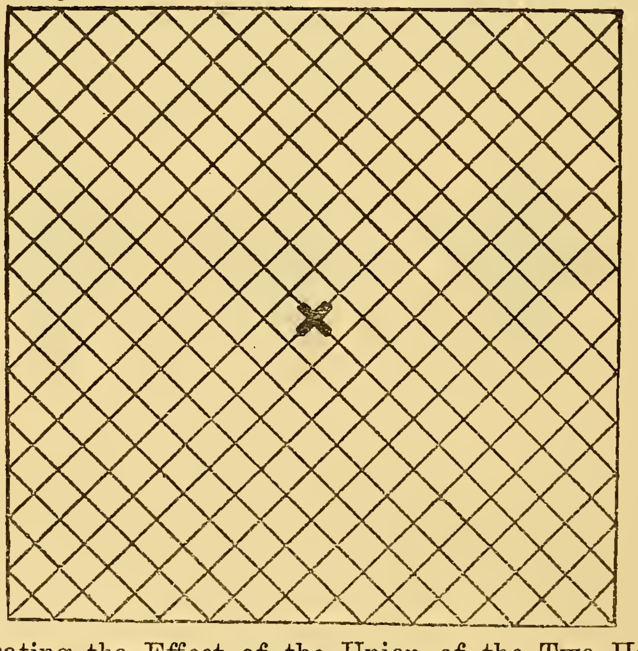

Fig. 194.—Illustrating the Effect of the Union of the Two Haploscopic Fields.

contains a series of black lines equally spaced and inclined at an angle of 45° with the visual plane. Upon uniting these haploscopically there is *not* seen a field of squares such as is shown in Fig. 194, but simply a wavering image whose separate parts correspond to the right-hand and to the left-hand member in turn. This occurs when no distinct direction is given to the attention. But if the attention is fixed upon either half of the diagram, then the portion which is under regard distinctly prevails for some moments over the other. Helmholtz explained this rivalry of visual fields as caused by a wandering of the attention. He finds in the influence of contours the effect of habit which leads one to examine particularly the contours of an object in order that the object may be recognized as quickly as possible. These phenomena indicate that we become conscious of the contents of each field of vision distinctly and separately from those of the other. They likewise prove to us that such fusion as does take place is not conditioned by the organic structure of the brain and also show that when a fusion of the sensations of the two fields does not occur in the interests of perception of the third dimension, each field of view preserves its own identity and independence.

389. *Theories of the production of relief.* The theories of corresponding or identical points and the theories as to the nature of the identity have been discussed in some detail in preceding sections. Briefly, then, in resumé it may be stated that we consider one point of the retina of one eye as corresponding to, or identical with, a point of the other eye when the images of the same exterior point, falling upon these two retinal points, are blended into a single point. It is evident that the two foveas are corresponding points since the object fixed is normally always seen single. Johannes Müller has given the following rule for finding other identical points: Suppose the retina divided into quadrants by a horizontal and a vertical meridian, both passing through the fovea. Two points, then, having the same longitude and latitude are identical. The question of why two points are corresponding while two others are not has been the subject of much discussion. Many advocates of the theory of identity suppose that there is an anatomical relation between two corresponding points, by assuming that the nerves conducting the impressions from two such points unite on their way to the chiasma into one which carries the impression to the brain. The theory of projections has been advocated by others: under this theory, for example, a point on the right retina five degrees to the left of the fovea localizes its impression in space as five degrees to the right of the point of fixation, while a point on the left retina situated five degrees to the left of the fovea localizes its impression in the same direction as in the case of the other eye; since the two impressions are projected in the same direction they are blended into one.

390. After the invention of the stereoscope and after various studies upon the production of relief had been carried out, the notion of corresponding points fell into disfavor since the experiments with the stereoscope seemed opposed to the doctrine of identical points. If we take, for instance, a stereoscopic presentation of two points, *A* and *B*, both of which are situated in the median plane but one more remote than the other, it will then be seen upon fixing the more remote object *A* that the images of *B* are not formed on identical points. For in one eye its image is to the right and in the other to the left of the fovea; however, it will be seen single and in relief and hence the single image of *B* will appear closer than *A*. On account of these contradictions Wheatstone favored the theory of projections. It remained to Javal to clear up, in large measure, this question and to give a satisfactory theory of relief.

391. In the *theory of the production of relief* as given by Javal we find two essential factors: (a) the neutralization or partial suppression of one of the images and (b) the influence of the ocular movements as emphasized by Brücke. We have previously referred to the rôle which the suppression of the images plays in binocular vision and comment has been made upon the suppression of one of the images which occurs when different images fall on corresponding points of the retinæ. Sometimes the image of one eye and sometimes that of the other is seen. While the image of one eye is seen, the corresponding part of the image of the other eye is lost. There is in normal cases ordinarily an alternation of suppression exhibiting itself as antagonism of the visual fields.

392. Tscherning, in writing on Javal's theory of the production of relief, says:—"Bruecke was the first who insisted on the great importance of the ocular movements for the perception of relief. Anyhow, it is certain that without them we could have only a very vague notion of it. Looking into a stereoscope, especially if the images are difficult to fuse, it is only after I have permitted my look to wander for some time on the figures, fusing sometimes the images of the distant objects, sometimes those of the near objects, that relief appears to me. As long as the sensation of relief is not produced I see double, sometimes the near objects, sometimes the distant ones; but at the moment when relief appears, I see all of them single. Certain authors claim that they have observed relief by illuminating the stereoscopic images with an electric spark, the duration of which light is so short that all ocular motion is necessarily excluded. This would certainly be impossible in my case for there always elapses a certain time before the real illusion, which does not prevent me from being able to form all at once a vague notion of relief.

"According to Javal, it is necessary, indeed, to distinguish between the *idea of relief,* which is produced by the fact that we see near objects in double cross images, and the *measurement of relief,* which depends on the *sensation* of the degree of innervation necessary to converge towards the near object. To account for the manner in which we come to obtain the sensation of relief, it is preferable to use images which are quite difficult to blend, the stereoscopic parallax of the objects represented being quite strong. We immediately fuse the images of distant objects, and all the others appear in double images. We then allow the look to stray on the figure, which forces convergence more or less, according as the object is represented more or less distant. After having continued thus for some time, relief manifests

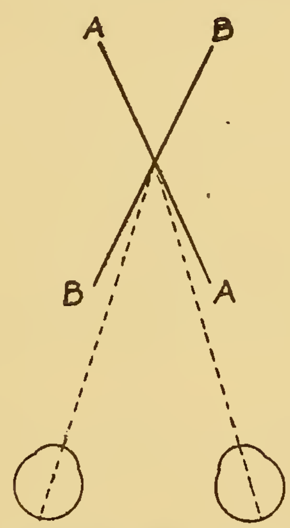

Fig. 195.—Illustrative of the Discussion as to the Part Played by a Directing Eye in the Production of Relief.

itself almost in the same way as we can, with closed eyes, obtain a very distinct idea of the form of an object by feeling it with the fingers. At the same time that relief appears, the double images disappear; the image of one or the other eye is suppressed. If one of the eyes plays the part of the *directing eye,* it is usually the images of the other eye which are suppressed, unless the image of the preponderating eye is much more peripheral than that of the other. In cases in which this preponderance is not developed, the double images seem to appear following the law of Javal: we suppress that one of the images which occupies the smallest retinal surface. We can account for the manner in which we suppress the images by looking at a rule which is held obliquely before the eyes, so that it presents a greater surface to one eye than to the other. Whether it occupies the position *AA,* Fig. 195, or the position *BB,* it seems to me, seen binocularly, to have the same appearance as when I close the left eye. Persons in whom the preponderance of one eye is not developed see the rule binocularly, as it is presented to the left eye, if it occupies the position *AA*. In the position *BB* they see it, on the contrary, as it presents itself to the right eye.

"The discussion of the two theories of binocular vision, that of *identity* and that of *projections,* has not yet closed. The explanation of Javal is applicable in reality as well to one as to the other. We *can* imagine the projection learned by experience; and even the fact of always projecting the images of the two foveas at the same place, the foundation stone of binocular vision, may be something learned. It is, perhaps, the superiority of the fovea, as to visual acuity, which causes us to always bring the images of the object which interests us to form themselves on both foveas, and we may thus have been led to always localize the impression of the two foveas at the same place. On the other hand, the advocates of the theory of identity take their stand on the anatomical observations of the semi-decussation in the chiasma, and especially on comparative anatomy, which shows that in many animals—fish, for example,—whose eyes are placed so as not to have a common visual field, the optic nerves cross completely. Clinical observations in hemianopsia, especially those of partial hemianopsia, are a further argument in favor of this theory. The study of the vision of strabismic patients, which is perhaps the best means of deciding the question finally, shows that, in consequence of a false position of the eyes, there may be developed a kind of correspondence between two retinal points which, under ordinary circumstances, are not corresponding: but this relation never assumes the character of true binocular vision with fusion, and it sometimes suffices, in a person who has squinted since childhood, to place the eyes in an approximately correct position, in order that, in the course of a fortnight, correct projection may gain the upper hand." (Tscherning, *Physiologic Optics,* translation by C. Weiland.)

393. *Tests for binocular vision.* Worth, in his book on *Squint,* describes a simple test which is an adaptation of Snellen's colored glasses and which he calls the "four-dot test." A piece of plain ground glass is covered on one side with black paper in which four round holes are cut. The lower hole is left clear. The upper one is covered with red glass and the two holes at the right and the left are covered with green glass. The whole arrangement is then mounted in a box which contains an electric lamp. The patient, seated at twenty feet, wears a trial frame in which a red glass is inserted before the right eye and a green glass before the left one. If he sees two dots, white and red,

he is using the right eye only; if he sees three dots, the white and two greens, he is using the left eye only; if he sees four dots he is using both eyes and enjoys binocular vision or simultaneous macular perception. If he sees five dots, one red, two greens and the white seen double, he has diplopia.

394. Another excellent device is the *amblyoscope* due to Worth. This instrument consists of two halves joined together by a hinge. Each half is a short brass tube joined to a longer tube at an angle of 120°. At the angle of junction there is a mirror: each half of the instrument has at its distant end an object carrier and at its near end a convex lens of five inches focal length. A brass arc connects the two parts of the instrument so that they can be brought together for a convergence of the visual axes up to 60° or separated for a divergence of 20°. The instrument should be adjusted for parallelism of the visual axes. In the object carriers are to be placed slides, one of which contains a slit and a small cross to its left while the other carrier has a similar slit and a dot to its right. If the patient sees the two slits as one and at the same time sees both the dot and the cross he has simultaneous macular perception. If in addition the tubes of the instrument can be converged or diverged and the slits still fused, the patient has true fusion with some degree of amplitude. The sense of perspective may likewise be tested by the use of proper slides.

395. The *Hering drop test* provides a method of examining the notion of relief rather than of measuring it; it is a test of the sense of perspective. The apparatus consists of a shallow box open at both ends or else two tubes suitably joined together. Two arms then project from this box, opened at both ends, or from the sides of the tubes respectively. The ends of these projecting pieces are joined by a thread on the middle of which is a bead. The patient places one end of the box or of the tubes close to his eyes and looks at the bead. The examiner then drops small objects, such as marbles, sometimes on one side of the thread and sometimes on the other. If stereoscopic vision exists, i. e., if the sense of perspective is developed, the patient will always give a correct answer as to whether the object is dropped on the near or far side of the bead. If not, approximately half the answers will be correct for they will be largely guesses. The principle upon which this test depends is simple: the box cuts off all view of surrounding objects and if the size of the falling objects is varied their apparent sizes can give no information as to their distance. The view of the falling object is too brief to permit of accommodation or convergence or any lateral movement of the patient's head. The patient

has, therefore, no aids to his judgment of distances and hence depends upon his sense of perspective entirely.

396. The well known *"bar-test"* of Javal, in which a pencil or similar obstacle is held midway between the patient's eyes and a page of print to see whether the observed party can read continuously without sudden movements of the head in order to avoid the obstacle, is a test of *rapid alternate binocular vision* and *not* a test of *stereoscopic* or *single binocular vision.* If both eyes have sufficient visual acuity but are not in co-ordination there will be either a bobbing of the head or a pause when the deviated eye has to take up fixation, followed, in turn, by another pause before the good eye can resume the reading.

397. *Unconscious conclusions and optic illusions.* We have seen that the psychical processes in vision come as a result of definite physical actions and we have pointed out that, by means of the stereoscope we are able to explain, by the comparisons which can be made between experiments made with such an instrument and our ordinary experiences, many of the phenomena of vision which would otherwise not be understood. The character of the perception obtained in vision is not apparently determined so much by the image formed upon the retina as by the movements of the eyes, although it does not follow that the line of sight must compass every detail of the figure seen. The apparent size, distance and even color of an object are influenced considerably by processes of mind and these mental processes differ in character under different circumstances. Conclusions drawn from experience, contrasts, comparisons of environment and a variety of psychical processes enter into the final conception which the mind entertains. The acts of motion of the eyes constitute the basis upon which the idea of space as recognized by the visual sense is founded: this does not mean, however, that motion is necessary in every instance of judgment of space but that the experience which has been gained by the acts of motion is essential. Hence, while it is probable that the visual notion of space is the result of movements, it is not possible to execute all the movements necessary to obtain an idea of the perspective of a complex body in the infinitely short time in which such ideas are formed; consciousness is not the result of a single process but actual movements and potential efforts, as Stevens states it, are combined in the formation of the idea of visual space. The experiment of Dove showed that an object illuminated by an electric spark may be located in space although the light of the spark has not persisted long enough to have enabled the eyes to adjust themselves for the object. Hence the knowledge of the distance and direction of the impression received by the retinæ permits of a judgment of the

extent and direction of the movement which would *have* to be executed in order to accomplish the adjustment.

398. A study of visual phenomena leads us to the conclusion that our knowledge of our surroundings is the result of mental deductions from physical signs and that the impressions received by the visual sense are simply so many symbols from which the mind draws certain conclusions. This method of drawing conclusions can be illustrated by the effects produced by certain combinations of diagrams the parts of which, when examined under ordinary circumstances, appear incapable of presenting the appearances which they assume under other circumstances.

Two ideas are in general entertained as to the manner in which objects are perceived by the eye. According to the popular conception, rays of light from the object pass into the eye and form an image of the object on the retina or cause an impression of an image which

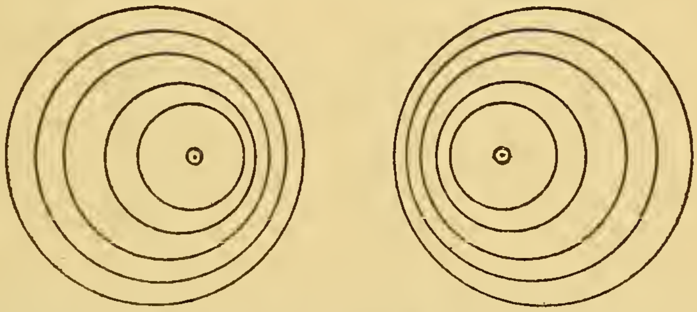

Fig. 196.—Diagram for Use with Stereoscope to Illustrate the Manner in Which Unconscious Conclusions are Reached.

is transmitted from the retina to the brain; this notion involves the transmission of the picture or impression as a whole. The second idea is that the impression caused by the light transmitted to the retina acts as a "finder" and that the retina, with its functions of recognizing light and color, acts as a guide to the muscles which move the eyes. These movements of these muscles in bringing the most sensitive portion of the retina into direct relation with various parts of the object constitutes an essential element in the ultimate mental impressions made.

399. Two series of rings, such as those shown in Fig. 196, when inserted in the stereoscope or when fused without its aid, give the impression of a solid truncated cone in space with the smaller base nearer the observer. The centers of all the circles, except those of the very small innermost ones, are displaced toward the point geometrically the center of the two sets of circles. Hence different amounts of muscular contraction are demanded for the convergence of the two

eyes observing the different pairs of rings. The cone does not appear complete, however, but has its apex cut off, while in the center of this section is a dot exactly in the plane of the section. The dot (or minute circle) does not complete the cone because each of the dots is exactly in the center of the smaller ring and the convergence of the eyes for the combined image of the dots is exactly the same as the average convergence or the convergence for the imaginary centers of the rings. This influence of muscular contraction and convergence makes itself felt if these two sets of circles are viewed in the plane of the page at any convenient distance, since the whole figure conveys the notion of objects in space tilted obliquely toward each other by virtue of the knowledge we possess that such objects in space would be seen under slightly different angles of convergence for different centers of the various contours upon the surfaces. We draw the same

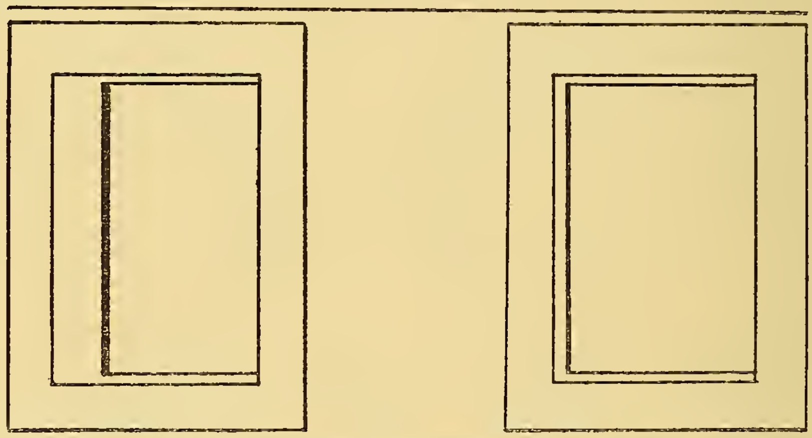

Fig. 197.—Illustrative of Mental Conclusions from Muscular Adjustment.

unconscious conclusions with respect to drawings and what they represent as we do with respect to stereoscopic diagrams under the stereoscope by reason of the knowledge we possess of the acts of motion of the eyes which are necessary to the formation of our judgment of the object and its position as actually existent in space.

400. Let us consider briefly the diagrams presented in Fig. 197. These figures are not symmetrical, but if they are observed through the stereoscope the framework is perfectly united at once and after a moment the heavy lines also but the latter swing backward like an open door. We can elucidate this phenomenon and give a process of reasoning by which these figures can induce this conception.

Suppose, in Fig. 198, the eyes *A* and *B* are directed toward *cd*, which represents the space of a door, and *ce*, which represents the door swung open. In looking at the door space the eye *A* moves through an angle *cAd* and the eye *B* makes an angle *cBd*. Directing in turn the eyes to the door proper it will be seen that the eye *A* will make a smaller excursion, (from *c* to *e′*) and the eye *B* will also make a movement less than *cd* but greater than *ce′* and measured by *cd′*. Hence, in looking at an open door as shown in the figure the eyes make equal movements for the open space but shorter and unequal movements for the opened door. These are the elements in the stereoscopic picture.

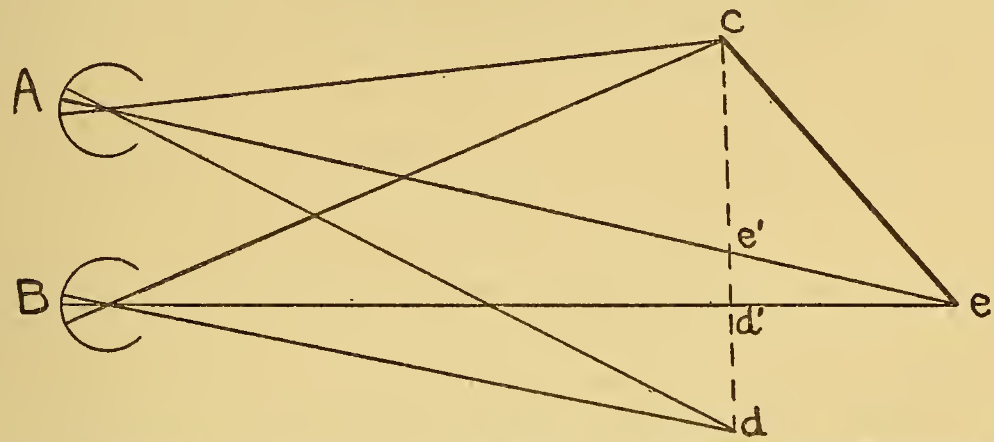

Fig. 198.—Diagram Explanatory of the Phenomena Discussed under Mental Conclusions from Muscular Adjustment.

These elements in turn conform to the everyday experience in ocular adjustments and hence the mind arrives at the conclusion that the object seen does not differ from what would actually induce such ocular movements. If we look at the two pictures presented in Fig. 197 we come unconsciously to the conclusion that the left hand diagram represents a door ajar while the right hand figure represents a door very

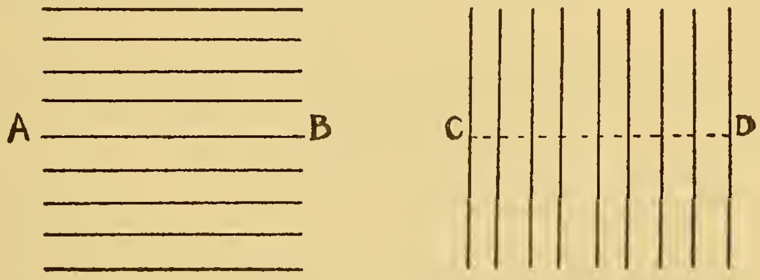

Fig. 199.—Illusion of Height and Breadth.

nearly completely closing up the door space. We can, then, from very simple line diagrams and crude drawings very readily gain a conception of space representations through our knowledge of the ocular movements and innervations which would be necessary if the objects outlined in the drawings were actually seen in space.

401. In Fig. 199 we have drawn two equal squares, one marked with horizontal and the other with vertical lines. The square with the horizontal lines appears appreciably higher than its mate, while the

square with the vertical lines is appreciably broader than the other. In neither case do the squares appear equal on all sides. The single effort of sweeping directly from one extreme of an object to the other without a halt or obstacle does not produce an effect in the course of the muscular impulse as when the eye passes from one side of the object to the other by a series of smaller exertions. That is, repeated small movements are of more consequence than a single muscular sweep of extent equal to the sum of the smaller movements. The passage

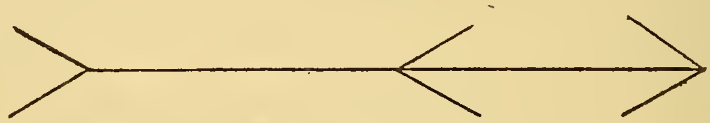

Fig. 200.—The Mueller-Lyer Illusion.

from A to B is uninterrupted, while the passage from C to D (Fig. 199), although equal to that from A to B, is interrupted and appears greater. This same phenomenon and explanation occurs in the Müller-Lyer illusion in Fig. 200. The two parts of the horizontal line are of equal lengths yet the line from which the prongs diverge in a direction partly continuous with the main line appears considerably longer than the part enclosed between those short containing lines which diverge backwards upon each other. The impression of the extent of the object is modified in the contrast in the muscular sense between

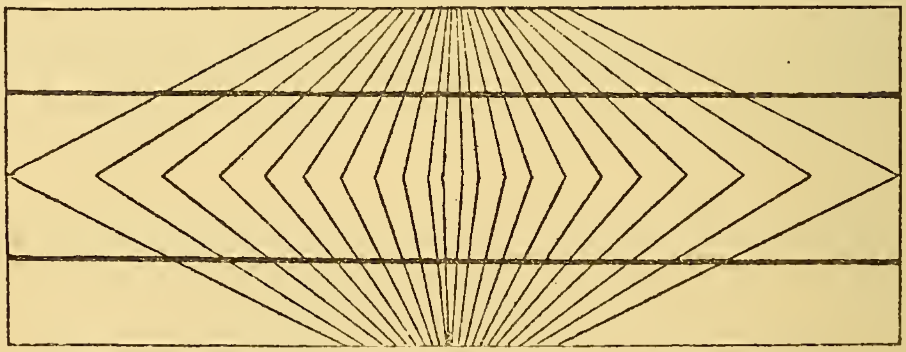

Fig. 201.—Convexity of Straight Lines. (After Hering.)

an action unimpeded and extended beyond the point of measurement and an action suddenly stopped and turned back upon itself. In looking at Fig. 200, then, we see that in the first place the eye follows the course of the main line and encounters the diverging lines with no sudden stoppage in its course and passes by a slight modification along one of the divergent lines. In the second case, however, in the ocular movements the attention is directed to the retrograde line and the movement is stopped before the extremity of the line is reached.

402. In the two accompanying diagrams, Figs. 201 and 202, a straight line does not appear as a straight line and parallel lines do

not seem to be parallel. In the first of the two diagrams the lines appear to bow away from the center point and in the second case they are apparently bowed toward the center of the figure. The explanation of these phenomena lies in the explanation already offered with respect to preceding optical illusions. If a movement be continued beyond the point of termination the distance appears greater than if the ocular movement is suddenly arrested and turned back. "In the case of Fig. 201 the angles on the outer sides of the lines permit the movement to slide without sudden arrest with the result that that side of the long line appears elongated and the line also approaches the branching lines, not because the picture on the retina brings the long line in closer relation to the branches, but because in the movements required the two lines forming the acute angles are brought in relation

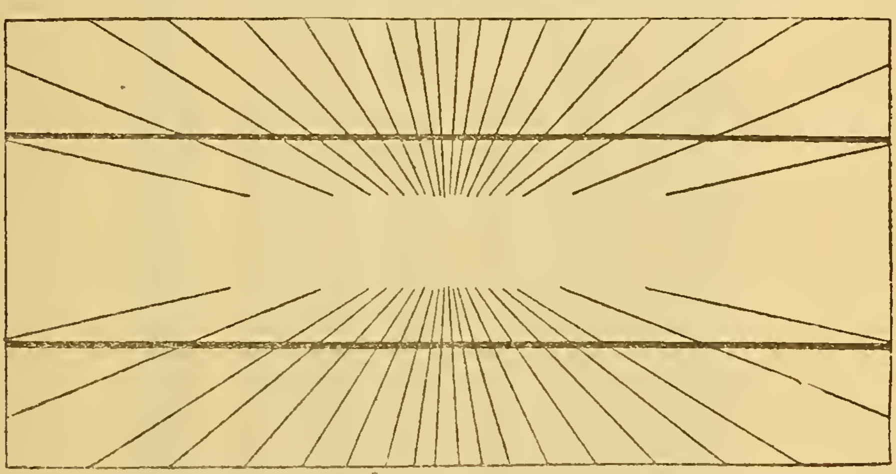

Fig. 202.—Concavity of Straight Lines. (After Wundt.)
From Stevens' Motor Apparatus of the Eyes. Copyright, F. A. Davis Company, Philadelphia.

to each other and, as Helmholtz remarks, there is a mental contrast of the angles between the direct and the oblique lines. The two long lines appear to approach toward the center and to diverge toward the extremities. The other figure shows the angles reversed, with the effect of changing the apparent curves of the really straight and parallel lines." (Stevens.) In explaining the geometric illusions of Hering (Fig. 201) and Wundt (Fig. 202), Hering argues that the separation of two points removed from each other by a small distance is estimated at a greater relative value than that of points further removed provided there is no dividing point between these latter. In a similar manner a small arc has a greater relative value than a greater one proportionately in perception; hence acute angles are overestimated and obtuse angles are underestimated. The movements of the eye for an acute angle are incomplete and suddenly arrested but for an obtuse angle they are nearer in unison with the full extent of the lines bounding the angles. That the eye is or may be arrested at a point along its course in the estimation of an acute angle and therefor overestimates its value is made plausible by investigations by Judd on the Müller-Lyer illusion. In these researches, photographs showed that the movement of the eye in the direction of the acute angle is arrested before the point of the angle is reached.

403. In the figure of Zöllner, (Fig. 203) the long straight lines which are parallel seem to converge and diverge upwards following the direction of the small oblique lines. The explanation lies again in the judgment giving too great a size to acute angles.

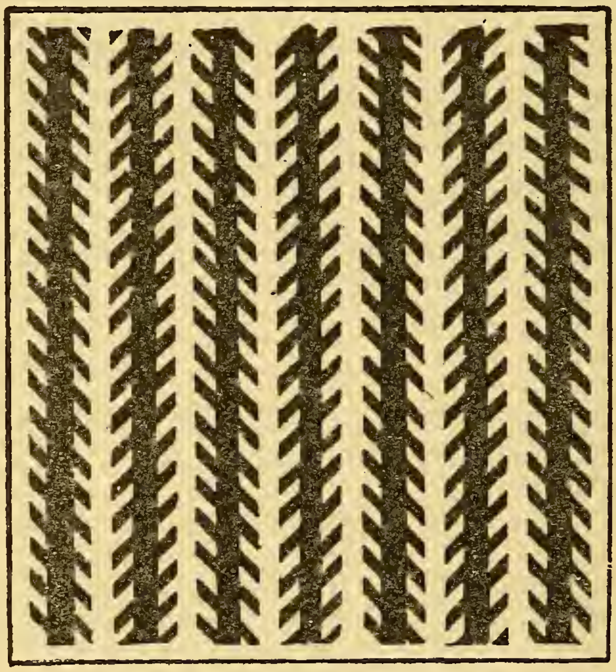

Fig. 203.—Zöllner's Figure.
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404. In many of these cases of optic or geometric illusions the phenomena of *irradiation* may furnish sufficiently satisfactory explanation. Helmholtz has pointed out how these phenomena, by the laws of contrast, might induce such effects. Quite recently Alfred Lehmann has investigated the influence of irradiation upon geometric illusions and asserts that the illusion represented in *B,* Fig. 204, must be entirely due to irradiation. The two lines in *B* appear to converge toward each other at the top of the figure, the fine connecting lines being transformed into a series of zig-zag lines as represented in *E.* Lehmann believes that the highly and feebly illuminated spaces, which are in close juxtaposition, permit irradiation from the clear to the obscure squares. In the case of the series of squares represented at *C* we have a connecting line in each set which is so broad that the effect of irradia-

tion does not extend to the black squares and hence there is no illusion. And again at *A*, where there is no marked contrast in the illumination, there is no illusion.

There are also illusions as to movements of exterior objects which often occur because of a false judgment of the movements which the observer is himself making. A familiar example is that of the apparent movements of objects as seen from the window of a train in motion: the observer does not take into account his own change of position and hence attributes the motion to the exterior objects.

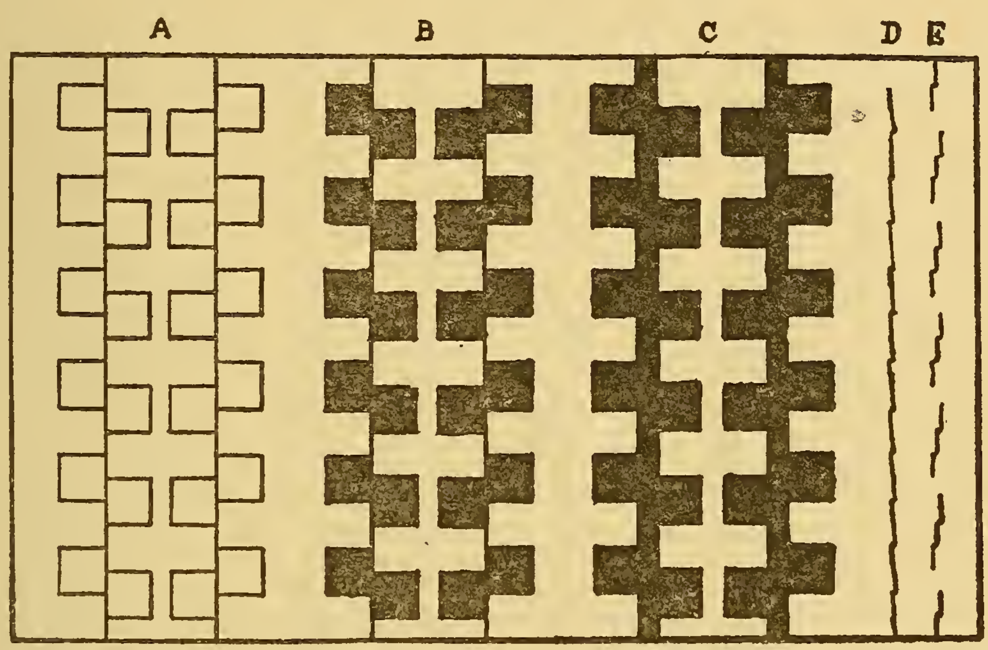

Fig. 204.—Irradiation as a Cause of Geometrical Illusions. (After Lehmann.) From Stevens' Motor Apparatus of the Eyes. Copyright, F. A. Davis Company, Philadelphia.

PART FIVE

BINOCULAR VISION AND DISORDERS OF THE MOTILITY

XXIII. DIPLOPIA

405. Binocular vision has been briefly defined as single vision with two eyes. The placing of the two eyes in such a position that the image of the object under regard shall fall upon the fovea of each eye is as voluntary as the adjustment of the accommodation. Normally, then, when both eyes "fix" the object, each eye has an image of the object on its fovea and these foveal images or impressions are transmitted to the brain and fused as one image in the visual centers: vision is single because the images impress corresponding portions of the two retinæ and are received by the brain as one. This condition is spoken of as equipoise or *orthophoria* and the eyes are said to be "in balance."

When, however, the image is focused upon portions of the retinæ that do not correspond as, for example, when one fovea only fixes an object and its mate receives the image of the same object on a part of its retina remote from the fovea, the brain is unable to fuse them and therefore is cognizant of two separate impressions; this condition is spoken of as *diplopia*. When the retinal impression of the non-fixing eye is close to the macula the patient is frequently conscious of blurred vision and of muscular efforts to make this eye also fix. Under such conditions the patient has headaches and other symptoms which are the usual accompaniment of asthenopia. In many cases where the two eyes fail to fix the object simultaneously diplopia does not occur since nature, abhorring doubleness of vision, allows one image to be ignored or suppressed. Diplopia may be removed by placing a prism before one eye (or divided between the two eyes) and thus causing the rays that pass through it to be deflected to such an extent as to fall upon the fovea of this eye which, under these conditions, corresponds to the other eye and two objects are no longer seen. And again, artificial diplopia may be produced by placing a prism before the eye in such a position and of such a degree as to prevent the transmitted rays from falling upon corresponding points.

406. *Homonymous and heteronymous diplopia.* There are two forms or varieties of diplopia known as *homonymous* and *heteronymous* (or crossed). In order to properly locate the positions of the images in space as seen under various conditions of diplopia one needs bear in mind the laws of projection and the laws of direction. Briefly, the underlying physical and physiological facts are:—(a) The image of an object formed upon the retina above the fovea is projected into space downward; objects situated below the horizontal visual line are recognized by that portion of the retina above the fovea: (b) the image of an object formed upon the retina below the fovea is projected upward: (c) the image of an object placed upon the retina to the nasal side of the fovea is projected toward the temporal side and (d) images formed on the temporal side of the fovea are projected toward the nasal side.

407. In homonymous diplopia the right image belongs to the right eye and the left image to the left eye. It is caused by an inward deviation of the eye as in esotropia. Fig. 205 shows the right eye (O. D.) fixing upon the object *F*, while the left eye (O. S.) is turned inward so that rays from *F* fall upon its retina to the nasal side of the fovea and are projected outward to the temporal side: the result is that the left eye sees a false object to the left of the real object. The right eye, as shown in the figure, fixes the point *F* which forms

its image at the center of the yellow spot, M_1, while the left eye, due to its excess of convergence, fixes the point f_1 which forms its image at the center of the fovea, whereas the point F forms its image at a point on the retina which is situated to the nasal side of the fovea at E. Let us give the left eye its relatively correct position by rotating it about A so that its visual axis, M_2Af_1, may take the direction M_2AF. In this new position, in order that an image may be produced at E, the point f_2 must be situated at a distance to the left of F equal to

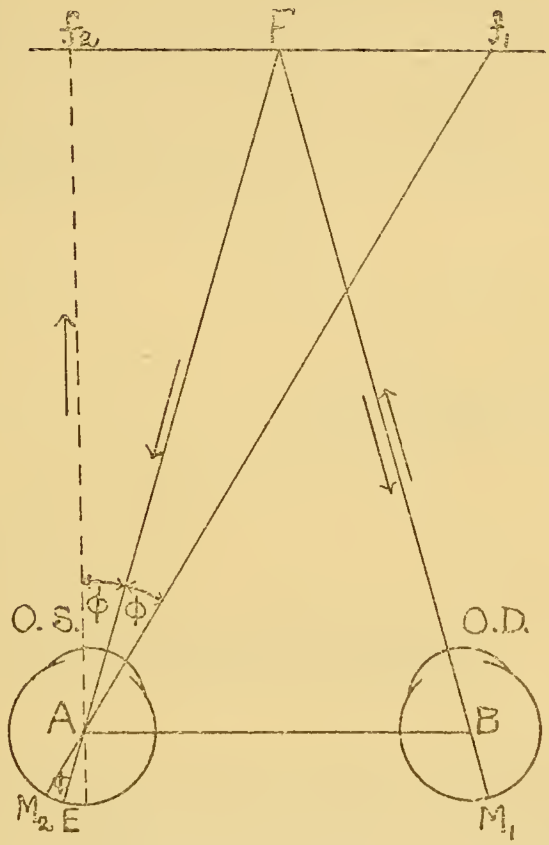

Fig. 205.—Illustrating the Phenomena of Diplopia.

the distance of f_1 to the right. This demonstration shows, in reality, that the left eye, deviated inwards by an amount represented by the angle Φ, projects its image of the point F outwards to f_2 such that the direction of projection forms with the line FE an angle equal to the angle of deviation Φ. We can so arrange matters by the use of prisms as to give the ray FE a direction such that it will coincide with the fovea M_2. A prism placed base *out* before the left eye will deviate the light toward its base and therefore so displace the ray within the dioptric apparatus of the eye as to place it upon the fovea and the image will then be projected to F and the diplopia disappear. In other words, the prism must give to the incident ray a deviation equal to the angle Φ, which is the angle of deviation of the left eye.

408. In heteronymous diplopia the right eye, for example, may fix the object and the left eye turn outward or exhibit a condition of exotropia. In this case the rays from the object fall upon the retina of the left eye to the temporal side of the fovea and are projected outward in space to the nasal side with the result that the left eye sees a false image to the right of the real object. This condition of affairs is often referred to as "crossed" diplopia.

409. At the risk of too much repetition at this point (for the writer has found that students and practitioners are too often at a loss as to the correct interpretation of these physiologic ocular phenomena) let us apply these principles to the location of images in cases of the deviation of the images in a vertical direction. Let it be first sup-

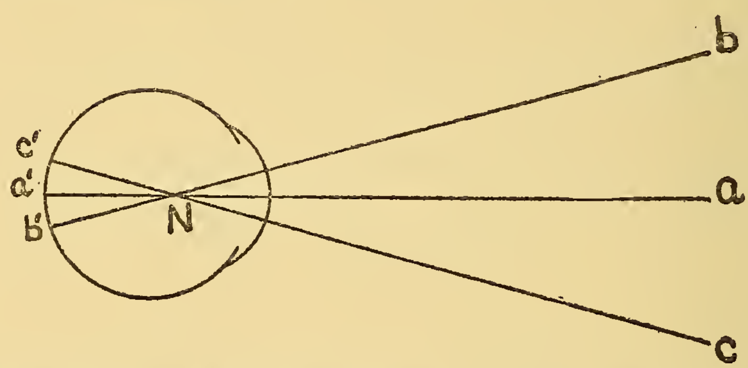

Fig. 206.—Optical Principles Involved in the Various Positions of the Eye and Images of a Given Object in Space.

posed that the images of objects fall upon the retina of a single eye. Let the objects, *b* and *c* in Fig. 206, be at the same height, one above and one below the line of primary fixation. Let *a* be the point of fixation. The image of *b*, the upper object, is then received at *b′* which is below the macula and the image of *c*, the lower object, is perceived by a point on the retina, *c′*, which is above the fovea. Hence, if an image is impressed at a point in the retina which is above the point at which another image is received, that which forms its impression above is mentally interpreted as being below and the object is, in the judgment of the observer, located at a point below the other object. According to the law of corresponding points the point *c′* in one retina will be removed in the vertical meridian of the retina as far upward as its corresponding point in the other retina. If the distance *bc* is represented by the distance *b′c′* on the retina, then if two images are received upon the retinæ, the distances of *b′* and *c′* from the macula on the lower quadrant of one eye would equal the distance from b' and c' from the macula in the lower quadrant of the other eye. Applying these principles, let O in Fig. 207, be an object seen by both eyes. If one eye, say the right, fixes the object the image will be perceived at the macula m_1. Let the left eye deviate downward. Then the image of O will not be produced at m_2 but at n, since the macula m_2 will have been rotated upward and the incident ray from O passing through the nodal point will meet the retina at the point n below the fovea. Accordingly, the image of O perceived at the retina at n will be mentally located and judged in space as at O', not below the object O but at a distance above O equal to the angle of deviation Φ.

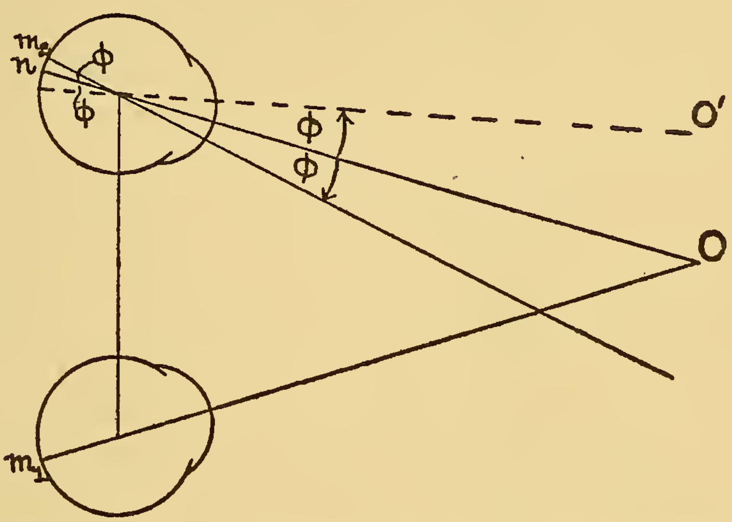

Fig. 207.—Illustrating the Optical and Visual Phenomena Involved in Homonymous Diplopia.

410. Binocular vision depends upon the blending or fusion of the images formed in each eye and the natural desire to preserve it is the origin of the impulse which directs the movements of the eyes and keeps them in association in the same direction. The subject of *strabismus,* therefore, not being able to readjust the nerve associations *assigns such position to the object of vision as an object stimulating the same part of the retina would have if perceived through an eye in its normal position. The displacement of the false image is always in the direction opposite to that of the false position of the eye.* This is certainly a correct rule fitting the majority of cases and we have used the word "always" in the above sentence with the one reservation that there are a very few anomalous conditions in which false interpretations arise from its application. It occasionally happens that a person having a divergent strabismus will, when caused to recognize his diplopia, insist that he sees the image homonymously and

again, when a convergent strabismus is present, he may describe the positions of the images as crossed as though the phenomena arose from a divergent condition. These phenomena are perplexing especially when there is a small convergence or divergence of the axes of the two eyes. Stevens claims that the anomaly presents itself only in cases in which there is not only the lateral but also a distinct vertical deviation as well as an extreme declination and as a general rule, to which there are a few exceptions, is observed after an operation for the correction of the lateral deviation in which the difference of height of the images or the extreme declination has been ignored.

411. Since a prism interposed between an eye and the point of fixation changes the path of the light which enters the eye from this point it is apparent that if, during binocular fixation, we place a prism before one eye, the light which then enters does not fall upon

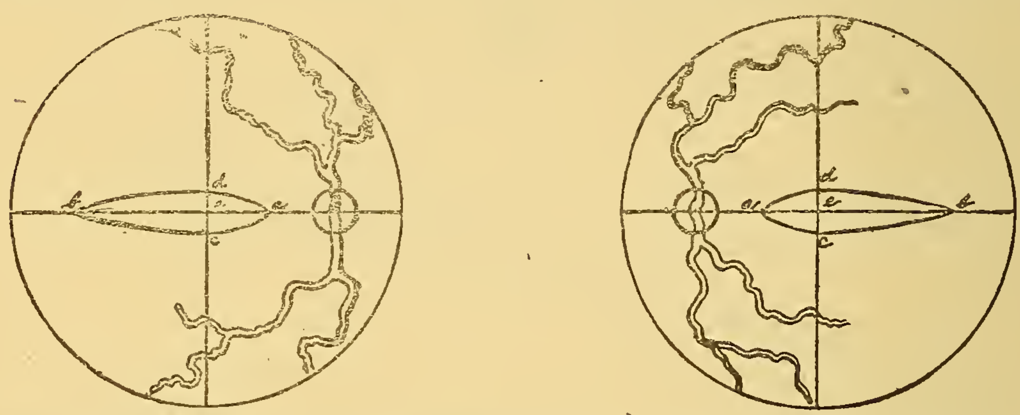

Fig. 208.—Field of Binocular Fusion in Each Retina. (After Savage.)

the macula of this eye and in consequence of this diplopia would result. But our natural and normal desire to avoid diplopia is so great that, in so far as the eye (or perhaps more accurately the eyes, since the innervation would be delivered to the muscles of both eyes engaged in the conjugate act of preserving single vision) is able to do so, it quickly readjusts its position and assumes that direction which causes the image to fall upon the macula. The strongest prism, with fixation at twenty or more feet, which can thus be overcome in the accomplishment of binocular vision measures the *breadth of fusion*. This *fusion power* varies greatly in different directions and, the same direction being considered, varies greatly with the individual and may be "sthenic" (or in strength) or "asthenic." The field of binocular fusion can be determined only by the use of prisms; it is necessary to determine the powers only in the four cardinal directions. The accompanying cut, Fig. 208, due to Savage, shows approximately the shape and size of this field of fusion. When an image is displaced by a prism to any point within this field, while the image in the other eye is on the macula, an effort at fusion will be made and if the muscle which is called into play, or the innervation, is strong enough, fusion will occur at once. When the image is thrown, however, by a sufficiently strong prism outside of the field of fusion, the "guiding sensation," as it has been so fitting called by Savage, which seems to reside in this area only, will not call on any muscle to move the eye for purposes of fusion. The power of the recti muscles for fusing images, taken as an average and expressed in prism degrees, is, for the internus (adduction, tested by prisms base *out*) 25°; for the externus (abduction, tested by prisms placed base *in*) about 6° to 8°; for the superior rectus (superduction, tested by prisms placed base *down*) 3°, and for the inferior rectus (subduction, tested by prisms base *up*) 2° to 3°. *Muscles having normal fusion powers should also possess normal verting powers:* when the one is abnormal the other is likely to be so also. The extent of these versions or rotations differ slightly amongst various authors: Stevens places the standard as follows: *out*, 48° to 53°; *in*, 48° to 53°; *down*, 50° and *up* 33°.

412. *Ocular palsies.* While a detailed description of the various ocular palsies and their diagnosis is not a subject to be dealt with in an article of this kind, a brief description is in order, however, to differentially indicate the variations between ordinary *squint* and *palsy* of an ocular muscle. The term *paralysis* is used to indicate a loss of power in one or more of the ocular muscles and should be regarded, not as a disease in and of itself, but as a *symptom* indicating lesions which may lie in the orbit or in some portions of the optic tract or brain. The symptoms produced by ocular paralysis may be listed as:

(1) *Diplopia.* This is usually the first symptom of which the patient is conscious. Two objects are seen because the images cannot be formed on corresponding points of the retina of each eye.

(2) *Impaired vision.* If the paralysis is slight there may be no real diplopia but simply an overlapping of the two images causing indistinct vision.

(3) *Vertigo.* This is largely due to faulty projection.

(4) *Impaired movements.* There is a noticeable inability of the eye to move in the direction of the paralyzed muscle, as shown when a test object is moved towards this side. The normal eye easily follows the movement, while the affected eye cannot turn beyond the median plane. The secondary deviation of the sound eye exceeds the primary deviation of the affected eye. When the good eye is covered by the

hand or with a screen it becomes the squinting eye and the uncovered eye is compelled to take up the work of fixation; the strabismus is thus transferred from the affected eye to the sound eye.

(5) *Vicarious inclination of the head.* In order to supply the function of the inactive ocular muscle the head may be turned in such a way as to cause the true and false images to be fused into one. When

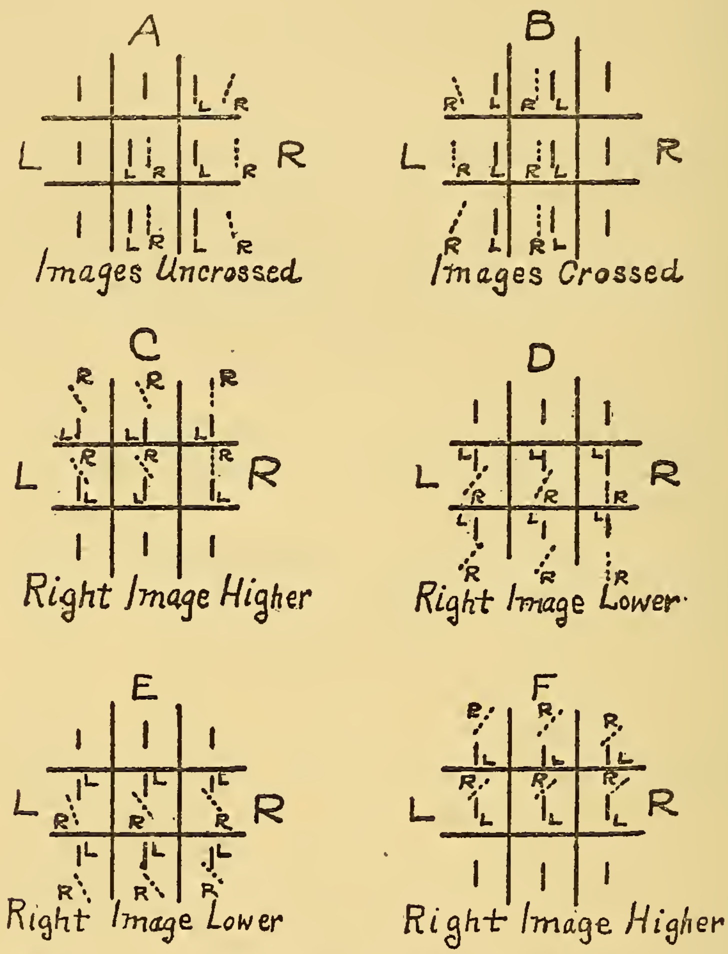

Fig. 209.—Positions of the Images in Palsy of the Individual Muscles of the Right Eye. A, Right external rectus; B, right internal rectus; C, right superior rectus; D, right inferior rectus; E, right superior oblique; F, right inferior oblique.

the eyes turn in the direction of the paralyzed muscle, diplopia is manifest and to avoid this the individual turns his head in the same direction. For example, in paralysis of the right externus or left internus, the face turns to the right.

413. Two points in general stand out pre-eminently in ocular palsies:—*(1) the displacement of the false image always corresponds to the direction of the normal action of the paralyzed muscle,* for a paralysis is followed by a turning of the eyeball opposite to that of the normal muscular action and we know that a false image is dis-

placed in a direction opposite to the turning of the eye; and, *(2) the patient's head turns toward the field of action of, or in the direction of, the paralyzed muscle.*

414. In making the diagnosis as to the seat of the paralysis, the main points to be carefully noted are: (1) whether the false image is seen with the right or with the left eye; (2) whether the diplopia is homonymous or heteronymous; (3) whether the two images are

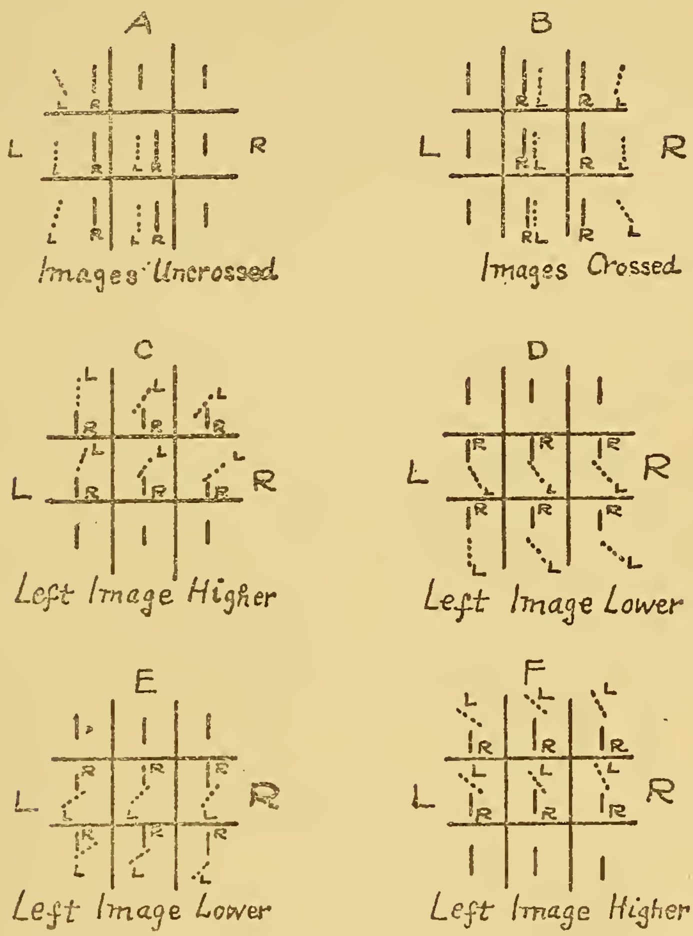

Fig. 210.—Position of the Images in Palsy of the Individual Muscles of the Left Eye. A, Left external rectus; B, left internal rectus; C, left superior rectus; D, left inferior rectus; E, right superior oblique; F, right inferior oblique.

parallel or obliquely inclined; (4) whether the two images are on a level or one higher than the other; in the latter case, whether the true or false image is the higher; and (5) the effect of varying the direction of gaze upon the lateral and vertical displacements and upon the obliquity. By careful study of these features in connection with the study of the physiological action of the ocular muscles and of the result of their paralysis a proper diagnosis may be reached. This procedure seems simple, yet the diagnosis of oculo-muscular paralysis is in many cases a very difficult task, for the reason that the test upon which most dependence must be placed is that of diplopia and even highly intelligent persons err in describing these unfamiliar visual sensations.

415. The diagrams presented in Figs. 209 and 210, taken from the work of Hansell and Reber on *"Ocular Muscles,"* are plots indicating the nature and the positions of the images seen under diplopia in various portions of the binocular visual field.

416. *Concomitant strabismus.* There are two forms of strabismus: *paralytic* strabismus which, as we have just seen, is due to a paralysis of one or more muscles, and *concomitant,* which, in a great majority of cases, is due to a defect of innervation. The differential diagnostic signs between these two forms of strabismus are well known: in cases of paralytic strabismus the excursion of the eye is less on the side of the paralyzed muscle and the secondary deviation is greater than the primary. In concomitant strabismus the deviation is the same for all directions which the line of regard may assume, although generally the convergence is more pronounced for the downward than the upward look. The patient does not complain of diplopia, but this condition may be artificially produced by any of the well-known dissociation tests dependent upon the displacement of one image, distortion of one image and those which neither displace nor distort, such as the cover test or the parallax test. The distance between the two images when diplopia is thus produced is the same everywhere; the squinting eye accompanies the straight eye in all its movements. Since both eyes cannot be turned toward the same point it is evident that only one eye can fix the object while the other must deviate. Naturally the patient gives preference to the eye with better vision, which then becomes the fixing eye, and the eye with the poorer vision becomes the deviating eye. But this does not mean that the strabismus is associated with one eye any more than with the other; each eye alone is normal as regards its muscular movements as a general rule, but when the two eyes attempt to act together strabismus occurs because there is not a satisfactory co-ordination.

There are three varieties of strabismus, namely: (1) *esotropia* (internal squint, concomitant convergent squint), (2) *exotropia* (external squint, concomitant divergent squint) and (3) *hypertropia* (vertical squint, up or down, and of the right or left eye). To these three commonly named we should add a fourth or (4) *cyclotropia.*

417. When from any cause the eyes tend to assume a degree of convergence greater than that required by the position of the point of fixation, an abnormal tax is placed upon the nervous system in order to maintain the visual lines in the proper directions for binocular

vision. If the tendency to excessive convergence is considerable, the great and continuous drain results in nervous exhaustion and beyond a certain limit of time the effort for binocular vision cannot be maintained. The *latent condition of esophoria* then gives place to a manifest error, *esotropia,* and the true vision is performed by one eye while its mate deviates inward. Latent excess of convergence may then become converted into *manifest* strabismus at times as when, for instance, the eyes are tired from excessive use and particularly in near vision, when spasm of convergence is frequently produced through the association with accommodation. In such cases the strabismus is *intermittent.* But when the effort necessary to give binocular vision is great, or when the fusion impulse is weak it usually happens that the person will abandon the effort to maintain proper convergence and will resort to monocular seeing. The strabismus then becomes constant or *permanent.* This condition usually arises in childhood, for convergent strabismus nearly always develops in childhood. If the vision is equally good in both eyes, either eye may be used for fixation while the other squints. This is known as *alternating* strabismus. But, in general, in strabismus one eye will have the preference over the other and fixation will be performed with the better eye. Although the strabismus is thus apparently confined to one eye it is not a uniocular affection. The excessive convergence is effected by innervation of both internal recti; but in order that the visual line of the fixing eye may be properly directed, adduction of this eye is prevented by suitable innervation of the externus, just as when convergence is maintained together with a lateral deviation of the two eyes (Landolt). In recently formed concomitant strabismus there is no abnormal limitation of the field of fixation in any direction, but when, from continued overaction, the interni have become (in effect) shortened while the externi have become weakened the power of abversion falls appreciably below that of the normal eye.

418. *Deficiency of convergence* is either *latent* or *manifest,* constituting respectively *exophoria* or *divergent strabismus.* As in excess of convergence, so in its deficiency, we find that binocular vision may give place to strabismus with monocular vision only at certain times (intermittent divergent strabismus). If vision is equally good in the two eyes, either eye may be used for fixation while the other squints (alternate divergent strabismus). But if, as is usually the case, the vision of one eye is inferior to that of the other the strabismus will be confined permanently to the inferior eye.

419. *Hypertropia.* In convergent strabismus of high degree there is ordinarily found in addition an upward tendency or deviation of the

squinting eye, while divergent strabismus is usually complicated with downward deviation. Aside from such cases non-paralytic imbalance is not common. Hypertropia, it is claimed by some, while generally considered with comitant errors, is in reality almost always of paralytic origin. Concomitant hyperphoria, or tendency to deviation upwards or downwards with respect to the primary isogonal circle, is quite common and is usually of a low amount, 1△ to 2△. Stevens believes that this form of strabismus (hypertropia), which has been regarded as rare, is in fact the most common of all forms. One may be subject to a degree of vertical squint which is sufficient to prevent the possibility of binocular vision and to result in a high degree of amblyopia while the defect does not attract attention. The defect shows mostly when the patient is looking at a considerable distance, but in many cases it manifests itself by an outward or inward squint when the hypertropic person looks at a near point. Stevens says: "In all cases in which a converging squint, not observable when the patient looks at a distance, occurs when he looks at a near object, either an actual vertical squint or at least a high degree of hyperphoria exists." This author also gives statistics to show that hypertropia occurs as the principal element of squint in about 25 per cent. of all the cases of strabismus (200 in number) which he records. It is more commonly accepted, however, that a true vertical deviation of one visual axis above the other is not often encountered but is usually associated with either esotropia or exotropia. The ciliary overaction in hyperopia often gives rise to a temporary esophoria or hyperphoria which may, under certain conditions, be carried over from heterophoric to heterotropic phases. Anatropia and catatropia are also explicable on this basis, for anatropia may be considered an overaction phenomenon and that in all probability it is closely related to esotropia. The excessive impulse which the ciliary muscle receives in hyperopia in the interests of accommodation also affects the remaining muscles supplied by the third nerve in consequence of which the eyes tend in the directions resulting from a combined action of those muscles, which is up and in. On the other hand, the absence of impulse to the ciliary muscle of myopes carries with it a diminished impulse to the other muscles innervated through the third nerve and their resultant inaction permits the eyes to be deviated in the direction resulting from the combined action of the remaining ocular muscles, the external rectus and superior oblique, which direction is down and out.

420. *Cyclotropia.* This is an actual loss of parallelism between the vertical axes of the eyes and the fixed median plane of the head. This condition probably never exists alone and is usually found in connec-

tion with other forms of heterotropia. There are two classes of cyclotropia known as *similar* and *dissimilar*. In the former the cyclotropia is plus or minus in both eyes, while in the latter class the error is plus in one eye and minus in the other. Savage has termed the first class "non-parallel cyclotropia," i. e., the vertical axes are either divergent or convergent; the latter class he has called "parallel cyclotropia" since the vertical axes are inclined one toward and one away from the median plane. Cyclotropia, like other forms of heterotropia, is alternating; i. e., the fixing eye will have its vertical axis parallel with the median plane of the head, while the vertical axis of the other eye will be torted in or out. It is also comitant, the angle being the same in all positions. Cyclotropia caused by paralysis or paresis and operation is non-comitant and is attended by very annoying symptoms.

421. *Causes of squint.* The causes of squint are many and varied. The chief causes, however, are (1) ametropia, which may produce a change in the normal relationship between accommodation and convergence: (2) amblyopia or impaired vision as evidenced by the number of strabismi that follow amblyopia and opacities of the cornea and of any of the refracting media: (3) anatomic anomalies, such as variations in the interpupillary distance, in the shape of the eyeball and in the divergence of the orbits and (4) mechanical anomalies, such as a difference in the length and strength of the several extra-ocular muscles.

422. The etiology of concomitant strabismus is a complex question upon which there are many differences of opinion. Boehm discussed the relation which exists between hyperopia and convergent squint. Donders, in his classic, portrayed the part that the anomalies of refraction exert in the etiology of squint. When an emmetrope fixes a near point the primary demand which regulates the position of his eyes is the necessity of seeing it single. If one eye is covered this necessity no longer exists, yet the observed person generally continues to converge toward the point fixed due to the relationship between accommodation and convergence. Even in cases of myopia in which, for example, no accommodation is demanded in fixing a specified point, the covered eye will converge somewhat, at least, for the object. This is due to what Hansen-Grut termed *sensation of distance;* for the patient, knowing that the object is a short distance away, converges because he is accustomed to do so in the interest of binocular vision even in cases in which this demand no longer exists.

423. Ametropia produces a change in the normal relationship between accommodation and convergence. While it is possible for accommodation to take place without convergence, or convergence

without accommodation, yet there is a relationship between the two processes which, if materially interfered with, will produce diplopia and ultimately squint. Thus in relative hyperopia it is known that the accommodative effort is accompanied by contraction of the internal recti muscles, or convergence. In a hyerope of, say, four diopters accommodating for infinity, convergence would be stimulated to a proportionate degree at the same time. If such a pair of eyes is accommodating for a near point, this hyperope must accommodate (and would thereby also converge) just that much in excess of what standard eyes would do. The result is that a person with a hyperopia of any considerable amount frequently squints inward in the effort to maintain binocular vision. If, again, one eye is more hyperopic than the other the difficulty of adjusting convergence and accommodation is increased. If one eye is hyperopic by three diopters while its mate has five diopters of error, then six diopters of accommodation is exerted to fix at thirteen inches. The eye having five diopters of hyperopia has, then, at the ordinary reading point two diopters of its error uncompensated with the result that the retinal image of that eye is not clear and accommodation is still further taxed, stimulating at the same time the internus, so that this eye deviates inward and may finally remain convergent. We thus see why a hyperope may become strabismic: it is not as easy to plausibly explain why the great majority of hyperopes do not squint. However, the amount of hyperopia is commonly the same in both eyes and there is apparently developed through habit the ability to accommodate without converging in a proportionate degree.

Myopic eyes, in contradistinction to hyperopic conditions, cannot accommodate beyond their far points but must converge for all points inside infinity. If the myopia is one of six diopters, then these eyes would have to converge six meter-angles to fix an object at that distance (7 inches) without any demand upon the accommodative mechanism. To converge this amount places a considerable burden upon the internal recti muscles; this force and effort cannot be maintained for any length of time without discomfort. The result is that convergence is relaxed and, one eye remaining fixed, the other is turned outward. This is more likely to occur if one eye is more myopic than the other. This explains the presence of divergent squint in cases of myopia: we may with safety say that one of the factors (accommodation) which sustains convergence is wanting. But all myopic eyes do not necessarily have squint as some of them have roomy orbits, strong internal recti and in many cases short interpupillary distances.

424. These are the commonly accepted views. The researches of von

Graefe, Worth, Howe and Maddox suggest, however, another and, to the writer, more satisfactory explanation of these phenomena. For convergence may be differentiated as *tonic, accommodative* and *reflex* or *fusional*. These are diagrammatically represented in Fig. 211 (due to Maddox), in which the eyes are fixing a point O at a distance of a quarter of a meter. In the figure, two lines marked R, R indicate the supposed position of the visual axes were all nervous impulse abolished.

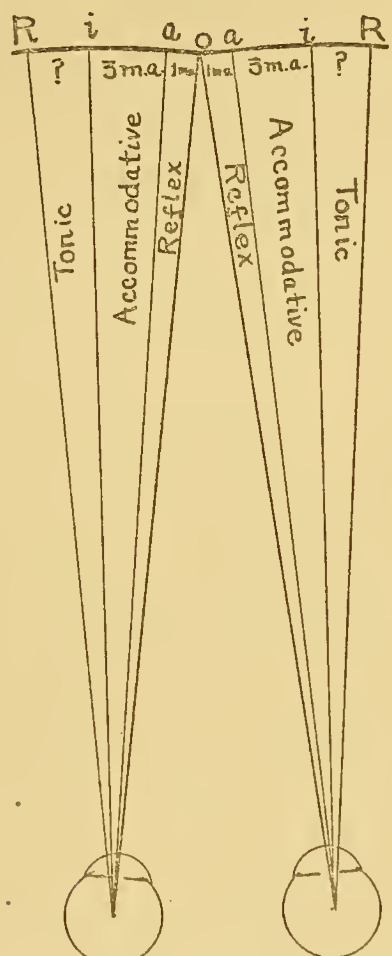

Fig. 211.—The Three Grades of Convergence in Vision at Twenty-five Centimeters. (After Maddox.)

The lines i, i indicate the *tonic convergence* during waking hours, due partially to muscular tone and partially to involuntary tonic action of the converging innervation. The ocular muscles no doubt possess a physiological tone; in addition to this common muscular tone there is a persistent activity of the converging influence, since all experimentation indicates that, if all innervations to an eye cease, the anatomical position of rest of the eyes is undoubtedly one of slight divergence. The visual axes of a pair of eyes are thus brought to the positions i, i

by tonic convergence; *it is the tonic convergence which is tested when von Graefe's equilibrium test at distance or any of the various phorometer, double prism or Maddox rod tests are applied.* For exophoria in distant vision indicates a deficiency and esophoria an excess of *tonic convergence* and the aberration is corrected, when both eyes are in use, by *reflex convergence* or *divergence* as the case may be. In *distant binocular vision* there are normally only two grades of convergence, the tonic and the fusional.

425. In *near vision* there is, however, what we have referred to as *accommodative convergence.* This second grade of convergence is added, of course, to the tonic convergence and depends necessarily upon the amount of accommodation in action. The investigations of von Graefe, supplemented by those of Howe, Worth, Maddox and others, have demonstrated that when, by means of prisms base up and down before each eye respectively or by means of a double prism *fusional convergence is eliminated* and the tests are made at relatively near points, we have a simple method of measuring the correlation between accommodation and its associated (hence properly called, *accommodative*) convergence. In other words, these tests make fusion passive and hence give a means of determining the convergence normally associated with the accommodation. As a rule, according to Maddox, each diopter of accommodation is accompanied by about three-quarters of a meter-angle of associated convergence. Hence, in a typical emmetrope, not presbyopic, the four diopters of accommodation in vogue for vision at a quarter of a meter are accompanied by three meter-angles of convergence, leaving a deficit of one meter-angle (approximately 6△) to be made up *reflexly* or *fusionally* as shown in Fig. 211. There is usually an exophoria at thirteen inches of about one (binocular) meter-angle and hence about 4°, but this varies in values assigned it up to 6° to 8°. This exophoria exhibited normally in near vision when the tests are made so that accommodation is active but fixation (involving fusional convergence) is passive is usually referred to as *physiological exophoria.* It is of necessity difficult to assign an exact limit between the physiological and the pathological and each case must be taken on its own merits, for this accommodative convergence test is but one of the many required to ultimately determine the lenticular assistance demanded in any case. Any cause which renders the ciliary muscle less responsive to its motor impulses thereby necessitates increased impulse to accommodation and with it an increase proportionately of the accommodative convergence. When, for example, the object fixed is brought near the punctum proximum of accommodation, the accommodative effort becomes so much greater than

the actual effect produced in the lenticular changes that the associated convergence exceeds its normal proportions and produces an esophoria. In hypermetropia, then, the accommodative convergence without a correction of the refractive error is greater than in emmetropia as a rule. Any condition, however, which renders accommodation easier, so that work is done with less effort, lessens the accommodative convergence. Myopia renders accommodation unnecessary in vision beyond the far point. There results, therefore, a diminution of accommodative convergence. Numerous experiments demonstrate that convergence is not affected by the actual accommodation produced but by the accommodative impulse in any case. (Some of these points are dealt with in a series of articles by J. C. Eberhardt on "The Dynamics and Economics of the Binocular Functions" appearing in the *Optical Journal* and *Review* 1916-17; also in a series of papers by Charles Sheard on "Dynamic Skiametry and Other Dynamic Methods for the Determination of the Co-ordination of Accommodation and Convergence," *The Keystone Magazine of Optics.*)

426. Reference has already been made to the third grade of convergence known as *fusional* or *reflex convergence.* This is the element which is, without doubt, most affected by ocular fatigue as is well illustrated in cases of periodic squint. There is generally no squint on arising in the morning but as the eyes tire the squint appears. Its manifestation is due in part to the diminishing strength, through fatigue, of the reflex muscular actions. When the amplitude of the visual reflex becomes smaller than the squint the latter cannot be overcome. Such cases as these must not be confused with "accommodative squints" in which the hyperopia may be absolute, or cases in which the squint appears only in near vision when accommodative effort becomes disproportionate to the actual work accomplished. In the preceding paragraphs reference has been made to the tonic and accommodative convergences. We have seen that the tonic convergence is measured by prisms when the tests are made at twenty or more feet and that the findings indicate, in general, the amount of "muscular insufficiency" existing when the eyes are allowed to assume their positions of monocular equilibrium. In binocular vision this excess or deficiency must be innervationally supplied in order to parallel the visual axes at twenty feet. Under standard conditions there is a slight deficiency which is supplied through positive fusion convergence. In turn, at near points, some device must be employed for measuring the accommodative convergence. Fig. 212 shows a simple method for obtaining such data. The test card, bearing a dot and a printed line of type, is usually held at the reading distance of thirteen inches. By

means of a double prism before one eye, its mate being uncovered, three dots and rows of printed letters will be seen: the upper and lower belong to the eye wearing the double prism while the middle line is seen by the eye under test. Let us assume that the right eye is under test. Then, if accommodation and accommodative convergence are associated in the ratio of approximately 2.25 or 2.5 to 1 instead of the commonly assumed orthophoric condition of 3 to 1, the dots and

● **Kindly read these words with care**

Fig. 212.—Dot and Line Test Object in Use in the Accommodative Convergence Test.

lines will appear as shown in Fig. 213 when the double prism is before the left eye. Some 4△ to 6△, base *in,* will be normally required to bring all these dots in a vertical line and the accommodation and its associated convergence would therefor be considered as properly correlated. Should the dots appear initially to be in a vertical line the interpretation would be that excessive accommodative *effort* (not excessive actual result, however) was demanded at the point for which the test was made, since the accommodative convergence was excessive.

● **Kindly read these words with care**

● **Kindly read these words with care**

● **Kindly read these words with care**

Fig. 213.—Images as Seen Under Fusional Dissociation Using the Maddox Double Prism.

The line of type is added beside the dot in order to assure that accommodation is taking place, the patient being requested to read it while the position of the middle dot is being located.

427. There is, therefore, under standard conditions under dissociation tests an exophoria or divergency at near points. This divergence does not exist in ordinary binocular vision since it is overcome or supplied by the fusion reflex which thus prevents doubleness of vision: the exophoria revealed by this test shows that the association of convergence with accommodation is not complete centrally but that a supplementary action is normally needed to supply the deficiency and fuse the two images. The fusion function is maintained in operation by the desire of the cerebral center to keep singleness of the two images.

The sensitiveness of this mechanism can be appreciated when we remember that if double images are produced artificially or through disease it is impossible for the mind to tell whether convergence should be positively or negatively increased in order to bring the two images together. Such an action, of this rather complex nature, must be in general more fatiguing than the mere overflow of one impulse into another. Fusion convergence must, therefore, involve a greater waste of nervous energy demanded for coördination than does accommodative convergence, for the latter is associated with the act of accommodation while the fusion convergence is a separate and independent function. Overtaxation of the reflex convergence may cause many of the complaints listed under the name of "muscular asthenopia," for it is

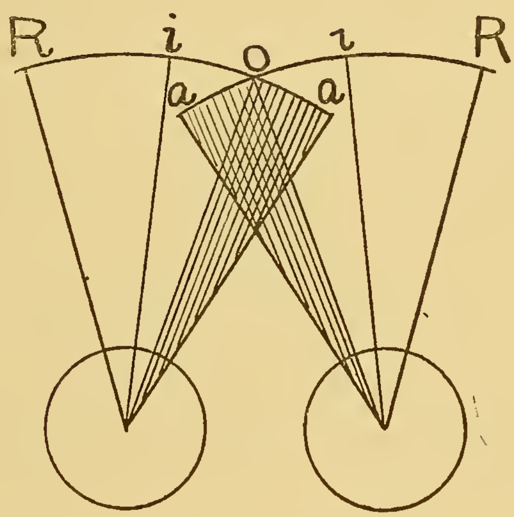

Fig. 214.—Representative of a Case of True Periodic Squint in its Latent Phase. (After Maddox.)

probable that many of these do not involve muscular elements but are, rather, central asthenopias.

428. Fig. 214 attempts diagrammatically to represent the relations between tonic, accommodative and fusion convergences in a case of periodic convergent strabismus. The first two kinds of convergence might be excessive if the subject were hyperopic and would bring the visual axes to *a, a* (over-convergence) instead of to *O,* the point under fixation. The visual axes will, however, be brought back to the point *O,* when possible, by negative or diverging fusion convergence. When, however, the negative reflex convergence becomes fatigued, squint will occur and that particularly at near points. If the hyperopia is corrected the accommodative convergence will be lessened, the tonic convergence will slowly tend to become less and the diverging centers will no longer be forced to function abnormally because of the abnormally

large accommodative innervation and associated accommodative convergence. Squints (high phorias) are thus often relieved and approximately normal convergence and accommodation relations established by the wearing of proper lenses. The effects of tenotomy in such cases are somewhat different; the anatomical divergence is increased and this thereby compensates and offsets the excessive tonic and accommodative convergences.

In cases of myopia, in which exophoria exists in both distant and near vision, an analysis similar to that which we have given above will show that the amount of positive fusion convergence demanded at near points is generally excessively large. The drain is therefore upon the positive fusional innervation; if this is unable to bear the load efforts to maintain binocular vision may cease and divergent strabismus ensue. When, however, the myopia is corrected, in part or in whole, accommodation is made active and the positive convergence associated with accommodation then enters as a factor to aid in the relief of the burden carried by the fusion convergence.

429. These analyses give a logical basis for the explanation of *why* all hyperopes do not have convergent squint and all myopes do not have divergent squint. For strabismus will not occur when the amount of either positive or negative fusion convergence demanded can be supplied and leave in addition an adequate reserve. The whole of the convergence is not supplied through the innervation associated with accommodation but the deficit is made up where possible through reflex convergence. Assume, for illustration, a case of myopia of 3 D. with an exophoria of 3△ at distance. Assume further an interpupillary distance of 64 mms. and that fixation is at 13 inches. A simple calculation shows that 20△ of convergence is demanded for fixation at this point: of this amount the convergence associated with accommodation furnishes nothing since the accommodation is nil. The fusion convergence must, therefore, under the conditions assumed, supply about 23△ of positive convergence in order to compensate for the losses occasioned by the tonic, accommodative and normally demanded fusional convergences. The whole responsibility for binocular vision is thrown in large measure upon the fusional centers and these may be unable to comfortably supply the demand and divergent strabismus therefore develops. If, again, we assume that the full error is corrected and hence 3 D. of accommodation demanded and produced at the near fixation point, then approximately 13△ of the convergence ought to be supplied through the accommodative convergence and leave but 10△ to be cared for through the medium of reflex convergence.

430. We have mentioned *amblyopia* as one of the possible causes

of squint. Statistics show that from 30 to 70 per cent. of all squinting eyes are amblyopic. Congenital amblyopia, due to imperfect development, doubtless plays an important part in many cases of squint: the cones of the fovea, the optic nerve or the visual centers in the brain may be at fault. This form of amblyopia is to be distinguished from that which is due to disuse and to the habitual suppression of the images in one eye known as amblyopia ex anopsia. Worth in his "*Squint*" says that congenital amblyopiæ exhibit certain peculiarities in common. He cites amongst others the following salient points: the fundus and media are normal in appearance; the fields of vision, both for white and for colors, are full; the peripheral form vision is normal up to within 20° of the fixation point so that "the defect would seem to consist in a want of due preponderance of the macular region and not in a general lowering of the sensibility of the visual apparatus." Worth found no case in which the vision of the amblyopic eye was less than 6/60. The most remarkable feature of these cases is that the defect is confined to one eye which almost invariably has a high degree of compound hyperopic astigmatism while the other eye has either emmetropic or low hyperopic conditions. In many cases the fusion faculty is well developed, as shown by examinations with the amblyoscope. In amblyopia ex anopsia, however, the blindness often reaches a degree which is never met with in the congenital form. In congenital amblyopia the central vision is never lower than 6/60 (Worth); this is the visual acuity normally found at 5° from the fixation point. In extreme cases of acquired reduction in acuity there is often a scotoma extending about 15° to 20° around the center of the field of vision.

431. These considerations form a logical basis for the determination of cause and effect; the possibility of attributing the strabismus to the amblyopia or the amblyopia to the strabismus is made possible. This is, of course, most important from a prognostic point of view. The "ex anopsia" variety is very clearly demonstrated by the rapid, though generally only partial, recovery of visual acuity which attends occlusion of the better eye. Javal has pointed out that if occlusion be indulged in for a considerable period of time, improvement takes place very often by sudden increases or steps, since the eye is at first not only lacking in acuity but is also awkward in seeing. This takes considerable practice to overcome.

432. Having confirmed what has been said about projection of retinal images and the conditions in muscle paralysis it might be supposed that in concomitant squint there would also be double images—homonymous in convergent and heteronymous in divergent squint. Experience

shows that this is not the case except in conditions of convergent strabismus of myopes, but that the patient does not see at all with the squinting eye the object fixed by the better eye or, in other words, that the retinal image of the squinting eye remains unperceived or suppressed. The squinting eye is not, however, entirely excluded from participation in the visual act. It can be shown that objects within the visual field of the squinting eye are partly seen and partly unseen, or that there is regional exclusion. The squinting eye may perceive everything lying within that part of the total visual field belonging to the squinting eye alone. The individual fields coincide in a smaller area than normal in divergent squint and the total visual field must then be greater than normal; in convergent squint, for the reverse reason, it must be smaller than normal. Objects lying within that portion of the visual field common to both eyes are not necessarily excluded from the squinting eye but only those that disturb the vision of the better eye. Many squinting persons testify that when reading they see double at the beginning or end of a line (i. e., that the image in the squinting eye is not suppressed if it appears on a background of white paper) but that it is not seen, or is suppressed, when it appears at a point where letters are seen by the better eye. Diplopia for objects seen at one side causes, however, little disturbance just as in the case of physiological double images. The fixing eye will conquer when there is a struggle between images of different objects for the same point in space. The nearer normal eye has, on the one hand, a greater visual acuity and, on the other hand, it uses in its assertion of supremacy its most sensitive retinal region while the squinting eye has only eccentric vision from a peripheral and less sensitive retinal area.

433. The *suppression of retinal images* is the means adopted by the eye for freeing itself from the disturbances of diplopia. The process must be a psychical one and wholly unconscious, accomplished by association of action of the two eyes. This can be demonstrated by the fact that the double images which are unnoticed in such cases are usually made perceptible by very simple means. Many who squint can see double as soon as their attention is called to the possibility of there being two images present; others may be made to see double by holding a piece of red glass in front of the better eye; or again a prism may be placed base up or down in front of the directing eye while the colored glass is worn before the better eye. There will then be formed upon the retina of the deviating eye an image of the object fixed by the other eye and this image will fall upon an area where retinal images have not as yet been suppressed because they arose from objects seen peripherally by the better eye also.

434. There also arise phenomena, termed *paradoxical diplopiæ,* discovered by von Graefe. In examining persons in whom convergent strabismus had been partially corrected by a tenotomy, he found crossed diplopia although the visual lines were still convergent and hence all tests should have shown homonymous or uncrossed diplopia. Javal explained these phenomena by saying that there is developed a vicarious fovea. The patient has first to learn to suppress the image in the strabismus eye; there is then gradually formed a notion of the false position of the squinting eye and the patient finally has to learn that an object which forms its image on the fovea of the good eye forms its image at a point inwards from the fovea of the strabismic eye and hence the image is localized at the place where the object to which it belongs is situated. It is, therefore, as if there were developed a correspondence between the vicarious fovea and the fovea of the good eye and such patients, under dissociation tests by means of vertical prisms, will see the two images almost on a vertical line instead of exhibiting widely separated images. If, then, in such cases a tenotomy is performed which does not completely correct the error, the image of the point fixed will be formed somewhere between the true and the vicarious foveæ. Patients first project the image according to the vicarious fovea and therefore see the images heteronymously. Later the true fovea may exert its supremacy and objects will be seen homonymously. It has been found that binocular *triplopia* can arise when, in the course of development of the change of fixation lines, the patient projects the image of the strabismic eye according to both foveæ and thereby sees one object to the right and one to the left of the true image.

XXIV. DIAGNOSIS OF STRABISMUS

435. A few tests for strabismus are briefly indicated below.

(1) *Inspection.* Simple inspection will often not only determine the presence of strabismus but will also locate the fixing eye and show the probable degree of deviation. The appearance of squint may, however, be illusory for myopic eyes frequently give the impression of a slight convergent squint while hyperopic eyes often appear divergent. This is due to the value of the *angle alpha* which, as has been pointed out, differs in value in various ametropic conditions of the eyes.

(2) *Ophthalmoscopic corneal images.* This test consists in reflecting the light, first on one eye and then on the other, from the mirror of an ophthalmoscope held about ten inches from the patient's face. The observer looks through the mirror aperture and directs the patient's gaze to the same aperture. A small circular reflection from the mirror

will then be visible in each eye. In emmetropic eyes the reflection will appear slightly to the inner side of the center of each cornea and if they are symmetrically disposed in the two eyes the existence of squint is made improbable. This method has been dealt with by Maddox in a chapter entitled "Ophthalmoscopic Images" in his work on *The Ocular Muscles.*

(3) *The cover test.* In any case of strabismus, whether manifest or latent, one eye fixes the object and the other deviates. The patient's attention is directed to some distinct distant object. If the fixing eye is occluded the patient is compelled to rotate the deviating eye into the proper position to fix the object. By means of a cover placed in turn before each of the eyes, the observer can watch the movements that occur behind it. If the cornea of either eye makes an excursion inward when its fellow is covered, it must previously have been deviated outward, and if the movement should be outward, the previous deviation must have been inward. Having the patient fix a pencil or other small object at eighteen inches we interpose a card before one eye. If the balance between convergence and accommodation is normal, the covered eye automatically holds its position. If convergence is in excess, the covered eye now having nothing to fix, turns *in* while if it is insufficient it turns *out.* These movements (or their absence) are often too slight to be definitely made out by observations carried out behind the screen. If, however, the covered eye is closely watched and the screen quietly but quickly withdrawn, it will fix the object by a movement of redress.

(4) *Linear strabismometry.* This takes account of the displacement of the pupil by measuring the amount of deviation in millimeters. It was at one time a popular method owing to the accepted theory that a displacement of the pupil of a certain number of millimeters could be rectified by setting back the tendon by an equal number of millimeters. The method is now practically obsolete.

(5) *Hirschberg's method.* A lighted candle is held about one foot in front of the patient's face, the operator placing his own eye near the candle. The operator looks over the flame at the eyes of the patient who, in turn, is told to fix his gaze upon the candle. The position of the corneal reflection in the squinting eye indicates roughly the amount of squint. The breadth of the cornea is about twelve millimeters. A squint which brings the reflection to the edge of the cornea then indicates a displacement of 6 mms. in the image and shows a deviation or squint of 3 mms. According to Hirschberg, if the reflex is at the margin of the pupil (3 mms. in diameter) a strabismus of 12° to 15°

is indicated: a reflex situated half-way between the center of the pupil and the corneal margin represents a strabismus of about 25°.

(6) *Javal's perimetric method.* In this method the patient is seated so as to bring the squinting eye into the center of the perimeter. At a distance of five or six meters a candle or luminous spot is placed for the fixing eye to regard. Another candle or small electric lamp is then moved along the arc of the perimeter, with the operator's eye constantly behind the light, until its reflection appears to occupy the center of the cornea. The squint is then measurable on the perimeter arc.

(7) *Charpentier's perimetric method.* The lamp is placed over the fixation spot of the perimeter and the operator's eye travels along the arc until the reflection appears to lie in the center of the cornea of the squinting eye. The squint is then measured as one-half of the angle of the arc.

(8) *The tangent strabismometer of Maddox.* This is in principle a flattened-out perimeter. The instrument is used by having the patient face the candle at the zero mark of the instrument at a meter's distance while the operator places his own head between the two but a little lower down and about a foot from the patient. The corneal reflections reveal which is the squinting eye and the amount of the squint is guessed at after the manner of Hirschberg's principle. The patient can then be directed to look at various figures upon the strabismometer until the reflection occupies its proper position. Maddox says that this instrument can be used for measuring and determining (a) concomitancy, (b) secondary deviation, (c) angle gamma, (d) degree of eccentric fixation, (e) imperfect duction, (f) vertical elements in the squint.

(9) *Worth's deviometer.* This is essentially a modification of the preceding method and is admirably adapted for testing small children. The instrument consists of an upright piece carrying an arm which can be swung to the right or to the left. On the back of the arm is a scale of tangents to degrees at sixty centimeters distance. Below the zero of the scale is an electric lamp of the cylindrical type. The operator flashes on the light and observes the corneal reflections. The position of the light on the cornea of the squinting eye enables a guess as to its deviation to be made. A brass carrier is then moved out on the horizontal arm; this is tapped with the finger to attract the child's attention. The light is then flashed on; if the line of light on the cornea of the squinting eye is in a corresponding position to that which it formerly occupied in the fixing eye, the angle of squint is read off on the scale on the back of the arm.

(10) *Priestley Smith's tape method.* In a darkened room the observer places himself at a meter from the patient. The patient gazes into distance. Light is then thrown by means of a retinoscopic mirror (or an ophthalmoscopic lamp) on the deviating eye. With the tape measure held at zero at the retinoscope, the observer moves his free hand horizontally away from the direction of the deviating eye, directing the patient to follow his moving hand, through which the tape measure slides, until the deviating eye is brought into such a position that the corneal image rests in the center of the pupil. The tangent relation enables the operator to calculate the angle of squint.

(11) *The diplopia test.* By determining the relation of the image of the squinting eye to that of the fixing eye a diagnosis of the actual position of the squinting eye and its degree of variance from parallelism can often be made. In amblyopia, as previously remarked, the operator will experience great difficulty in forcing a recognition of the image of the amblyopic eye since such a patient does not complain of diplopia nor is he conscious of the false image of any object. The use of prisms, base in or out, before the squinting eye and a colored glass before the other eye in order to render the true light dull and indistinct will often permit of the recognition of two images. The prism power before the squinting eye which permits of a fusion of images is the prismatic measure of the deviation. Another important factor is often brought out by this method, and that is the presence of a vertical difference in level of the two images. The determination of the presence or absence of hypertropia is very necessary.

(12) *The tropometer.* This is a supplementary test and the information furnished by this instrument is of value in showing the exact power of temporal rotation of esotropic eyes, nasal rotation of exotropic eyes and so on.

XXV. TREATMENT OF STRABISMUS

436. The treatment of strabismus may be divided into *operative* and *non-operative.* We shall treat briefly of the latter class only and pass over the operative method with the simple statement that when various methods such as the use of cycloplegics, convex or concave lenses, prisms for relief or exercise, prism exercises and efforts to improve vision, fail to remove the causes of the squint operative measures alone remain and the choice lies between a tenotomy or an advancement.

The non-operative methods of treatment of strabismus involve (1) re-establishment of diplopia, (2) correction of the refractive errors, (3) prisms, (4) cycloplegics, (5) exclusion of the good eye, (6) bar reading, (7) exercises with the stereoscope or amblyoscope, (8) exercises without the stereoscope. These methods are designated as orthoptic or educative; one or more of them may be used at the same time as the nature of the case seems to demand.

(1) *Re-establishment of diplopia* and of the vision of the strabismic eye should presumably be first attempted when such are not already existent to a fair degree. The non-squinting eye is kept covered by means of a blinder or under the influence of atropine. During this period of treatment the two eyes should not be left uncovered at the same time since the neutralization of error accomplished may disappear and the strabismus may even increase.

(2) *Correction of the refractive errors.* The essential factor (or one of the essential factors at least) which permits an ocular deviation to occur is a defect of the fusion faculty. The eyes, lacking sufficient fusional innervation, are for the time being kept approximately correlated by their motor co-ordinations. They are, however, in a state of unstable equilibrium and are easily persuaded to squint under influences which would have no effect on them ordinarily. In a majority of cases it is the state of the refraction which chiefly determines whether the eyes shall deviate inwards or outwards. Since hyperopia and hyperopic astigmatism demand abnormal accommodation and hence abnormally large associated convergences, it is a rational treatment to give convex lenses (either spheres or cylinders) to the full limit possible. For in such cases these lenses serve a double purpose in that they not only improve vision by correcting the refractive error and thus afford a stimulus to binocular vision but they also lessen the need for accommodation and in so doing diminish the over-convergence and relieve the burden upon the divergent fusional centers. Divergent strabismus, being ordinarily associated with myopia, arises in large measure from the lack of accommodative effort and its associated convergence, thus throwing the burden of positive convergence at near points upon the convergent fusional centers. All myopes under 6 diopters should ordinarily be given a full correction of their optical defects so that accommodation may be normally brought into action, while in cases of myopia of higher degree the fullest possible correction which will afford a fair degree of comfort in work at the near point should be prescribed. Hyperopes having divergent squint should, in general, be given the weakest spherical correction consistent with comfort in near work; presbyopes, if myopes, should be given the strongest minus corrections or, if hyperopes, the weakest plus corrections possible for use at thirteen inches.

With respect to the lens corrections, the important points are: (1) that the spherical correction should be as full as possible (convex lenses in convergent squint coupled with hyperopia and concave lenses in divergent squint coupled with myopia) and any existing astigmatism be fully corrected, and (2) the lenses should equalize the accommodative action and, as far as possible, the visual acuities of the eyes. This last point brings up for consideration the equalization of accommodation and the obtainance of as nearly equal visual acuity in both eyes as possible. This the writer believes to be a very essential point. Accommodative action, or rather the relief from such action, can be readily accomplished by various methods for twenty feet or infinity. Assume a case of convergent strabismus in which static refraction at 20 feet evidences as the full error O. D. + 4 D. S. and O. S. + 2 D. S. In such a case as this let us further assume that visual acuity and other tests demonstrate that the right eye is the squinting member; likewise, let it be amblyopic and fail to give, let us say, any indication of visual acuity greater than 5/10 wearing the lens which gives the best vision and which approximates in value that determined by skiametric methods. In the case of the left eye we shall say that + 2 D. S. brings 20/20 vision. Tests made upon the accommodation at 13 inches, however, are likely to disclose in such a case some such data as follows: right eye, wearing distance correction, cannot read anything on the near test card but by the addition of + 3 D. S. to the static correction, making + 7 D. S. as the full near-working correction, the patient can read No. 3 or No. 4 Jaeger type with fair ease; for the left eye, however, tests upon the accommodation at thirteen inches, the patient reading No. 2 Jaeger type through concave lenses, demonstrate a total accommodative amplitude of 7 D. Experience has shown that in many cases such as this hypothetical one various occlusion, bar reading and stereoscopic exercises fail to develop a normal relationship between accommodation and convergence and establish normal conditions when the *distance* correction only is prescribed for the amblyopic eye. The writer believes that many such ocular conditions which are often pronounced hopeless would be improved and ultimately saved if the treatment were instituted from the near or reading point rather than from the distance findings only. Certainly accommodation and convergence are normally demanded at fixation points inside of infinity: the accommodative powers must logically, then, be developed and the visual acuity improved before harmonious correlation of the two eyes can take place. This development of accommodative power can be best accomplished at the reading point; since this may have become weakened or partially lost through lack of use it is logical to supply such an eye with such a lenticular correction as will enable it to see as nearly normally as possible at near points and to give it such optical assistance as will make it a normally functioning, useful organism and stimulate development. As the accommodation, etc., develops, this near correction can be cut down in value until ultimately in many cases nearly (if not full) normal accommodative amplitude will have been restored and the distance correction prove satisfactory for all purposes. The writer believes, therefore, that such cases can be handled to advantage by (1) prescribing full distance corrections for general wear with occlusion of the better eye from time to time for stated periods and (2) by prescribing a pair of glasses for near work in which the amblyopic or weaker eye is given such lenticular assistance as will equalize if possible the apparent accommodative powers and acuities of the two eyes and if this is impossible (as is generally the case) to give such lenses as will afford the poorer eye the best working conditions possible in the hope of developing the accommodation. The patient should then be instructed to cover up the good eye with a blinder and engage in reading coarse print for short periods of time very frequently.

(3) *Prisms* do not correct strabismus: they simply aid in securing binocular vision in spite of the deviation due to the squint by optically producing a deviation of the rays within the eye of such an amount as to produce an image upon corresponding points and thereby re-establish binocular vision. The base of the prism is placed opposite to the ocular deviation; hence, bases *out* in cases of convergent strabismus and bases *in* in cases of divergency. Strictly speaking such optical assistance in no wise corrects the strabismus but leaves the eyes in their positions of weakness. As to the advantages and disadvantages of the use of relieving prisms and the amounts which should be prescribed under various conditions, the reader is referred to the various standard treatises dealing with ocular muscles. We have remarked that, for the cure of squint, prisms are useless. There is one possible exception, however. If diplopia is elicited but the two images are too far apart to be fused and one of these images is faint owing to its position on the retina of the deviating eye, prisms may be employed with profit for the purpose of bringing this image nearer to the macula so that it may become clearer and capable of being fused with the other image. Such prisms must not, of course, cause fusion directly but are simply the indirect cause or instigators of it. They are temporary measures only and should be reduced in strength as the muscles become more nearly balanced in action. In divergent squint prisms base in are much more likely to prove valuable than are prisms bases out in convergent squint.

(4) *Cycloplegics* are employed for the paralysis of accommodation: they are applicable in that large class of cases of convergent strabismus

occurring in young children having hyperopia. By a suspension of the act of accommodation there will be a suspension of the convergence associated with the former. During the continuance of the cycloplegic the lenses which correct the ametropia and of which the strength is as nearly as possible the full correction are worn constantly during waking hours. In many cases of young children the squint begins to decrease as soon as the cycloplegic takes effect and the prognosis is favorable for good vision with glasses when this occurs. In other cases, while the drops are in the eyes and glasses worn constantly, the squint entirely disappears but reappears as soon as the effect of the cycloplegic wears off. In such cases the drops are at present ordinarily put in one eye only and that the good eye. The possessor can then use this eye for distant vision as before, but on account of the abolition of accommodation will not be able to use it for near work. For this purpose he will be compelled to bring into action the squinting eye in which the function of accommodation (whatever power it may have) has not been artificially (i. e., through the use of cycloplegics, et al.) interfered with. In this manner the vision of this eye can be improved under proper optical correction, fixation preserved and amblyopia from disuse avoided.

(5) *Exclusion of the good eye.* This method consists in covering the eye by placing over it a patch or putting before it an opaque lens and thus compelling the patient to use the squinting eye for all visual purposes. This method of treatment has somewhat the same effect as the use of a cycloplegic, but few parents will take the trouble to do this every day and continue the practice for some length of time, and few children are willing to bear the inconvenience. A point of difference between the two methods, however, is that when the eye is simply covered it will still be able to accommodate in sympathy with the uncovered eye. By exclusion of the good eye the inferior or squinting eye preserves the faculty of fixation and develops and increases its vision. In alternating strabismus, where first one eye and then the other is used, each eye is able to maintain its vision and power unimpaired. Nature surely points the way in this decided manner and it seems wisdom, therefore, to follow and to transpose every case of fixed unilateral squint into an artificially created alternating one by exclusion of the good eye for several hours each day and compelling it to assume the squint for that period.

(6) *Bar or controlled reading.* A pencil or ruler is interposed in front of the eyes in a position practically parallel to the printed page which the patient is instructed to read. If he does not possess binocular vision but is making use of one eye only, when the line of

vision of this eye comes to the pencil or ruler a certain portion of the page will be cut off and the patient will be compelled to skip a word or two. If the two eyes are co-ordinated, however, no portion of the print will be occluded and there will be no interruption in the reading. By persistent practice, covering months in many cases, the patient may be taught to use the two eyes together. Considerable effort and patience may be required at first but gradually the function of binocular vision may bcome established. It is only after a considerable time subsequent to the re-establishment of binocular vision that the patient can see stereoscopic relief.

(7) *Exercises with the stereoscope and amblyoscope.* To exercise fusion a stereoscopic box may be employed without prisms but with lenses whose focal length is equal to the depth of the box. Partial pictures are placed in the two sides of the instrument in such a way that the complete image can be obtained only by combining them. At the commencement of these exercises they must be sufficiently near to each other for fusion to occur, after which gradual separation and approximation of the cards serves as a gymnastic exercise. There is, in these cases, no accommodation present and the effect on the muscles is increased if the lenses be gradually lessened in strength to permit of accommodation during the exercise. Instead of moving the card, varying prismatic power can be obtained by altering the distance between the lenses.

The Worth-Black amblyoscope is an ingenious instrument devised for stimulating and exercising the fusion faculty. It consists of two bent tubes through which two pictures are seen and fused. One set of pictures is designed specially for inducing stereoscopic impression, a second for stimulating binocular vision and a third for stimulating fusion. The distances between the pictures can be varied or the angle of convergence or divergence changed in the instrument; the intensity of illumination can also be varied. When there is convergent strabismus in children, the period of life during which an impaired fusion sense can best be remedied is that between three and six years of age (Worth). After the age of six or seven years the fusion faculty may have become too depleted to be restored.

(8) *Ocular exercises.* If harmony between the accommodation, accommodative convergence and fusional convergence is restored by lenses embodying the corrections of the ametropic errors, then the ordinary exercise of convergence and the lateral movements of the eyes usually serve to strengthen weakened muscular functions or to restore relative equality of power between antagonistic muscles. This is natural training; muscle training, frequently called *orthoptic exer-*

cising, consists of exercises for the purpose of strengthening weak muscles or weakened functions of muscles. Various methods have been recommended by the authorities who deal with this subject, but there is no agreement as to the best method. Probably there is no single superior method, but natural gymnastic exercises, such as in the alternate contraction methods, generally prove the most satisfactory. Likewise, it is true that a great deal depends, in the success of these various exercises, upon the personality and skill of the operator and upon the patience and faithfulness in the carrying out of instructions on the part of the patient. In all cases the refractive corrections are worn during the exercises and the prisms used are *adverse;* that is, the base is toward the more efficient muscles, thus causing increased action on the part of the defective muscles. Hence, for training the internal recti the prisms are bases *out* and for the external recti they are bases *in:* for the right superior or left inferior the right prism is base *down* and the left prism base *up;* for the other vertical muscles the prisms are reversed.

The three classes of exercises are in general differentiated as (a) varied contraction, (b) maintained contraction and (c) alternate contraction exercises.

In sustained contraction exercises adverse prisms are worn continuously for specified periods.

(a) Weak adverse prisms, $1\triangle$ to $4\triangle$, are given to be worn frequently every day for short periods. The time of wearing is increased slowly until they can be worn without discomfort for an hour. The same procedure is then instituted at the reading point.

(b) Nearly the highest prisms that can be overcome are first used and their power increased little by little so long as diplopia does not occur. They are used daily for a few minutes, with intervals of rest, some two or three times and for distance only.

In *varied contraction exercises* the action of the muscles to be trained is constantly varied.

(a) By means of rotary prisms, the prismatic power is gradually increased to the extent of the power of the inefficient muscles without diplopia ensuing: a few seconds interval is given in order to obtain complete fusion of the images before each increase of power is made.

(b) The internal recti can be exercised by slowly approaching a pencil towards the eyes until the pencil appears double and then slowly removing it and repeating the exercise. The external recti may, in turn, be each separately exercised to excellent advantage according to Thorington by covering one eye and following an object, such as a pencil, held about two feet from the eye, as the object is carried out from the median plane into the field of action of the externus. This class or kind of exercise may be spoken of as the natural gymnastic method and is probably one of the best.

(c) Change of distance of the object by a change of the distance of the person viewing it while adverse prisms are being worn. An excellent way is to view the object from a near point and then to slowly move away from it, returning towards it again and so on.

In *alternate contraction exercises* there are produced a series of alternate contractions and relaxations of the deficient muscles obtained by the alternate use and non-use of adverse prisms.

(a) A power slightly weaker than the strongest that can be overcome is used: its power is increased slightly and then again reduced by alternately adding to and removing an additional prism. The strength of the first prism and that of the additional prism are gradually augmented. The duration of the exercises is, at the commencement, two to three minutes daily, the period of time being increased as the muscles become stronger.

(b) A distant plane or luminous spot is viewed alternately *without and through* a pair of weak adverse prisms, each for a brief period. This is the essential feature of Savage's rhythmic exercises. The object is first viewed without the prisms for a few seconds (three to five) and then with the prisms for the same period. In the latter case additional action of the defective muscles is obtained. The relaxation of this additional action results from the removal of the prisms. The prismatic power for esophoria is $2\triangle$ increasing to $12\triangle$: for exophoria $1\triangle$ increasing to $4\triangle$: for hyperphoria $\frac{1}{2}\triangle$ increasing to $2\triangle$.

(c) A variation of this last exercise is to alternate distance vision with that at the reading distance, both of them alternately with and without the prisms, each period of fixation to last for about five seconds.

XXVI. HETEROPHORIA OR LATENT SQUINT

437. Latent deviations of the eyes involve the same principles as do the manifest deviations which are recognized as squints. Heterophoria is the condition of muscular imbalance when perfect fusion is maintained only at the expense of extra effort on the part of the relatively weak muscles or weak innervations. Latent deviations are liberated from the superior influence of desire for binocular vision in the dark, or when one is sleepy or abstracted, when the muscles are not under the control of the single vision centers or are, again, made manifest when the vision of the two eyes is dissociated by making single vision impossible or undesirable with prisms in the one case and Maddox rod and so forth in the second case. Suppressed deviation

may be demonstrated by the exclusion of one eye or by means of an artificial diplopia of some sort so as to dissociate the two eyes or by the arrangement of two objects so that each is seen by one eye only. By dissociation of the two eyes is not meant that any of the innervations are made to cease to be conjugate but that the desire for single vision is removed so that the eyes assume their positions of equilibrium. If a strong prism, base down, is held before one eye, everything appears double and the distances between the double images of an object is so great that the cerebral centers concerned make little if any attempt to unite them since such centers are wholly unaccustomed to so great a separation between the images of a single object. The eyes are therefore dissociated and if any latent deviation exists it will show itself by a movement of one image to the right or left of an imaginary vertical line passing through the other.

438. Heterophoria is as common as ametropia and is in fact the common accompaniment of ametropic conditions. But heterophoria of sufficient degree to cause trouble is not as common. In those who suffer from asthenopic symptoms it is found that in a small proportion of cases the trouble is due to heterophoria simply but that, in the majority of cases, it is due to conditions of ametropia coupled with the associated innervational excesses or deficiencies. Latent strabismus may be due to a muscle or group of muscles being too weak or too strong for their antagonists, or to an abnormal position of insertion of tendons whereby a muscle possesses a greater or lesser mechanical advantage than normally, or to a muscle or group of muscles which are abnormally innervated. We are in many cases unable to determine whether the fault lies in the muscles themselves or in their innervations. It is not commonly found, however, that there is a deficiency or excess in the limits of rotation of either eye. The differentiation between muscular or innervational abnormalities must lie, in so far as ophthalmic science has given us a clue, in a determination as to the relation which the versions and ductions bear to each other. Various methods and tests which can be applied as to the accommodation, the convergence as associated with the accommodation, the fusional convergence and the reserve convergence at the reading point, such as have been discussed in outline under the caption *Causes of squint,* demonstrate that a large majority of cases of heterophoria, particularly those in which asthenopic symptoms arise, are due to excessive or deficient innervation. It is probably true that if all investigations upon the muscular equipoise are made at twenty feet only that we are then entitled to conclude simply that the eyes tend to assume certain abnormal relative directions and we are seldom able to analyze the

defect further than this by such tests. It is likewise true that within certain limits suppressed deviations are physiological, for though the accommodating and converging centers are functionally connected in an intimate manner they are not indivisibly one. We have previously pointed out that, when the eyes fix a near object, they converge less than they accommodate with each approach of the object and that, in the light of the latest researches, there is a physiologic exophoria of 3△ to 5△ at the normal reading point. Worth says that "Heterophoria is essentially a defect of motor balance. Squint, on the other hand, is essentially due to a defect of the fusion faculty." This is doubtless a very fair statement of the facts if the ciliary muscle is included in the list of those taking part in the motor balance.

439. The distinctive names which are employed to indicate the direction of the tendencies are: (a) esophoria, or tendency to abnormal convergence of the visual axes, (b) exophoria, or divergent tendency, (c) hyperphoria (or hypophoria) in which one visual axis tends to lie in a higher plane than the other and (d) cyclophoria, or a tendency to abnormal rotation of the eyes around a fore-and-aft axis so that what should be the vertical axis of the eye is no longer parallel to the median plane of the head.

440. *Pseudo-heterophoria,* as it has been termed, arises in cases of uncorrected ametropia and frequently disappears very quickly when appropriate correcting glasses are worn. It seems, therefore, a wise procedure on the part of the practitioner to determine the heterophorias before corrections are applied to the eyes and also while the patient is wearing the exact correction of his ametropia at distance. This will enable the operator the better to form a judgment as to whether or not changes can be made in the interests of the ocular economy by an increase or decrease of the correcting lenses.

441. The symptoms of heterophoria are those of "eye-strain" in general: frontal headaches coming on particularly toward night, pain through the eyes after close observation of objects, pains at the temples, between the brows, over one or the other of the brows: migraine, dizziness (particularly associated with hyperphoria) and so on. Hyperphoria, or a vertical imbalance tendency, is the form of heterophoria which is most likely to cause trouble, for few persons can bear a hyperphoria of more than one degree without inconvenience. Esophoria is said by many to be the least troublesome. This is probably true when the vocation is an out-of-doors one or is of such a nature as to demand distant-seeing largely, but it is probably one of the most aggravating of heterophoric conditions when an excessive amount of near work is engaged in. It is a question in the writer's mind whether

or not modern civilization, with its excessive demands upon convergence and accommodation, will not develop hyperopia with excessive esophoria and premature presbyopia and that there need not be as great a fear of increase in myopia in school children of the oncoming generations as of highly hyperopic and latent strabismic conditions. In other words, it is the experience of a large number of practitioners that esophoria in greater amount at near than at distant points is very common amongst high school and college students and that this esophoria is functional and tied up with an overly innervated but not overly actuating accommodation.

442. *Methods of testing the relative motor balance of the eyes.* The methods which have been discussed in connection with ocular paresis and strabismus are quite largely applicable also to investigations upon latent deviations. Among the best tests are the following:

(a) *The objective screen test.* This is made by having the patient fix a very definite test-object and screening one eye for fully half a minute. Suddenly withdrawing the screen, watch whether the eye makes an instantaneous movement (corrective movement) and if so in what direction. A corrective movement inward indicates a condition of latent divergence or exophoria. The movement of redress is in the direction of the weak muscles or muscle insufficiently energized.

(b) *The subjective screen test.* A sudden screening of the fixing eye makes the object fixed appear to the patient to move as the deviating eye makes a corrective movement in order to take up fixation. Thus, if the right eye be the one first screened and the object fixed moves to the right there is esophoria. Parallax is measured in terms of the prism which causes its abolition.

(c) *Prism tests.* The methods involving dissociation of the eyes by means of prisms base up or down before one eye and strong enough to produce insuperable vertical diplopia or the Maddox double prism have been briefly described in other paragraphs. These methods permit of the measurement of lateral deviations by the amount of prism, base in or out, necessary to bring the various images in a vertical line. The double prism also permits of the measurement of vertical imbalances since the second, or central, dot or light seen by the eye under test may not be geometrically placed half-way between the upper and lower images due to the double prism, and prisms base up or down are required to produce this vertical balance. The optical and physiological principles involved as to the relation between the direction of the deviation of the eye and the direction of projection have been treated under the heading *Diplopia.*

(d) *Glass-rod test.* This single glass rod or high-powered cylinder test does not depend, like the prism tests, on the separation of the images of an object but on alteration of the shape of one of the images so that it is no longer recognized as in anywise similar to or belonging to the same object. The explanaton of the optical action of such a rod is worthy of comment. When a point of light is seen through a high-powered cylinder it appears as a ribbon of light at right angles to the axis of the cylinder. The reason is that the light source is no longer in focus for the retina in this (the power) meridian but is in focus for the meridian corresponding to the axis, or no power meridian, of the cylinder. The light which emerges from the cylinder is therefore drawn into a line of light parallel to its axis from which it again spreads out to be ultimately collected into a line, perpendicular to the former line, on the retina. The length of the diffusion line on the retina is exactly equal to the diameter of the diffusion circle created by a spherical lens of the same power.

Whatever form of rod or cylinder is employed, a horizontal streak will be produced when the axis of the rod is vertical and a vertical streak, in turn, when the rod is placed horizontally. In testing latent deviations one eye is left uncovered, except for distance corrections which the practitioner may desire to have worn during these tests, and the rod is inserted before the other eye. The patient is instructed to fix a luminous spot distant some 5 or 6 meters. He should then see the spot and the ribbon of light. If the line appears to pass through the light there is orthophoria: in lateral deviations, if the streak is on the same side of the luminous spot as is the glass rod the diplopia is evidently homonymous, indicating latent convergence, and if on the other side the diplopia is crossed showing exophoria. In the same manner, by making the axis of the rod vertical, a horizontal line of light is produced and latent downward or upward deviations determined.

The question is often asked as to whether the patient fixes the luminous spot (flame) or the streak (line of light). He is free to fix either. In the von Graefe prism test the patient can at will fix either the direct object or the prismatic one, since either eye is able to move so as to receive an image on the fovea of its retina, although such a movement displaces the fixing point of the other eye away from its image. If the alternate fixation makes the line of light shift to considerably different amounts the case is one of anisometropia or paresis. Maddox remarks that alternate fixation can generally be secured by transferring the rods from one eye to the other and "that so delicate a revealer of anisometropia is this procedure sometimes that so small a difference as a quarter of a diopter was once detected (before the refraction was tried) in a person with one eye emmetropic and the other hyperopic by 0.25 D.''

(e) *Multiple Maddox rods (one before each eye) in tests for cyclophoria.* Imbalances of the oblique muscles, giving rise to cyclophoria, can be detected by the use of a red multiple Maddox rod before one eye and a white Maddox rod before the other, enough prism (base up or down) being inserted before one eye (assume the right) to produce vertical diplopia. (See, however, in this connection the monograph on *The Modern Phorometer* by DeZeng, in which the latest device shows the use of two white multiple rods.) With the axes of the rods vertical two separate and distinct streaks or lines of light will be seen lying in an approximately horizontal plane, the white streak lying below the red one when the white Maddox rod is in front of the right eye. If the streaks appear parallel with each other and horizontal there is no cyclophoria. Should the red streak as seen by the left eye appear horizontal and the white streak seen by the right eye appear at an angle, cyclophoria of the right eye would be shown. Should the white streak tip upwards at the temporal side and approach the red streak the case is one of right plus cyclophoria, whereas right minus cyclophoria would be indicated should the white streak tip temporally downward or away from the upper streak. Such tests may be repeated by placing the prism before the other eye. The degree of either plus or minus cyclophoria may be accurately measured by rotating the respective Maddox rod to such a position as will bring the tilting line into a horizontal plane. The reason for this will be apparent from what follows in the next paragraph.

443. Another most excellent method and used by many practitioners in tests at six meters and one-third of a meter is the double prism with a single horizontal line as a test-object. Before testing the obliques the writer believes that any vertical and horizontal insufficiencies should be corrected. The double prism is then placed, bases horizontal, before one eye. The eye under test is the one before which the double prism is not placed, i. e., the eye seeing the central line. The tests should also be made with and without correcting lenses especially if these contain oblique cylinders. For purposes of emphasizing the physiological and optical principles involved, there are given in Fig. 215 various diagrams to explain the phenomena as subjectively seen by a person having plus cyclophoria of the right eye, i. e., insufficiency of the right superior oblique. Let AB be an arrow used as the object. Under the action of the double prism before the left eye, assuming no cyclophoria of this eye, two horizontal, reversed and inverted images will be received upon the retina as B_1A_1 and B_2A_2 in Fig. 215 (B).

The dotted lines indicate that the horizontal and vertical meridians of this eye are properly held in position. These two images will, therefore, be projected into space and seen as A_4B_4 and A_3B_3 as shown in E. The right eye, under test, having an insufficiency of the superior oblique will, then, under dissociation show an extorsion at the top and an intorsion at the bottom with the result that the vertical and horizontal meridians will be rotated as shown in C. The horizontal arrow will then be imaged upon the retina in C in the position A_5B_5 but will be projected into space as the line A_6B_6 tilting upward at the temporal side as shown in diagram D. Diagram E shows the positions of the three images as viewed by the patient. Other similar analyses can be

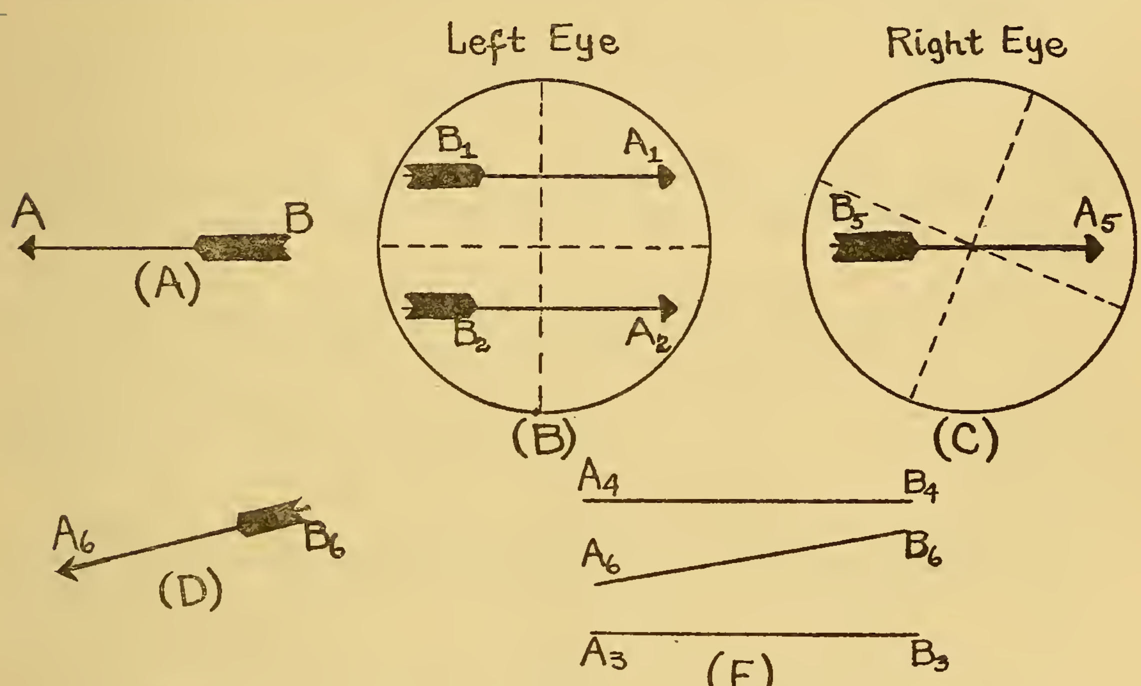

Fig. 215.—Retinal Images and Spatial Projections in Cyclophoria.

made or the indications given by the test memorized as follows: *The central line inclines on the temporal side toward the weaker of the obliques or it points on the nasal side in the direction of the stronger oblique muscle.*

444. In cyclophoria it is difficult to determine whether there is actual insufficiency of the muscles or malposition of the images due to oblique astigmatism when such exists, since it can be demonstrated (*vide* the writings of Savage) that, in astigmatism, there is a displacement of the images of all lines not parallel with the one or the other of the principal meridians and *that these displacements are always toward the meridian of greatest curvature.* False torsion increases with convergence and this may augment or tend to rectify the real torsion caused by deficient oblique muscles. Hence torsion tests made at near points should be

considered in connection with those made at distance. If the torsion is found only at near points (i. e., plus cyclophoria) it may generally be neglected unless the torsion be rather large: in fact the writer believes that, since hyperopia accompanied with esophoria constitutes the major portion of all refractive findings, the slight plus cyclophoria usually found at near points is wholly false as indicative of weaknesses of the obliques. *The condition of ex-cyclophoria in near vision is so common as to deserve being regarded as physiological.* Cyclophoria results chiefly from oblique astigmatism and cannot be measured or corrected with prisms. Steele's rules for the shifting of cylinders for the relief of cyclophoria are given in composite form in Savage's *Ophthalmic Neuro-Myology.*

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Summary of points relative to the diagnosis and treatment of heterophoria.

445. As a brief summary of some of the important points with reference to the diagnosis and treatment of heterophoria we cite the following:

A. Diagnosis. (1) The measurement of imbalances at six meters by the use of the Maddox rod or double prism and so forth.

(2) Repetition of the measurements with the head placed in different attitudes to make sure, by proving concomitancy, that no paralysis is present.

(3) The amplitude of convergence, or the convergence near point.

(4) The investigations upon the muscular conditions at the reading point: (a) amplitude of accommodation, (b) accommodation and associated accommodative convergence with fusional convergence passive, (c) reserve convergence, (d) conditions of vertical balance and cyclophoria.

(5) Tests upon the breadth of fusion, or prism duction, at six meters; especially the abduction in esophoria and the super- and subduction in hyperphoria to confirm the existence of the heterophoria.

B. Treatment. (1) As a rule prisms are not required and should not be given because heterophoria exists: their necessity must be clearly indicated: their use is often unsatisfactory and harmful.

(2) To determine their necessity, tests must be made with and without corrective lenses in both distant and near vision and on different occasions.

(3) Dependence should be placed largely on tests made with corrective lenses. One is never quite certain, however, as to the true muscular balance until correcting sphericals and cylinders have been worn.

(4) Exophoria and cyclophoria of small amounts in near vision are relatively unimportant, while in distant vision hyperphoria is "at least four times more worthy of notice for each degree than horizontal deviations" (*Maddox*).

(5) If the imbalances cause no symptoms, which is usually the case in low degrees, it is generally better to leave them alone. These indications should, however, aid in the choice of corrective glasses. Hyperopes with esophoria should be fully corrected since the relief of accommodation relaxes the convergence associated with it: hyperopes with exophoria should be undercorrected in many cases (this is the commonly accepted doctrine upon this point but is not satisfactorily substantiated according to the writer's notions upon this subject), and myopes with exophoria should be as fully corrected as seems feasible. Regard must be had in this respect, however, as to whether the correction is for near or distant use since, for example, a myopia with exophoria at distance may exist which exhibits an esophoric condition at near points.

(6) Deviations which demand considerable effort to overcome and where feelings of strain, frontal or occipital headaches and so forth exist, require special attention and investigation. In many cases the writer is confident that the seat of such troubles lies in a weakened fusional convergence, whether this be diverging or converging as the case may be. If these fusion powers (or centers) cannot be further developed by exercise it will often be found advantageous to include small prismatic elements in the corrections prescribed, particularly in those for near work.

(7) In general, as to the amounts of prismatic aid to be given, authorities practically agree upon

(a) Two-thirds of a persistent hyperphoria.

(b) Not over one-half for distance to two-thirds for near in esophoria.

(c) Not over one-third for distance to one-quarter for near in exophoria.

PART SIX

RETINAL AND CHIASMAL IMAGES

XXVII. RETINAL IMAGES

446. *Calculations for sizes of the retinal images in hyperopia and myopia.* This topic has been in part discussed under the caption *Relations between the refractive condition of an eye and its visual acuity* which forms a portion of the discussion on the topic *Form Sense.* We are, however, in the closing section of this treatise desirous of con-

sidering in particular the rôle played by the form and position of the retinal images upon binocular vision.

It has been previously shown that a lens placed in the anterior focal plane of an eye has no effect on the size of the image formed, the latter being merely moved forwards or backwards as the case may be. The image is then the same size as in emmetropia. Hence, in a case of axial anisometropia, if we could place the correcting lens exactly at the anterior focus of each eye, the retinal images would be identical in size. Under these conditions the effect of convex lenses is merely to reduce the divergence of light and that of concave lenses to increase its divergence from each point of the object incident on the optical system of the eye. It is impossible in many cases to make a comparison of the sizes of the images formed because they may not be sharply formed at the retina. In the following statements, therefore, a distinction must be drawn between the image, *I*, actually formed by the dioptric system in the vitreous in front of the retina or that formed back of the retina and the image formed upon the retina in any of these ametropic conditions. For if the retina is not coincident with *I*, the blurred retinal image, owing to the confusion circles, is larger than the ocular image (*I*). As a brief summary we may state that:

(a) In axial hyperopia and axial myopia, the ocular image (*I*) is the same as in emmetropia.

(b) In refractive hyperopia, *I* is larger than in emmetropia.

(c) In refractive myopia, *I* is smaller than in emmetropia.

(d) In refractive hyperopia, *I* is larger than in axial hyperopia when both are accommodated for clear vision.

(e) In refractive myopia, *I* is smaller than in axial myopia when both see clearly the same near object.

(f) In axial hyperopia, with accommodation, *I* is smaller than in emmetropia.

(g) In refractive hyperopia, with accommodation, *I* is nearly the same as in emmetropia.

(h) In axial myopia, the image of a near object seen clearly is larger than in emmetropia.

(i) In refractive myopia, the ocular image of a near object clearly seen is smaller than in emmetropia.

(j) In axial hyperopia and axial myopia corrected by a lens at the anterior focal point, the ocular image is the same as in emmetropia.

(k) In refractive myopia, similarly corrected, *I* is larger than in emmetropia.

(l) In emmetropia, for near vision, *I* is larger with a convex lens than when accommodated.

(m) In hyperopia, I is larger with a convex lens than when accommodated.

(n) In myopia, I is smaller with a concave lens and accommodated than without the lens.

447. *Size of retinal image as affected by position of correcting lens.* Ametropia can, of course, be corrected by a lens which is not coincident with F_A, the anterior focal point of the eye, but, for the same error of refraction, a convex lens must be weaker and a concave lens stronger the farther it is withdrawn. Without entering into the geometrical proof of this and converse theorems, let it suffice for the moment to say that, if the image is formed at the same distance by whatever correcting lens is used and in whatever position it may be placed, we know that

(a) A convex lens placed in front of the anterior focal point, F_A, the image is larger.

(b) A convex lens placed behind F_A, the image is smaller.

(c) A concave lens placed in front of F_A, the image is smaller.

(d) A concave lens placed behind F_A, the image is larger.

The size of the image after refraction through any optical system is obtained from the equation

$$\frac{I}{O} = \frac{F_A}{a} \quad \text{.............................(1)}$$

where I is the size of the image, O the size of the object, F_A is the anterior focal length and a is the distance of the object from the anterior focus of the system.

If a lens be introduced before the eye we have a new optical system and as before, the size of the image formed by this new system will be found from the equation

$$\frac{I_1}{O} = \frac{F}{a_1} \quad \text{.............................(2)}$$

in which a_1 represents the distance of the object from the new anterior focus and F represents the new focal distance. Hence the relation of the size of the image with the lens to that without the lens is expressed by the equation

$$\frac{I_1}{I} = \frac{a \times F}{a_1 \times F_A} \quad \text{.............................(3)}$$

When the distance of the object is great in comparison with the change in position of the anterior focus caused by adding the new lens, then a and a_1 may be considered identical and hence

$$\frac{I_1}{I} = \frac{F}{F_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

But F, the focal length of the new or combined system of eye and lens, is derived from the general equation

$$F = \frac{F_1 F_A}{F_1 + F_A - e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

and making this substitution, we have

$$\frac{I_1}{I} = \frac{F_1}{F_1 + F_A - e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

In this equation F_1 is the focal length of the lens, F_A is the anterior focal length of the eye and e is the distance between the eye and the lens. This formula is sufficiently accurate to determine the effect of correcting lenses upon retinal images except in the case that the object is near the eye. We shall now examine briefly the condition when a and a_1 cannot be considered identical. By a somewhat detailed but not difficult process of analytical reasoning it can be shown that

$$\frac{I_1}{I} = \frac{F_1}{F_1 + (F_A - e) + \left(\frac{F_A - e}{a}\right)^2} \quad \ldots\ldots\ldots\ldots (7)$$

This is the general expression for the magnifying power of any lens in combination with the eye or with any other optical system. If we examine this expression we see that if $F_A = e$, i. e., if the lens be placed at the anterior focus of the eye, then $\frac{I_1}{I} = 1$. If, again, the the lens be convex and F_A be less than e, that is, if the lens be without the anterior focus of the eye, then $F_A - e$ will be negative, but $(F_A - e)^2$ will be positive. Varying values of these two quantities and of the value of a give rise to the conclusions in the form of the four statements which we have made in the preceding paragraph. For example, we observe from a study of equation (7) that as e, the distance between the lens and the eye, varies, the magnifying power varies: when this distance becomes such that $e - F_A = F_1 + \frac{(F_A - e)^2}{a}$, the denominator of the expression for the magnifying power becomes zero and $\frac{I_1}{I}$ equals infinity. If we solve the equation

$$e - F_A = F_1 + \frac{(F_A - e)^2}{a} \quad \text{.......................} (8)$$

we obtain the relation between $e - F_A$ and a, which exists when I_1/I is infinity. This relation becomes, by the solution of equation (8),

$$e - F_A = \frac{a}{2}\left(1 \pm \sqrt{1 - \frac{4F_1}{a}}\right) \quad \text{.................} (9)$$

Since the square root of a negative quantity cannot be actually extracted, the function $\frac{4F_1}{a}$ must be less than unity or equal to it. Hence the least value which a can have and satisfy the condition that $\frac{I_1}{I}$ = infinity is that $a = 4F_1$. When $a = 4F_1$, we find the corresponding value of $e - F_A$ to be $\frac{a}{2}$. We thus obtain a basis for the following rule relative to the effect of changing the position of the lens, to wit:—As a convex lens is removed from the eye, the magnifying power increases so long as the distance of the object from the lens is more than twice the focal length of the lens and when the distance between object and lens is less than twice the focal length of the lens the magnifying power is diminished by further removal of the lens from the eye. If, with a convex lens, $F_A - e$ is positive, which means that the lens is placed within the anterior focus of the eye, then I_1 will be less than I. If this lens is concave, i. e., F_1 is negative, our equation shows that when F_A is less than e, or when the lens is without the anterior focus of the eye, I_1 is less than I except when $a = e - F_A$; in this case, as with convex lenses, I_1 and I are equal.

448. *Some simple conclusions as to retinal images in ametropia.* The object and its image subtend equal angles at the nodal point, so that the image of an object which subtends a given angle at the nodal point depends on the distance the axial rays travel before the image is formed. If the latter is at the retina the size of the retinal image is to the size of the object as the distance between the nodal point and the retina is to the distance between the nodal point and the object.

Taking the nodal point of the reduced eye as 15 mms. from the retina and letting I represent the size of the image, O the size of the object and f_1 the distance of the object from the nodal point we have,

$$I = 15 \frac{O}{f_1} \qquad (10)$$

This formula gives us a simple means of calculating the approximate sizes of the retinal images in ametropia, always assumed to be axial however. For the number 15 mms. in equation (10) is assumed to be the distance from the nodal point to the retina in emmetropia and in ametropia it is known that 3 D. of axial error are equivalent to an increase of 1 mm. depth of the eye dependent upon whether it is myopia or hyperopia under consideration. In a hyperopic (axial) condition of 6 D. the constant would be 13 mms. and in myopia (axial) the constant would be 17 mms. and the ratio of the retinal images I_1 and I_2 would be expressed as

$$\frac{I_1}{I_2} = \frac{13 \frac{O}{f_1}}{17 \frac{O}{f_1}} = \frac{13}{17}.$$

Such calculations, as far as exactness is concerned, are of little utility, however. For in myopia, the image would be extremely blurred unless the object were at the punctum remotum and in hyperopia with accommodation relaxed it would be similarly blurred and if accommodation is in vogue the optical system is considerably changed; this is true also for a myopic eye accommodating within its far point.

Equation (4) may be written as

$$\frac{I_1}{I_2} = \frac{F_{A1}}{F_{A2}} \qquad (11)$$

in which the subscripts indicate the sizes of images or the anterior focal lengths in two conditions of the ocular system fixing the same point presumably at a large distance from the eye. We can, therefore, find the approximate values of F_{A1} and F_{A2}, the anterior focal lengths in two ametropic conditions, and hence find the relative sizes approximately of the retinal images. Taking the equations of the reduced eye we find that the relation between posterior focal length, F_B, anterior focal length, F_A, and indices of the media, $n_1 = 1$ and n_4 (assumed water) $= 1.33$, is given as

$$\frac{F_A}{F_B} = \frac{n_1}{n_4} = \frac{3}{4} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (12)$$

A double use of equation (12) will give the relation that

$$\frac{F_{B1}}{F_{B2}} = \frac{F_{A1}}{F_{A2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (13)$$

This equation, taken in conjunction with equation (11), gives the solution that

$$\frac{I_1}{I_2} = \frac{F_{A1}}{F_{A2}} = \frac{F_{B1}}{F_{B2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (14)$$

This is in accord with the statement given in conjunction with equation (10). If, for example, the posterior focal lengths in two cases

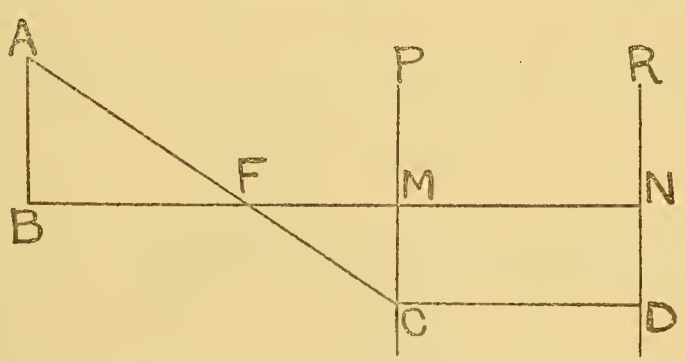


Fig. 216.—Illustrating the Method of Determining the Size of the Retinal Image.

are 18 mms. (6 D. of hyperopia) and 22 mms. (6 D. myopia), assuming 20 mms. as normal, we find that the ratio of the sizes of the retinal images according to equation (14) is 18/22.

449. The foregoing calculations are not, of course, strictly accurate. The formula for calculating the size of the retinal image deduced from the relation existing between the distance of the object to the nodal point and the nodal point to the retina is only true providing the object is at such a distance that no accommodation is exerted and the length from the cornea to the nodal point is so small as to be negligible in comparison with the object distance. When the object is brought sufficiently close these assumptions no longer hold, since either the image is no longer sharply formed at the retina or the image is formed sharply at the retina by means of accommodation. The focal lengths of the system in the latter case are shortened, the

nodal point is advanced and the distance of the image from the cornea is now a posterior conjugate focal distance and not a principal focal distance. The following demonstration includes the necessary corrective factors so that the final result, based upon the influence of the anterior focal length upon the size of the retinal image, is of general application. Let *BMN* be the principal axis of a reduced eye, in Fig. 216, of which *P* is the cornea or refracting plane and *R* the retina. Let *AB* be the object at a comparatively short distance such that accommodation is necessary in order to retain the image on the retina. A ray, *AC*, passing through the anterior focus, F, of the accommodated eye will proceed after refraction parallel to the principal axis and determine the size of the retinal image *ND*. Let the *distance of the object from the cornea* be f_1 and let f_2 be the posterior conjugate focus, *MN*, which is a fixed value in any particular case, since the image is to be formed at the retina. In order that *MN* may remain upon the retina, the value of *F*, the anterior focal length, can be found from the equation

$$\frac{1}{f_1} + \frac{\mu}{f_2} = \frac{1}{F} \qquad (15)$$

$$\text{or} \quad F = \frac{f_1 f_2}{\mu f_1 + f_2}$$

When $= 4/3$ and $f_2 = 20$ mms., we find that

$$F = \frac{20f_1}{4f_1/3 + 20} = \frac{15f_1}{15 + f_1}$$

$$\text{But} \quad \frac{MC}{AB} = \frac{FM}{BF} = \frac{F}{f_1 - F} \qquad (16)$$

and by substitution of the value of *F* in this last equation we have

$$ND = \text{retinal image size} = \frac{15 \times \text{size of object}}{f_1}.$$

This is strictly accurate for all values of f_1 *measured from the cornea.*

450. The equation (15), $\frac{1}{f_1} + \frac{\mu}{f_2} = \frac{1}{F}$, is of service in enabling us to calculate and compare the sizes of the retinal images when a near object is seen by an emmetropic, hyperopic or myopic eye: in each

instance the object viewed is to be taken close enough to the eye so that accommodation is involved and the assumption made that the image is in each case formed upon the retina. As an ilustration, let it be required to find the relative sizes of the retinal images when an object at 30 cms. is viewed by (a) an emmetropic eye, (b) an eye myopic 3 D. and (c) an eye hyperopic 3 D. Again assuming the emmetropic eye to have a posterior focal length, f_2, equal to 20 mms. and that 3 D. of refractive error are to be considered as indicating an axial change of one diopter, the equations are:—

(a) Emmetropia $\frac{1}{30} + \frac{4/3}{2} = \frac{1}{F}$ or $F_E = 1.43$ cm.

(b) Myopia $\frac{1}{30} + \frac{4}{6.3} = \frac{1}{F}$ or $F_M = 1.49$ cm.

(c) Hyperopia $\frac{1}{30} + \frac{4}{5.7} = \frac{1}{F}$ or $F_H = 1.36$ cm.

Referring to Fig. 216 and employing equation (16) we find that the retinal images, in order of their sizes, are:—myopia, $\frac{1.5}{28.5} \cdot O$, emmetropia $\frac{1.43}{28.57} \cdot O$ and hyperopia $\frac{1.36}{28.64} \cdot O$, in which O represents the size of the object. We thus see that if an eye which is myopic 3 D. and an emmetropic eye which is accommodated 3 D. observe an object at 33 cms. the retinal image is larger in the former condition. We likewise see the basis for the following statements: (a) in emmetropia, for near vision the image is larger with a convex lens than when accommodated; (b) in hyperopia, the image is larger with a convex lens than when accommodated and (c) in myopia, the image is smaller with a concave lens and accommodated than without the lens.

451. *Retinal images and binocular vision in anisometropia.* In this connection we shall discuss the accomplishment of vision in anisometropia when this inequality in refractive condition is myopic or hyperopic and shall reserve for subsequent consideration the rôle played by astigmatism in altering the forms of retinal images and the effects thereby produced in monocular and binocular vision. The ideal condition of a pair of eyes which are not refractively normal is known as isometropia: there is in a goodly proportion of persons,

however, an appreciable difference of refraction in the two eyes or what is specified as *anisometropia.* But anisometropia is to be considered a defect only when it is sufficient to cause some disturbance either visual or nervous. The least refractive difference which may be regarded as an anomaly varies with the degree of refractive error in the eyes. For example, if one eye is emmetropic while the other has 2 D. of myopia, there would be no hesitancy in classing this inequality as a defect capable of giving rise to very great disturbance, especially in binocular vision: but if one eye has 8 D. and the other 10 D. of myopia the same anisometropia (2 D.) is a subordinate factor. The chief reason why such an anisometropia can cause disturbance in the one case and not in the other should be apparent to the reader from the discussion which has preceded and dealt with the relative sizes of retinal images in various ametropic conditions, both at distant and near points, when accommodation may or may not be demanded. Certainly the sizes of the retinal images in the case of one eye emmetropic and its mate fairly myopic or hyperopic will be proportionately much more different and have much greater differences in sharpness of outline than in the case of two eyes, both highly myopic or hyperopic, and yet differing by one or two diopters, in which the sizes of the retinal images and their distinctness will not be appreciably varied the one from the other.

Vision in anisometropia may be accomplished in one of three ways. (1) There may be binocular vision: (2) vision may be monocular, either eye being used alternately, and (3) vision may be monocular to the exclusion of the other eye.

452. When it has been ascertained by means of the stereoscope or other test that an anisometrope possesses binocular vision, the question arises as to how such vision is accomplished in these cases. There are two possibilities: either by the exercise of a greater amount of accommodation in one eye than in the other or by the mental fusion of the clear image as formed in the adapted eye with the blurred image present in the other. The majority of practitioners and ocular experimentalists consider the premise that ocular adjustment in both eyes for an object lying within the range of accommodation of each eye may be attained by accommodating unequally for each eye as impossible. This theory of unequal accommodation has, however, received support from Fick, who cites a number of cases in evidence of his opinion that the refraction is equalized by unequal action of the ciliary muscles. He, in turn, has been refuted by Hess, who, from a number of experiments, concludes that there is no evidence in support of unequal accommodation. Fick, in his *Diseases of the Eye and Oph-*

thalmoscopy, cites the following case in support of his contention:— "The shadow-test disclosed (in a certain case) compound hyperopic astigmatism: the test letters showed this condition only in the left eye, while the right eye accepted a cylindrical but no spherical lens. I concluded that in the right and more acute eye there was latent hyperopia but in the left eye manifest hyperopia as well. Two doses of homatropin proved that my assumption was correct, for not only the left but the right eye also accepted a spherical lens—on the right a lens of + 3 D. If this clearly indicated unequal accommodation, it became a certainty when I had occasion eight days later to test the glasses prescribed by me. I examined the patient again with the following result: while the right eye, with a simple cylindrical lens, was fixing letters (D = 4) at 4 meters, the refractive condition of the left eye was determined by skiascopy; then the test letters were removed, and as the right eye was gazing into space, the left eye was again tested by skiascopy; in both cases the refractive condition of the left eye remained the same, that is, unchanged, while the refractive condition of the right eye had varied to the extent of 3 D." This problem seems to be similar to that of dynamic compensatory astigmatism. We do not have, however, good reasons for believing that the ciliary muscle of one eye can be innervated alone, or that when both muscles are innervated one can receive a greater impulse volitionally than the other. It is possible that Fick's results may be interpreted otherwise than as supporting a view of unequal accommodation; for a stimulation of the accommodation-center may give rise to greater actual contraction of the muscle in one eye than in the other. It is likewise possible that because of unequal sclerosis the same impulse may produce a greater change in the curvature of the lens in one eye than in the other. The writer believes that these two explanations are wholly adequate to explain these unequal accommodations.

453. Alternate vision in anisometropia generally occurs when one eye is emmetropic—or practically so—the other eye having a diopter or two of myopia (or hyperopia) and possessed of good visual acuity. Such an anisometrope enjoys a certain advantage in that his distant seeing is done with the emmetropic (or hyperopic) eye and his near work by the myopic eye.

In the majority of cases demanding the services of a refractionist the ametropia of one eye will differ slightly from that of its fellow. Appropriate corrections are ordered in such cases. It is equally logical to attempt to restore normal relationships when the dissimilarity is more marked. Such corrections cannot be tolerated by many persons; if, however, the patient is young and unhindered by latent squint the anisometropia should in general be totally neutralized for the patient will grow accustomed to the glasses after a short period of discomfort. The explanation of this intolerance and discomfort is found, in part, in the nerve disturbance occasioned when an eye which has previously acted solely in a subordinate capacity in vision is suddenly put in condition to co-operate with its mate and again, in part, in the secondary effects of lenses. For, if a correcting lens is worn in the anterior focal plane of the axially ametropic eye there will be formed an image equal in size to that obtained in emmetropia. If both eyes are properly corrected their retinal images should be of equal size. The disturbance cannot, therefore, be attributed in this case to unequal retinal images but rather to a change in ocular habits of seeing to which the person has become accustomed.

454. Convex lenses lead an observer to suppose that the object is

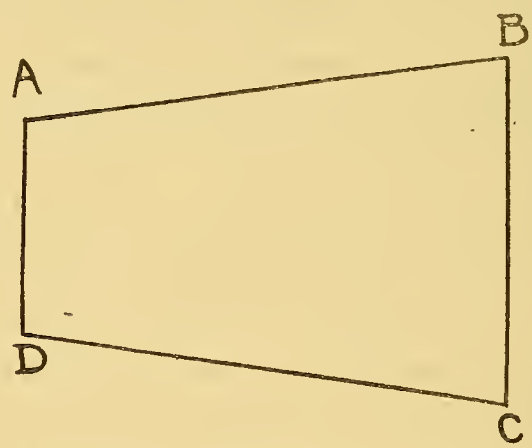


Fig. 217.—Apparent Alteration in the Size of an Object Produced by Anisometropia.

more remote than it is by reason of the diminution of accommodation required to see an object distinctly and consequently they make the object appear larger than it is as seen with the naked eye. This apparent alteration in the size of an object produces in anisometropia a one-sided disturbance. If a rectangular diagram is placed in front of, and equidistant from, the two eyes (assumed equal or, for example, emmetropic) and viewed binocularly with a convex lens before the right eye the rectangular shape of the object will be destroyed, for the right side will appear broader than the left as illustrated in Fig. 217. If a concave lens is used before the right eye, the right side of the diagram will appear smaller than the left. This disparity arises because of the fact that the right eye is chiefly concerned in looking at the right side of the object. The apparent alteration is due to a disturbance of accommodation; for the effort of accommodation demanded of the left eye in adjusting itself for the left side of the figure is greater than is demanded of, or is more than sufficient for, the right eye wearing a convex lens, consequently the impression arises that the right side is farther away and larger than the left side.

455. *Effects of cylindrical lenses upon the sizes of retinal images.* The formulæ which have been deduced for spherical lenses are also applicable to cylindrical corrections. The action of this latter class of lenses is confined to the meridian at right angles to the axis of the lens; hence the remarks made as to the effects of spherical lenses upon the size of retinal images apply also to cylinders with the restriction that the effect is confined to the refracting meridian since no effect is produced by a cylindrical lens in the meridian of its axis. Astigmatism in general is a curvature defect while hyperopia and myopia are largely axial defects. The effect of astigmatism upon retinal images

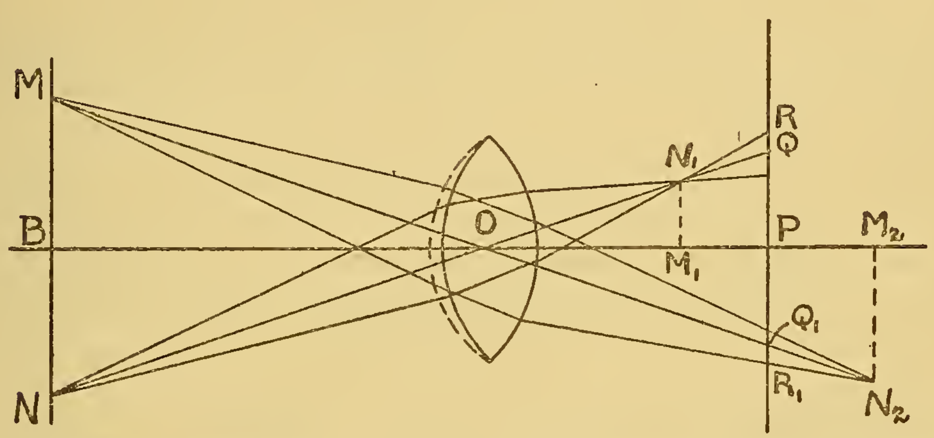


Fig. 218.—Effects of Astigmatic Errors Upon the Size and Position of the Retinal Images.

will, therefore, not be analogous to that of axial ametropia. For if an eye is hyperopic in one meridian and emmetropic in the meridian at right angles thereto, the defect in curvature in the hyperopic meridian is the same as though a concave cylinder were placed in contact with a normal cornea. Such a lens would be within the anterior focus of the eye and its effect would be an enlargement of images in the refracting meridian of the lens. Since the curvature of the eye is less in the hyperopic than in the emmetropic meridian, the anterior focal length is greater in the abnormal than in the normal meridian. Consequently the image of an object will be too large in the former meridian, for the size of the image is proportional to the anterior focal distance. In this same way we find that, in myopic astigmatism, the image is too small in the myopic meridian. Fig. 218 shows the effect of the faulty position of the retina, for it must be remembered that the retina is not in the proper position to receive an accurately focused image in the faulty meridian. Let B and P be conjugate points and hence the image of BN be at PQ. If now the retina remain at P while the lens is changed in curvature, and therefor power, so that M_1 and B are conjugates, then the true ocular image M_1N_1 will be smaller than PQ, but the indistinct image as received upon the retina will be larger as represented by PR. In a similar manner, if B and M_2 are conjugates, it is apparent that both the focussed and blurred images will be larger than normal but the blurred image will be less enlarged than the other.

456. We can now understand the influence of cylindrical lenses used for spectacle purposes upon retinal images. A properly selected convex cylindrical lens brings the image of an object to an accurate focus on the retina but this image is enlarged in the meridian of maximum power of the cylinder or in the meridian at right angles to the axis of the cylinder. For if the lens is placed at the anterior focus of the eye, the new image will be of the same size as M_2N_2 (Fig. 218) since the effect of the lens is to bring the image forward without changing its size. If the correcting lens is worn without the anterior focal point of the eye the new image will be larger than M_2N_2. In either of the specified lens positions, therefore, the retinal image will be larger than the blurred image PR_1 which is received upon the retina when no correcting lens is worn and larger than $PQ_1 = PQ$, the normal image. A properly selected concave cylinder, in turn, throws the image back upon the retina and causes a decrease in the size of the image in the refracting meridian of the lens. If the correcting concave cylinder is worn at the anterior focal point of the eye, M_1N_1 (Fig. 218) will represent the size of the new image. Under any conditions, practically, of lens position the retinal image will be smaller than the blurred image PR_1 which the eye receives without lenses and smaller than PQ, the normal image. We come, therefore, to the general conclusion that cylindrical lenses, worn as spectacles, do not under any circumstances produce normal retinal images, for all objects are magnified in the refracting meridian of a convex lens and minified in this meridian by a concave cylindrical glass. If the correcting cylinder could be worn in contact with the cornea, the seat as a general rule of defective curvature, normal images would result.

In Fig. 219, let A represent a rectangular object. Then if it be viewed through a spherical lens held *beyond its focal length* from the eye, B will represent the appearance of the object. If a cylindrical lens is used with axis vertical, C will represent the object as it will appear to the observer; the cylindrical lens has the same effect as the spherical one in deviating the rays in the meridian at right angles to the axis of the lens, i. e., rays from the right of the object are made to cross over and intersect on the left and vice versa. The object is therefore reversed in this direction but is not so affected in the meridian parallel to the axis of the lens for in this meridian the rays are not deviated by the lens. In a like manner *D* represents the object

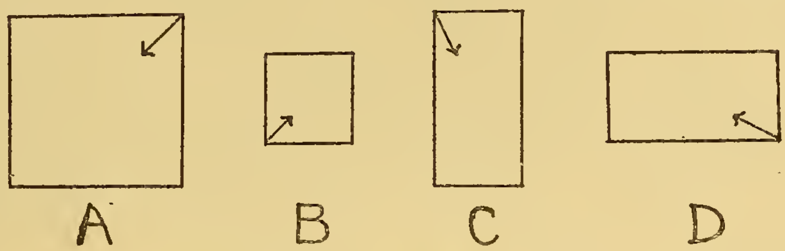


Fig. 219.—Rectangular Object.

A, As seen normally; B, as seen through a spherical lens held beyond its focal length; C, as seen through a cylindrical lens similarly placed with axis vertical and D, as viewed through a cylindrical lens with axis horizontal and similarly placed.

as it would appear when viewed through a cylindrical lens with axis horizontal.

457. *The twisting properties of cylindrical lenses and ocular astigmia.* In view of the preceding considerations as to the effects of cylindrical lenses upon retinal images it follows that if a person holds

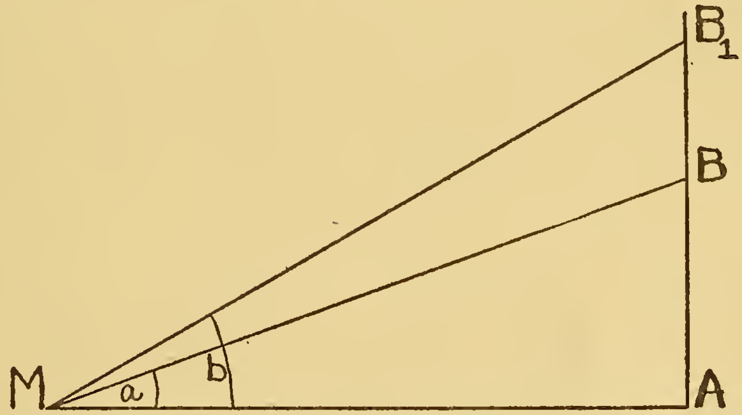


Fig. 220.—Illustrative of the Twisting Properties of a Cylinder.

such a lens in front of the eye and views a distant rectangular object—such as a window-frame or picture—through it there will be formed a distortion of the object which will change with every variation of the position of the lens. If the axis of the cylinder is parallel to one of the sides of the object the rectangular form of the object will remain but the ratio of the sides will be altered (see Fig. 219). If the lens is rotated in its own plane, the distortion will not now be confined simply to the apparent size of the object but will also affect the direction of the lines forming the sides so that the rectangular object will assume the form of an oblique parallelogram.

Let *MA,* Fig. 220, be a line parallel to the axis of a cylindrical lens and let *AB* be perpendicular to its axis. The retinal image of the line *MA* will be the same with or without the cylinder. If one looks at the line *AB* and if the lens is convex, then *AB* will be magnified and will appear, for example, as AB_1. If, therefore, any oblique line, *MB,* is looked at, its direction will be changed and it will assume the position MB_1. Hence, any line not parallel or perpendicular to the axis of a cylindrical lens undergoes an angular deviation when viewed through such a lens. If *a* is the angle which the line makes with the axis and *b* is the angle which the line apparently makes with the axis, we may write

$$\frac{\tan b}{\tan a} = \frac{AB_1}{AB} = m,$$

in which *m* represents the magnifying power of the lens. When the lens is a concave cylinder, *m* is less than unity and the line MB_1 appears to be in the position *MB.*

458. We have seen that retinal images in astigmatic eyes are not normal in their proportions; we have seen also that the effect of astigmatism upon retinal images is analogous to that of cylindrical lenses placed in contact with a normal cornea. A cylindrical lens thus placed would have a magnifying or minimizing action on images in the direction of the refracting meridian of the cylinder; it is upon this property that the apparent deviation of lines depends. It is, therefore, clear that in astigmatic eyes all lines not parallel or perpendicular to the axis of the astigmatism are twisted out of their proper relations. A rectangle whose sides do not correspond in direction with the meridians of greatest and least refraction appears as an oblique parallelogram. Since the dioptric power of the eye is relatively great in comparison with the amount of astigmatism, however, the distortion is small. The defect is not appreciable to a person whose eyes are astigmatic, even in astigmia of high degree; when, however, the astigmatism is corrected by a suitable lens complaint is frequently made of annoying distortion of lines (metamorphopsia). In general this disturbance is transient: since a cylindrical lens as worn before the eyes cannot reduce the retinal image to its proper proportions it is evident that it cannot correct the distortion of lines and it is easy to see why annoyance should arise when glasses are first worn. (See article by J. A. Lippincott on ''Binocular Meta-

morphopsia Produced by Optical Means,'' *Archives of Ophthalmology,* 1917.)

459. G. C. Savage, in his books on *Ophthalmic Myology* and *Ophthalmic Neuro-Myology* as well as his ''Oblique Astigmatism,'' (*The American Encyclopedia of Ophthalmology,* Vol. XI) has dealt with the subject of retinal images produced under various conditions of astigmia and has shown their influence upon fusion and binocular vision in a most delightfully analytical and scientific manner. We shall quote, in the succeeding paragraphs, quite freely from his writings: at least many of the essential ideas herein involved are taken from this writer.

460. In emmetropia, hyperopia and myopia there is no displacement of the images and, therefore, the law of corresponding retinal points is satisfied when the oblique muscles obey the subordinate law governing them, i. e., when they parallel the vertical axes with the median plane of the head. In vertical and horizontal astigmatism there is no displacement of images of vertical and horizontal lines but images of oblique lines are displaced: this displacement is in the same direction and to the same extent in the two eyes, hence the law of corresponding retinal points can be satisfied only when the law governing the obliques is satisfied and obeyed. In oblique astigmatism, however, in which the meridians of greatest curvature either diverge or converge above, the images of vertical and horizontal lines are displaced so that they no longer bear a proper relationship to the lines themselves; hence the images must fall on non-corresponding retinal points or, more properly, non-corresponding lines. As a result, in such eyes no line in space can have both images properly related to it, for a line that would be parallel with the meridian of greatest curvature of one eye would not be parallel with the meridian of greatest curvature of the other. Hence, the two images of any line cannot fall on corresponding retinal parts when in oblique astigmatism the meridians of greatest curvature are not parallel. In order, therefore, to harmonize these images and to satisfy as perfectly as possible the law of corresponding retinal points, ''the individual law governing the obliques must be suspended and the vertical axes of the eyes must be made either to converge or diverge above,—the former if the meridians of greatest curvature diverge above, the latter if these meridians converge above. The same is true when the principal meridians of one eye are vertical and horizontal, while those of the other are oblique.''

In astigmatism there is displacement of the images of all lines not parallel with the one or the other of the two principal meridians. The obliquity of retinal images was first demonstrated in 1890 by Savage and his collaborator Lowry by the production of artificial oblique astigmatism and at that time the following important law was formulated: *"The retinal image is displaced toward the meridian of greatest curvature."* "This being true—and there is no exception to the rule—the image of a vertical or horizontal line is displaced toward the meridian of best curvature in oblique hyperopic astigmatism, from the best meridian in oblique myopic astigmatism and toward the myopic meridian in oblique mixed astigmatism."

461. Fig. 221 shows a square as seen by a non-astigmatic eye, as seen by an eye astigmatic according to the rule and as seen by the latter after the astigmatism has been corrected by a plus cylinder.

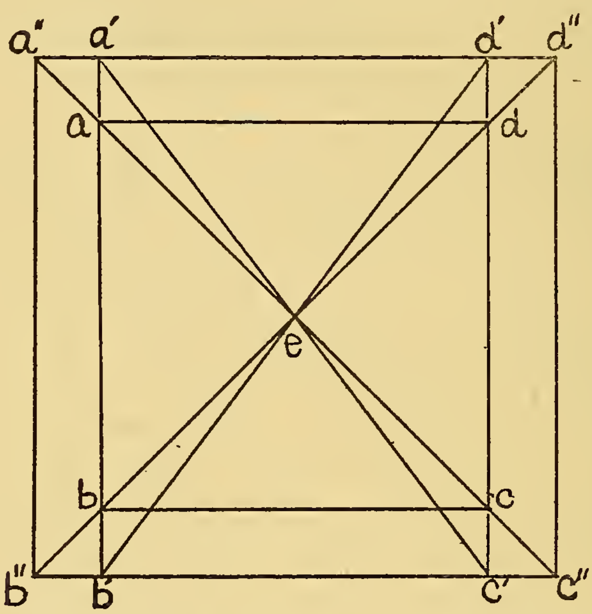


Fig. 221.—Representing a Square as Seen by (a) Non-Astigmatic Eye; (b) an Eye Astigmatic According to the Rule, and (c) as Seen by the Latter After Correction by a Plus Cylinder. (After Savage.)

The rectangle *abcd* is the square seen by a non-astigmatic eye and *ac* and *db* represent the diagonals. The rectangle *a'b'c'd'* is the square as seen by the astigmatic eye with the meridian of greatest curvature vertical. The refraction of the axial rays from *a* and *b* by the astigmatic cornea is such as to make them cross each other on the way back to the retina sooner than they would have in the absence of astigmia: hence their points of impingement on the retina are more widely separated and the line itself is proportionately increased. Therefore the line *ab* must become the line *a'b'* and the line *dc* the line *d'c'* and, since *ad* and *bc* are not affected in size, the original square becomes a rectangular parallelogram *a'b'c'd'*. The diagonal *ac* has been rotated toward the vertical and become *a'c'*, while the diagonal *db* has been rotated in the opposite direction, but also toward the vertical, and becomes *d′b′*. *They have both been rotated by the refraction of the astigmatic cornea toward the meridian of greatest curvature.* The image changes, therefore, effected by the astigmatic cornea are: (a) an increase in the length of the lines parallel with the meridian of greatest curvature, (b) an increase in the distance between the lines parallel with the meridian of least curvature and (c) a corresponding rotation of the diagonal toward the meridian of greatest curvature. The giving of the proper lens, i. e., plus cylinder, to this eye affords such aid as to make its refractive power in the least curved meridian equal to the unaided refractive power of the meridian of greatest

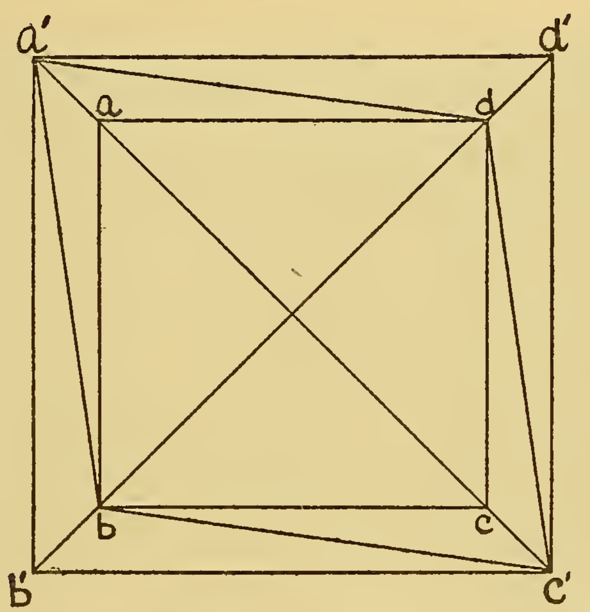


Fig. 222.—Representing the Image Changes when the Astigmatism is Oblique, the Meridian of Greatest Curvature Being at 135 Degrees. (After Savage.)

curvature. The result will be a lengthening of the horizontal lines *a′d′* and *b′c′* into the lines *a″d″* and *b″c″* and a displacement of the lines *a′b′* and *d′c′* until they become *a″b″* and *d″c″*. The final figure *a″b″c″d″* as seen by the corrected astigmatic eye is a square. The cylinders, by changing the rectangular parallelogram *a′b′c′d′* to the square *a″b″c″d″*, has also rotated the diagonals *a′c′* and *d′b′* back to their original positions.

462. Fig. 222 shows the image changes when the astigmatism is oblique, the meridian of greatest curvature being at 135°. That portion of the diagram lettered *abcd* shows an original square for a non-astigmatic eye. In an obliquely astigmatic eye the diagonal *ac*, being at an angle of 135°, is in the plane with the meridian of greatest curvature, while the diagonal *db* is in the plane with the meridian of

least curvature. The diagonal *ac* is increased in length by the astigmatism into *a′c′* for reasons already given, while the diagonal *db* is neither altered in size nor direction. The sides of the square, not being parallel with the principal meridians, must be rotated toward the meridian of greatest curvature, *ab* becoming *a′b*, *ad* becoming *a′d*, *bc* becoming *bc′* and *dc* becoming *dc′*. The figure *a′b′c′d* is a non-rectangular parallelogram which leans down and to the right. A plus cylinder correcting the astigmatism will increase the length of the diagonal *db* into *d′b′* to the exact length of the diagonal *a′c′* and at the same time will so rotate the sides *a′b*, *a′d* and so forth as to convert the non-rectangular parallelogram *a′bc′d* into the magnified square *a′b′c′d′*.

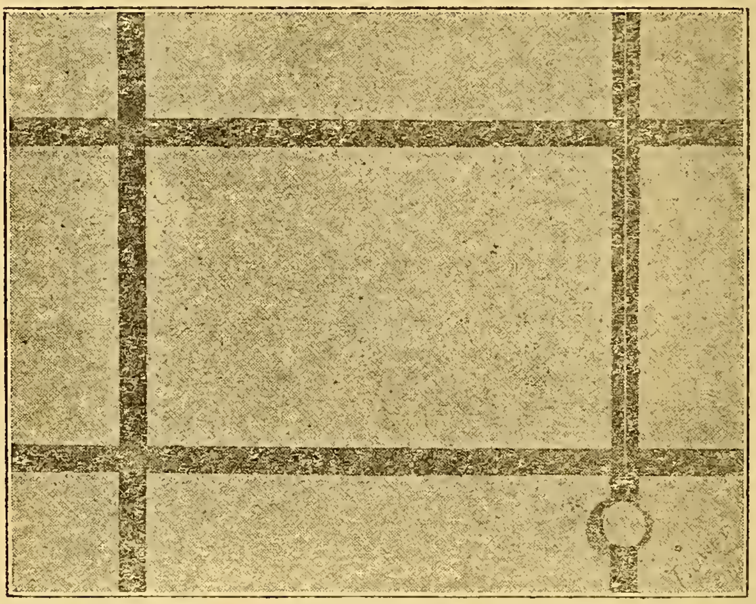


Fig. 223.—Photograph of Rectangular Frame through High Grade Camera Lens. (After Savage and Lowry.)

463. It is evident that, if the astigmatism is equal and of the same kind in the two eyes, the meridians of greatest curvature being parallel, though oblique, the two images of a square held vertically will be distorted alike and hence can be fused readily and completely. If the meridian of greatest curvature in the right eye is at 135° and in the left eye at 45°, the image in each eye will be a parallelogram leaning in the opposite direction from the image in the other eye. The two cannot be perfectly fused though an attempt at fusion will be made in an effort on the part of the eyes to obey the law of corresponding retinal points which is the supreme law of binocular single vision. The fusion of two images in astigmia at axes 135° and 45° respectively, would give an objective square the appearance of a

trapezoid with the longer base uppermost. The fusion would be effected by the superior obliques converging the vertical axes of the eyes.

464. The obliquity of retinal images can be demonstrated by the results obtained with a camera. If the camera is properly focussed we have a representative emmetropic eye. By placing a concave cylinder in apposition to the camera lens an artificial hyperopic astigmatism is created and the effects upon the images can be investigated when the astigmatism is "with" or "against" the rule or oblique. Lowry carried out such a series of investigations and they are reproduced in Savage's writings. Fig. 223 shows that there is

Fig. 224.—Photograph of Rectangular Frame Through Camera Lens Carrying in Front of Itself a Cylinder Producing Simple Vertical Hyperopic Astigmatism. (After Savage and Lowry.)

obtained a perfect rectangle, sharp and distinct, such as would be seen by an emmetropic eye, when the focus is accurately adjusted.

465. In Fig. 224 the axis of a minus 3 D. cylinder was placed at 90° in apposition to the camera lens, hence producing simple vertical hypermetropic astigmatism. The meridian of greatest curvature is at 90° and of least curvature is at 180°. The reproduced photograph shows a perfect rectangle with its horizontal line sharply defined and the vertical very indistinct.

466. If, however, there is placed before the camera a cylinder of about 3 D. power with its axis at 135° and in addition a + 1.50 D. sphere—in order to give the middle of the focal interval without changing the focus of the camera—a non-rectangular parallelogram is formed as shown in Fig. 225. Every point is equally indistinct and

nowhere are the lines at right angles as in the original. The vertical lines deviate to the right at the top and to the left at the bottom, while

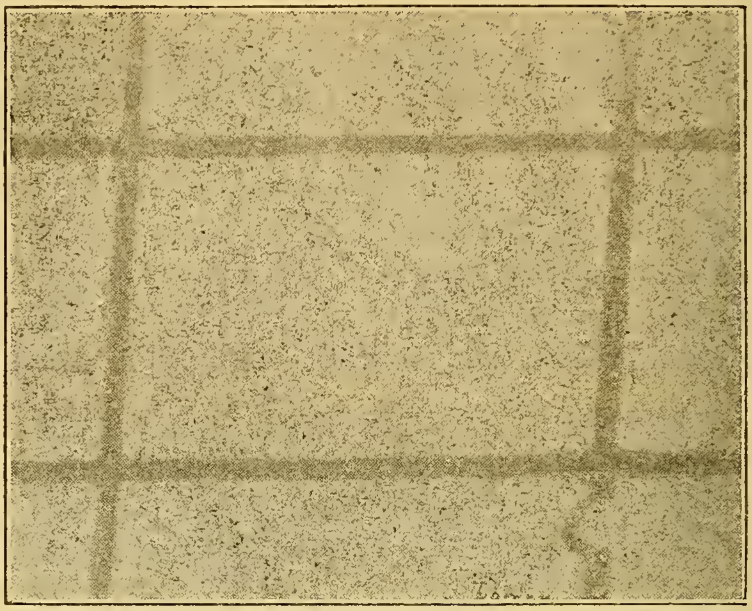


Fig. 225.—Photograph of Rectangular Frame Through a Camera Lens Carrying in Front of Itself a Cylinder at Axis 135 Degrees. (After Savage and Lowry.)

the horizontal lines are elevated at the right and depressed at the left. (Note:—These points are all the reverse of the images, therefore the reverse of the object as it would be seen.)

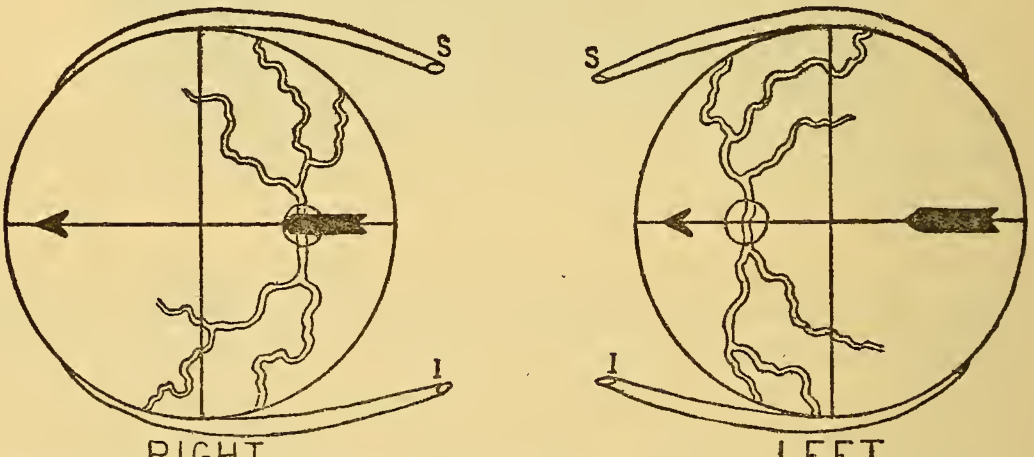


Fig. 226.—Representative of a Pair of Eyes in which the Two Principal Meridians are Vertical and Horizontal. (After Savage.)

Any reader may at his pleasure test out these monocular effects as well as those to be briefly considered from the binocular single vision viewpoint by making himself astigmatic by the addition of a 4 or 5 diopter cylinder (either plus or minus) and viewing a rectangular card through the cylinder placed at various angles.

467. The degree of the displacement or distortion (*Fr.* dénivellation) is proportional to the degree of astigmatism and to the inclination of the principal meridians with respect to the object fixed. Fig. 226 represents the retinal image of a horizontal arrow formed by a pair of non-astigmatic eyes or by eyes having astigmatism in vertical—horizontal meridians. In either case there will be no displacement of

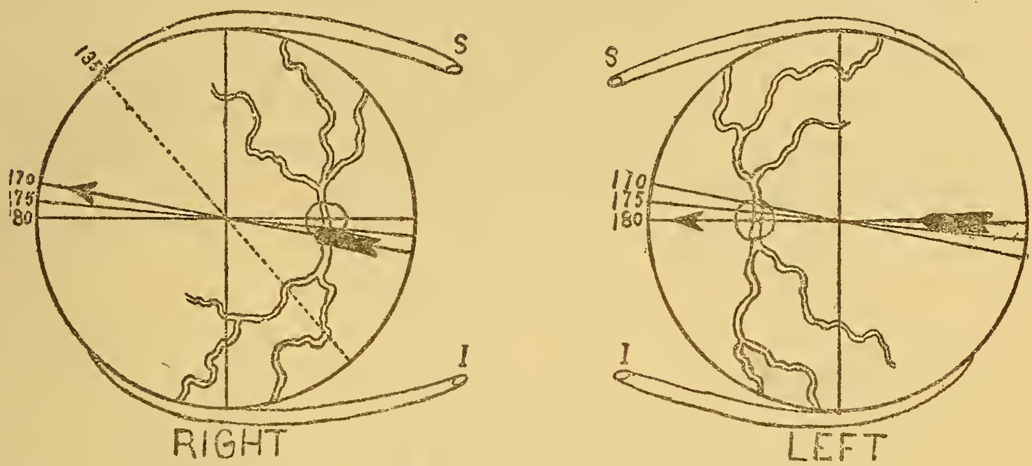


Fig. 227.—Representative of a Pair of Eyes.

Left eye has its best meridian at 90° while the right eye has its best meridian at 135° as shown by the dotted line. (After Savage.)

vertical or horizontal objects. For in either anastigmatic eyes or those having their principal meridians vertical and horizontal all points of an object situated in the plane of the principal meridians will have their images formed in the same plane.

468. Fig. 227 "represents a pair of eyes in which there is hyperopic astigmatism, either simple or compound. The left eye has its best

Fig. 228.—Unaided Eyes Diagrammed in Figure 227 Would See the Arrow Double as Illustrated Above.

meridian vertical. In this eye, the arrow, held as before, throws its image on the horizontal meridian of the retina, hence in the same plane with it. In the right eye the best meridian is at 135°, as shown by the dotted line. In obedience to the well known law of refraction by curved surfaces, the image of the same arrow must be oblique in this eye, and hence not in the same plane with the object. The obliquity of the image will be greater or less, depending on the quantity of the astigmatism. It is represented as falling on the meridian

31

170° of the retina. The horizontal image in the left eye and the oblique image in the right eye do not fall on parts of the two retinas that harmonize. The direction of either image in relation to the other cannot be changed except by artificial means—a proper cylindrical lens. This being true, the pair of unaided astigmatic eyes, represented by Fig. 227, must see the arrow double as shown in Fig. 228 unless something is done for the purpose of harmonizing the images."—(*Savage.*)

469. For an inclination of the astigmatic axis of 45° the angle of displacement—i. e., the angle which is formed between the linear object and its image—is practically at its maximum. This angle can be obtained from the formula

$$\tan \theta = \tan a \frac{\frac{1}{h} - \frac{1}{v}}{\frac{1}{z} + \frac{1}{v} + \left(\frac{1}{z} + \frac{1}{h}\right) \tan^2 a}$$

in which the symbols have the following significances:

a = angle which the linear object makes with the principal (horizontal) meridian H having a radius of curvature h.

v = radius of curvature of the vertical meridian, V.

z = distance of the object from the reflecting and refracting surface.

θ = angle which the image of the linear object makes with the direction of the linear object itself.

By assuming that the line object is at infinity, 1/z becomes zero and the above formula takes the following simplified form:

$$\tan \theta = \tan a \frac{\left(\frac{1}{h} - \frac{1}{v}\right)}{\frac{1}{v} + \frac{1}{h} \tan^2 a}$$

In this expression h and v being the radii of curvature in the two principal meridians, $\frac{1}{h}$ and $\frac{1}{v}$ represent the refraction of these meridians expressed in diopters. Designating R_h as the refraction of the meridian of lesser curvature and R_v as the refraction of the meridian of greater curvature, we can write

$$\tan \theta = \tan a \frac{R_v - R_h}{R_v + R_h \tan^2 a}.$$

By assigning to R_h the value of 60 diopters and to R_v, in succession, values of 61, 62, 63, 64 and 65 diopters and to a (angle of astigmatic inclination) successive values of 0°, 15°, 30° and 45° we obtain the following table:

Degree of Astigmatism	*Inclination (a)* (Inclination of object with reference to principal meridian of least curvature)	*Angle (θ)* (Angle formed between the linear object and its retinal image)
1 diopter	0°	0° 0′ 0″
	15°	0° 14′ 8″
	30°	0° 24′ 32″
	45°	0° 28′ 25″
2 diopters	0°	0° 0′ 0″
	15°	0° 27′ 47″
	30°	0° 46′ 28″
	45°	0° 56′ 26″
3 diopters	0°	0° 0′ 0″
	15°	0° 41′ 4″
	30°	1° 11′ 44″
	45°	1° 23′ 50″
4 diopters	0°	0° 0′ 0″
	15°	0° 49′ 40″
	30°	1° 34′ 30″
	45°	1° 50′ 51″
5 diopters	0°	0° 0′ 0″
	15°	1° 6′ 28″
	30°	1° 56′ 36″
	45°	2° 17′ 26″

(Note:—The formulæ quoted above and the accompanying table are taken from the essay entitled Astigmie, by D. E. Sulzer, in Volume 3, *Enclycopédie française d'Ophthalmologie.*)

470. Let us consider for a moment the case of a pair of eyes each having an astigmatism of five diopters, the two principal meridians of minimum curvatures being inclined at 45° on the temporal side in each case (i. e., O. D. 135°, O. S. 45°). An image of a horizontal line formed upon the right retina will make an angle of 2° 17′ 26″ with the object in a certain direction (i. e., either above or below it), while the image formed of the same straight line by the left eye will make an angle of equal amount but in the opposite direction. The two images will, therefore, form an angle of 4° 35′ between themselves. By a superposition of the two images so that they fall on identical

points of the retinas, binocular single vision will ensue. The question arises as to how this can be accomplished. There are, according to Savage, but two ways of accounting for the absence of double vision in such cases as the one detailed above or diagrammed in Fig. 227. Sectional ciliary contraction would account for it; but experimentation shows that, when all ciliary power has been suspended by atropine or by age, the eyes are still able to do something by means of which the double vision is prevented. Such eyes must, therefore, execute rotations about the antero-posterior axes: it is sufficient that the principal meridians of the two eyes shall be parallel, for then all corresponding parts of the two retinal images will fall on identical points of the two retinas. Demands must, therefore, be made upon and met, in large measure, by the oblique muscles in binocular single vision.

In concluding this rather abbreviated presentation on this very important topic we are pleased to quote the following from Savage, together with two plates taken from his *Ophthalmic Myology.*

471. Fig. 229 may be taken for study. "Both eyes have oblique astigmatism of the same kind and quantity. In the right eye the meridian of greatest curvature is at 135° and in the left at 45°. If a rectangular figure be presented to the eyes represented in Fig. 229 it would not be seen with one eye alone or with both together as a rectangle. The rectangle, when held before the right eye in Fig. 229, instead of throwing a rectangular, would throw a non-rectangular, parallelogram image on the right retina; the same rectangle would also throw a non-rectangular parallelogram image on the left retina. The state of refraction of the right eye would make the distorted image lean down and toward the left side, while the distorted image in the left eye would lean down and toward the right side. Cutting off the view of the left eye, the law of direction would have full sway, while the law of corresponding points would be suspended. Since in one eye alone the law of direction is unalterable, all lines of direction must cross in the center of retinal curvature; and the right eye, with the parallelogram image leaning down and to the left, must see the figure casting the image, not as a rectangle, but as a parallelogram leaning down and to the left. Screening the right eye while the left eye looks on the rectangle, it is seen, not as a rectangle, but as a parallelogram leaning down and to the right, the law of direction determining the shape of the figure seen by the left eye, just as it fixed the shape of the figure seen by the right eye. Diagram 1 — 2 — 3 — 4 is what is seen with the right eye alone; diagram 1′ — 2′ — 3′ — 4′ is what is seen by the left eye alone. The moment these two eyes are allowed to look at the rectangular figure, the law of correspond-

ing retinal points is brought into conflict with the law of direction, and the latter is modified by the former. There is no necessity for

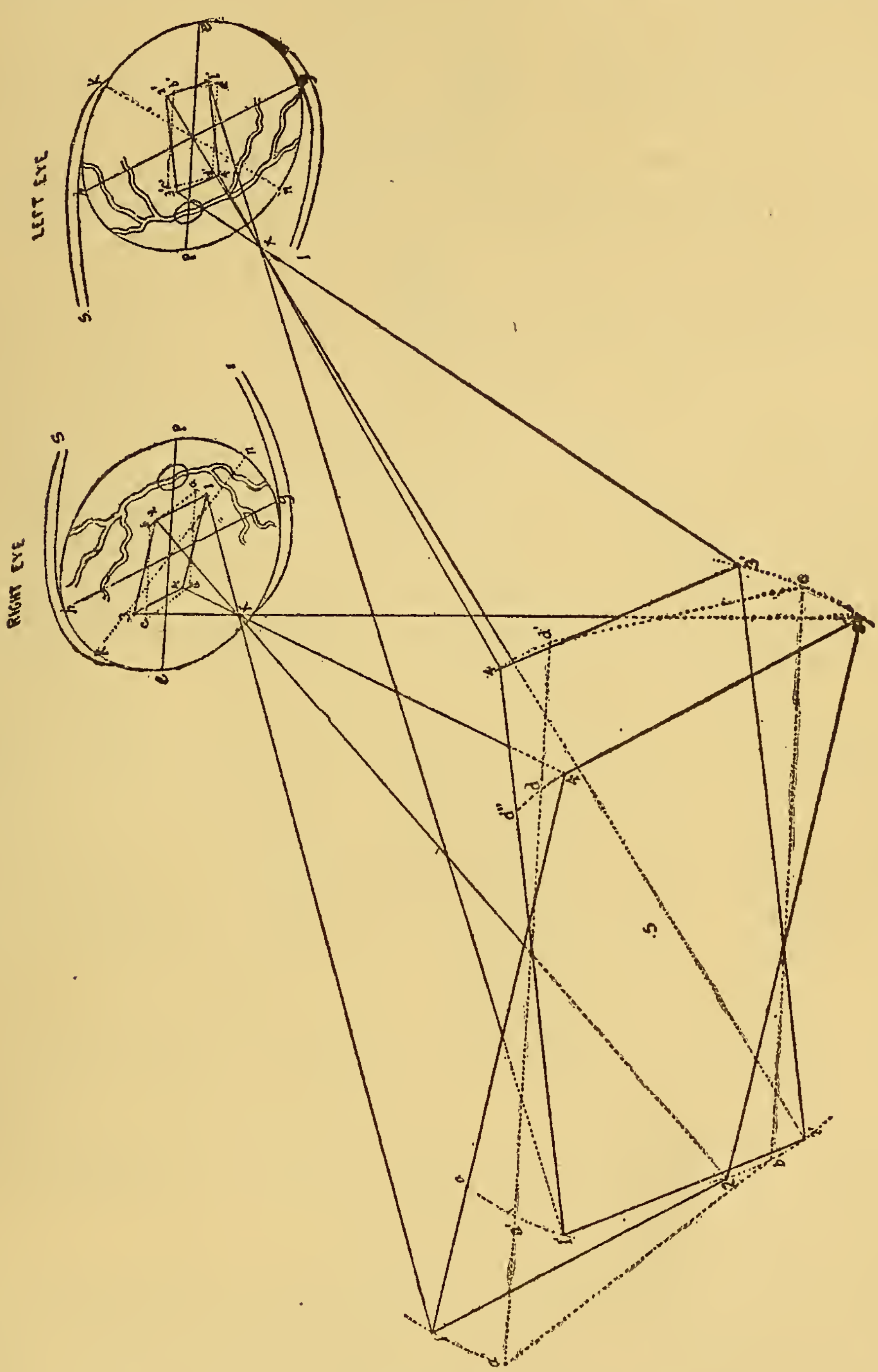


Fig. 229.—Showing the Retinal Images in Non-parallel Oblique Astigmatism. (After Savage.)

changing the visual axes when looking at the rectangle with these two eyes; but, unless some change is effected in some way, each eye would see its own parallelogram leaning down and toward the opposite side. Instantly a change does take place in both eyes, so that the two see together, not a rectangle nor a parallelogram, but a trapezoid with the longer side above. A clear understanding of what this change is and how it is effected may be had by a further study of Fig. 229. In the right eye is shown a dotted parallelogram *abcd* of precisely the same form as the parallelogram image 1 — 2 — 3 — 4: but in the former the upper and lower lines are parallel with the horizontal meridian. In the left eye also is shown a dotted parallelogram *a'b'c'd'* of the same form as the parallelogram 1′ — 2′ — 3′ — 4′, with its upper and lower lines parallel with the horizontal meridian of this eye. The line *cb* in the right eye bears throughout the same relation to the macula, the horizontal and vertical meridians of this eye, that the line *c'b'* does to the same parts of the left eye, and they, therefore, correspond. The greater part of the line *da* in the right eye also corresponds with the greater part of the line *d'a'* in the left eye, the parts of these lines not corresponding being their extremities. But the line *cd* in the right eye nowhere corresponds with the line *c'd'* in the left eye, except at the points of beginning above; and the same is true of lines *ba* and *b'a'*, in their respective eyes. If the dotted parallelograms could be made to coincide with the parallelogram images, the result would be that the two eyes together would see the figure *abcd'*, a trapezoid, with the longer side above. How this is effected is shown in Fig. 230, where each eye has been revolved on its visual axis by its superior oblique muscle, so that the horizontal meridian is made parallel with the upper and lower borders of the parallelogram image; and thus, as far as possible, corresponding parts of the two retinas are brought under the two dissimilar images, and the figure seen binocularly is *abcd'*. The part of the trapezoid seen in common by the two eyes is *a'bcd*, the part seen by the right eye alone is *aba'*, and that seen by the left eye alone is *dcd'*. As will be seen, the law of corresponding points has so modified the law of projection that the visual lines no longer have a common crossing point. This is anarchy, so far as projection is concerned, in these eyes."

472. "When the law of direction is interfered with, as a result of the conflict between it and the more imperious law of corresponding retinal points, the object seen is always in the position that it would have been in, had the images primarily fallen on the parts of the two retinas that have been rotated under them, in obedience to the supreme law of binocular single vision—the law of corresponding retinal points. The displaced images, as a result of either natural or artificial means, cover areas of the two retinas that do not correspond. In order to have binocular single vision, retinal areas that more nearly correspond, and are of the same shape and size as the images, must be brought

under them. The object will be seen as though no rotation had taken place, as if the images had primarily fallen on these parts, in perfect

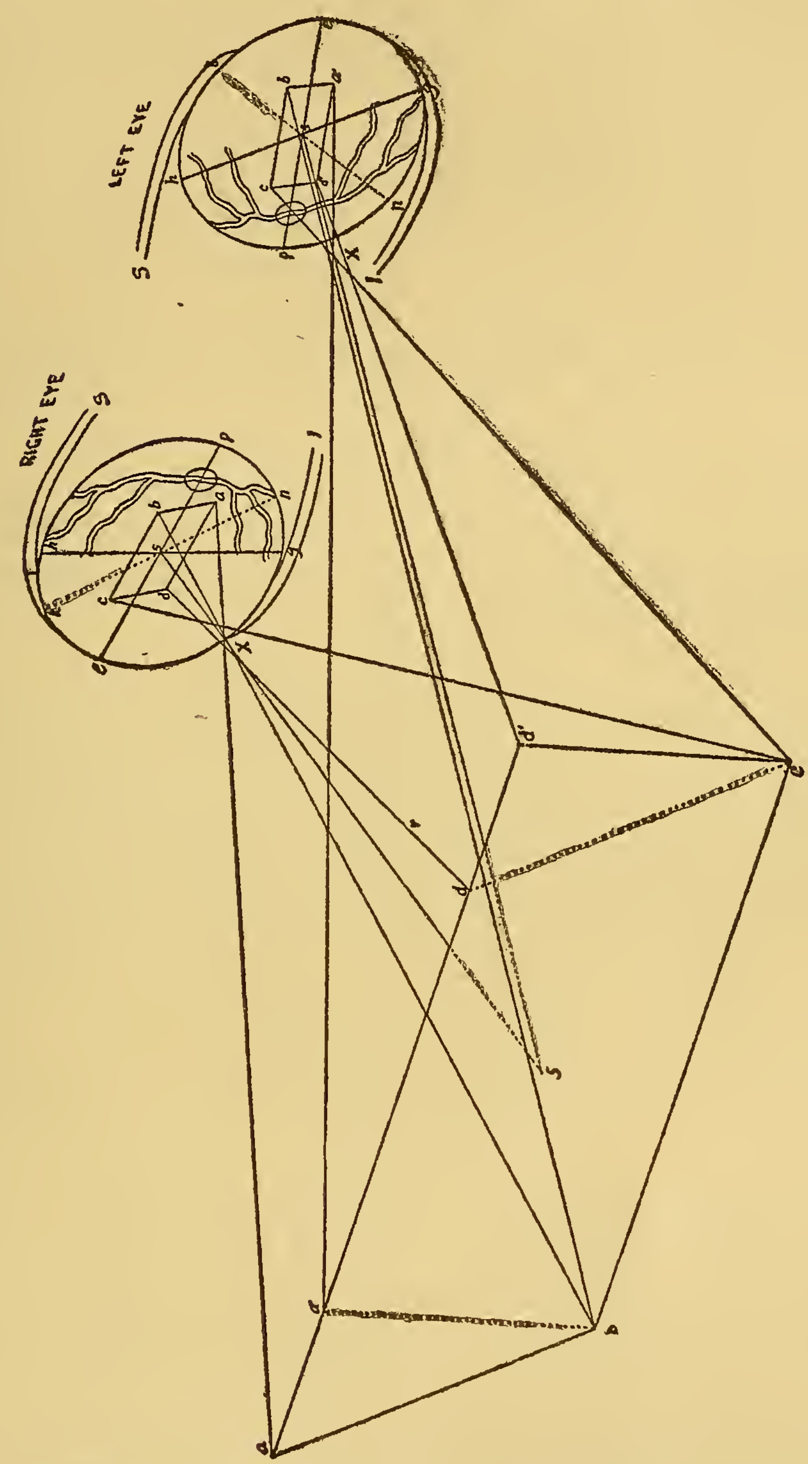


Fig. 230.—The Retinal Images in Certain Instances of Non-parallel Oblique Astigmatism. (After Savage.)

obedience to the law of projection, although the lines of direction drawn from the images to the single object will not cross at the center of retinal curvature. In cases of decentration of the maculas, and in displaced images by means of prisms, all lines of direction will cross at one point, but that point will be above, below, to the outer or inner side of the true point; while in oblique astigmatism, and when the axes of correcting cylinders are displaced, no three lines of direction cross at the same point.''

473. ''Imperfect as is binocular single vision in uncorrected oblique astigmatism, the meridians of greatest curvature either diverging or converging above, it could be effected in no other way than by a revolution of the eyes by the symmetric harmonious action of the oblique muscles. It is true that Nature has one other method of preventing diplopia—namely, mental suppression of one of the displaced images. It may be that amblyopia resulting from oblique astigmatism high in degree, and from insufficiency of the obliques, is more common than one would at first think. Certainly, if the obliques cannot do their proper work in effecting binocular single vision, in the first years of life, nothing is more reasonable than to suppose that amblopia *ex anopsia* would develop.''

XXVIII. CHIASMAL IMAGES

474. In 1914 C. F. Prentice, M. E., presented in the pages of *The Ophthalmic Record* a paper entitled The Prism-dioptry Establishes a Dimensional Unit at the Optic Chiasm. In this essay he postulates the hypothesis of a *chiasmal image* which, as Casey Wood says, ''is new and if accepted will be a real addition to the subject, but some of us will find it difficult to accept it, not only because it is difficult to conceive of an image anywhere obtained except at the primary sensitive-plate of the retina, where it is (physically) formed, or within the central area about the calcarine fissure where it is (psychically) interpreted: . . . but we know that there are other neuronic points and areas within the cranium that greatly influence the character of the visual image, and as there is no valid reason why one should conceive of image-formation at any point along the optic radiational lines, the concept of a conjoint image within the chiasma is not only a thinkable but a useful idea.'' No better or briefer presentation of this subject can be made than that given by Prentice. (See *The American Encyclopedia of Ophthalmology,* Vol. III, pages 2047-2055; Vol. VIII, pages 6170-6172, and Vol. X, pages 7296-7302.

475. ''In *physiologic optics,* the chiasmal image is a strictly figurative image consisting of that orderly assemblage of the optic nerve fibrils, within the cross sectional and comparatively small area of the optic chiasm, which receive their individual stimuli from corresponding points in each retinal image. Of course, this also implies that the

supposed chiasmal image should be proportionately smaller than the retinal images, in order that they may appear in their entirety within the more circumscribed area of the chiasm. However, this may remain a matter of conjecture, and is quite immaterial, when merely comparing the positions of the images projected from the retinæ into the parallel area of the optic commissure, and that may or may not there constitute a *single* chiasmal image of the size conceived by the brain,

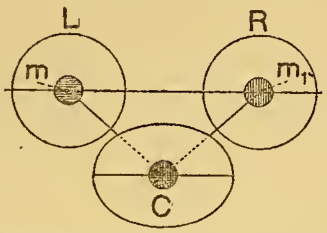


Fig. 231.—Chiasmal Image. (After Prentice.)

in either orthophoric (orthoscopic) or heterophoric vision, respectively. Therefore, the *figurative* chiasmal image may be safely accepted, since it, at least, makes it easy to produce diagrams in which the retinal images and the image conceived by the brain are separately pictured to illustrate the phenomenon of binocular vision. With this conception of the chiasmal image, orthoscopic binocular vision may be said to require absolute equality in the dimensions of the retinal images, in order that these identical images, when conveyed by the optic nerves,

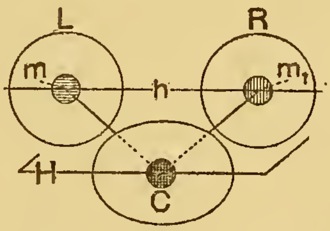


Fig. 232.—Chiasmal Image. (After Prentice.)

may exactly cover each other at the optic chiasm. In other words, the axial image-points m and m_1, Fig. 231, being separately transmitted by their respective optic-nerve fibrils, must exactly cover each other at the center C of the chiasmal image, which is, therefore, of two-fold light and shade intensity, provided that the light and color perceptions, respectively, are the same for each eye. In Fig. 231 this increased intensity of the chiasmal image at C is graphically illustrated, being achieved in the drawing through superposition of the vertical and horizontal shadings used respectively to distinguish the right and left retinal images in R and L from each other."

476. "This single chiasmal image at C corresponds to that equipoise of the extrinsic ocular muscles which is associated with the normal directions of both visual axes, called orthophoria. In fact, it is that condition in which the centers m, m_1 and C of their respective images are all located in the same horizontal plane hH, thereby insuring a common horizon for them as shown in Fig. 232. Therefore, the chiasmal horizon, in the absence of a better term, may be said to be the horizontal diameter of the chiasmal field in whose center C are located the superposed chiasmal images transmitted from the macular centers m and m_1, the foveæ of both eyes. In fact, as will be later shown, the axial image-points m and m_1, the centers of the maculæ, are always projected to the center of the chiasmal field, C, regardless of the directions of the visual axes. It is evident that a faulty projection of the visual axes, such as in hyperphoria, for instance, will cause a change in the elevation of at least one of the axial image-points, m, m_1, with respect to a common horizon H, thus creating two vertically displaced

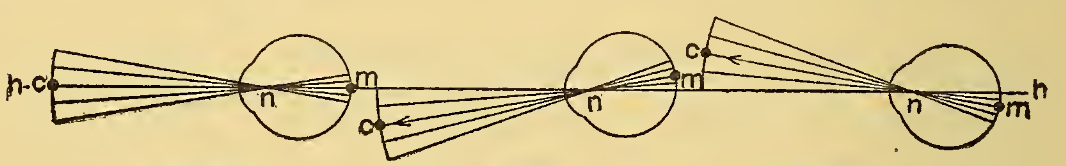


Fig. 233. Fig. 234. Fig. 235.
Chiasmal Images.

images of the same object somewhere within the field of the optic chiasm. In order to become familiar with the location of such dual chiasmal images it is necessary first to determine the positions of their corresponding retinal images. A fundamental law in physiologic optics teaches that the center of the object and the center of the image are in line with the nodal-points of the eye; in other words, the center of the object, the nodal points and the macular center are points upon the same line, the visual axis, located in a plane coincident with, or that may be inclined to, the horizon. In the following diagrams the single nodal point of Donders' reduced eye is applied."

"In Fig. 233, the visual axis cm is coincident with the horizontal plane hh. In. Fig. 234, the visual axis cm is directed below, whereas, in Fig. 235 it is directed above the horizontal plane hh."

477. "In each of these figures the visual axis is directed to the center c of its own object-space, irrespective of the dimensions of the object that may be located upon the horizon, so that the visual axis may also be said to be a line connecting the center c of the object-space with the center m of the image-space. Therefore, each change in the direction

of the visual axis merely establishes a new center *c* in the object-space of the deviating eye which is in line with the fovea *m*, the center of the image-space. In short, for the deviating eye, Fig. 237, *D* is the displacement of the visual axis cm_1 from the object-center O_1, whereas *d* is the corresponding displacement of the fovea m_1 from the image-center I_1."

"It is here proposed to confine the discussion to hyperphoria, be-

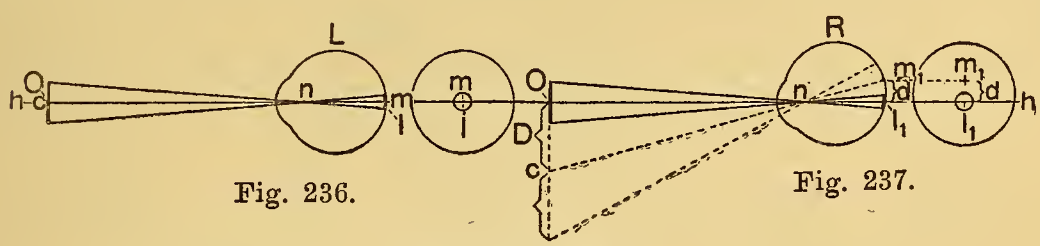


Fig. 236.

Fig. 237.

Chiasmal Images.

cause it may be more effectually counteracted through the use of prisms than any other muscular imbalance. For convenience, the left eye, Fig. 236, is pictured from the temporal side with its visual axis *cm* horizontal; therefore, the centers of the object *O* and its retinal image *I* are located in the plane of the horizon. The right eye is shown in Fig. 237, from the nasal side, with its visual axis cm_1 in the object-space below the horizon; wherefore, the center of the same object *O* is above the visual axis cm_1, and the center of the image I_1 is correspondingly

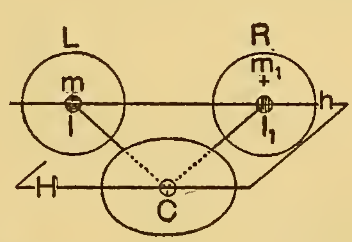


Fig. 238.—Chiasmal Image. (After Prentice.)

below it at a distance *d* from the fovea m_1. Comparison of the figures shows that the centers of the retinal images *I* and I_1 in both eyes are located in the same horizontal plane *hh*, but that they occupy quite different positions with respect to their associated axial image-points, *m* and m_1. The fovea *m* of the left eye and the center of its retinal image *I* are coincident, whereas, in the right eye its retinal image-center I_1 is below the fovea m_1. Moreover, it is quite evident that there would not be any consciousness of a difference in the elevations of the retinal images, *I* and I_1, if they were transmitted by the optic nerves to the chiasm in the same horizontal plane *hH,* as shown in the *incorrect* Fig. 238. That two images are produced which effect consciousness of a difference in their elevations with respect to a common horizon *H* is proof that the phenomenon must be explained through the associated functions of the optic nerves and the chiasm. In fact, this consciousness of two images can only be accounted for by the assumption that the axial image-point of each eye at the macula is conveyed by its

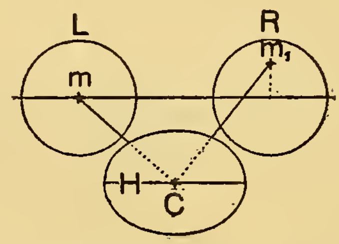


Fig. 239.—Chiasmal Image. (After Prentice.)

corresponding optic-nerve fibril to the center of the chiasmal field. In other words, both macular centers *m* and m_1 are transmitted by their respective nerve fibrils so as to produce superposed images of themselves at the center *C* of the chiasm, Fig. 239, regardless of any deviation that may exist between the visual axes."

478. "Such being the case, the center *m* of the retinal image *I,* when transmitted through the left optic nerve by its macular fibril *mC,* Fig. 240, is located in the center *C* of the chiasm upon the chiasmal horizon

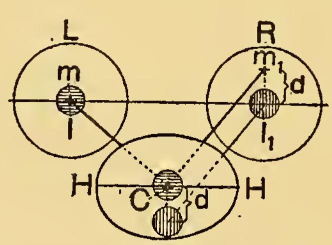


Fig. 240.—Chiasmal Image. (After Prentice.)

HH; whereas, the center I_1 of the retinal image in the right eye is conveyed along the right optic nerve below the macular fibril of m_1 to a point at the same distance *d* below the chiasmal center *C.* In short, the vertical separation of the chiasmal image-centers is equal to the distance *d* between the macular center m_1 and the center I_1 of the image in the deviating eye. It is further evident that, if the vertical displacement *d* of the fovea m_1 in the deviating eye is greater than or equal to the diameter of the retinal image itself, two separated chiasmal images are formed, Fig. 241 and Fig. 242; whereas, if the displacement of the fovea m_1 in the deviating eye is less than the diameter of its retinal image, two eccentrically superposed chiasmal images are produced, Fig. 243. Therefore, the nature of these dual images, as to whether they are separated or overlap, depends upon the proportion of the displacement d of the fovea m_1 in the deviating eye to the diameter of its retinal image I_1 centered in the horizontal plane h."

479. "Moreover the foveal displacement d and the diameter of the retinal image may both be determined, provided the distance of the nodal point from the retina is known. For example, Donders' reduced eye being chosen, and in which the distance between the nodal point

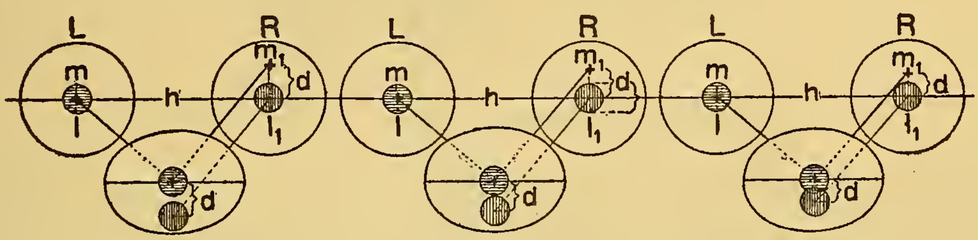


Fig. 241. Fig. 242. Fig. 243.

Chiasmal Images. (After Prentice.)

and the retina is equal to 15 mm., or 1.5 cm., it is apparent from Fig. 244 that:

$$\frac{d}{1.5} = \frac{D}{100} \therefore d = \frac{1.5D}{100},$$ in which D is a dimension in the tangent plane Oc at a distance of 100 cm. from the nodal point n. Therefore, D is synonymous with the prism-dioptry when it is made equal to 1 cm., and, if introduced in the above equation, gives $d = \frac{1.5}{100} = 0.015$ cm., or 0.15 mm. as the separation of the chiasmal image-centers for a deviation of the visual axes corresponding to 1 prism-dioptry. The above equation also proves that:

"In manifest hyperphoria of 1 prism-dioptry the distance between the chiasmal image-centers is equal to one hundredth part of the distance between the nodal point and the retina in the deviating eye. It is also apparent that the separation of the chiasmal image-centers will increase in proportion to the deviation between the visual axes, wherefore, 1^{Δ} 2^{Δ} 3^{Δ} 4^{Δ} of deviation between the visual axes correspondingly produce 0.15 0.3 0.45 0.6 mm. separation of the chiasmal image-centers. It is next necessary to determine the size of the retinal images, in order to ascertain if the chiasmal images are separated or overlap for a particular deviation of the visual axes."

480. "As the size of the retinal image depends upon the size of the object, it is convenient to select as the object one of the small letters that represent the conventional unit of visual acuity in Snellen's test types, and whose vertical and horizontal dimensions are embraced by the visual angle of 5 minutes. Therefore, the height of this object corresponds to the tangent of 5′, which, if computed at a distance of 1 meter, is 0.001455 M., or 1.455 mm.; consequently, the object-letter is

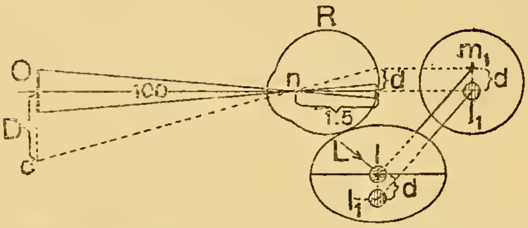


Fig. 244.—Chiasmal Image. (After Prentice.)

8.73 mm. square, at 6 meters, the distance at which the type is used. Incidentally it may be stated that this dimension is rarely exactly reproduced in modern editions of Test Types, and which are often found to be correspondingly faulty in all of the letters. The size of the retinal image, I, produced by a letter 8.73 mm. square, when placed at 6 meters from the reduced eye, may be deduced from Fig. 245 as follows:

$$\frac{I}{15} = \frac{8.73}{6000} \therefore I = \frac{15 \times 8.73}{6000} = 0.02182 \text{ mm.},$$

which is such a minute dimension that, even if mechanically reproduced for inspection, it

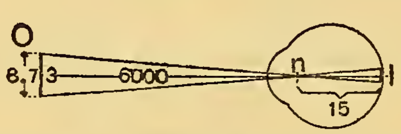


Fig. 245.—Size of Retinal Image.

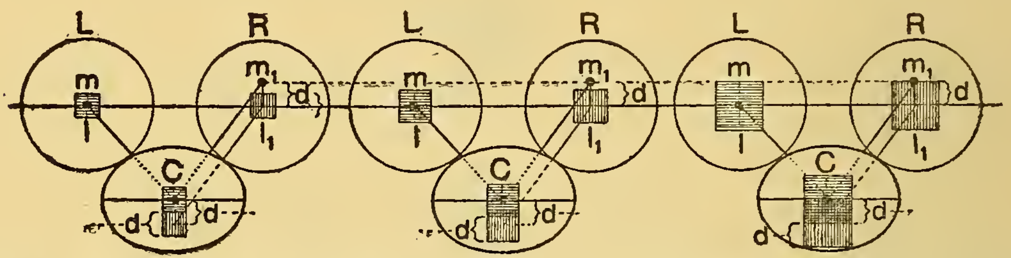
could scarcely be differentiated by the eye without the use of a microscope. As 1^Δ is known to produce a separation of the chiasmal image-centers equal to 0.15 mm., and the vertical dimension of the retinal image in the deviating eye of normal visual acuity is 0.02182 mm., it follows that the dual chiasmal images are separated, since $d = 0.15$ mm. is greater than I or $v_1 = 0.02182$ mm. In order that the dual chiasmal images may just touch each other peripherally, the diameter of the image I would have to be equal to $d = 0.15$ mm. This value for I being introduced in the equation $\frac{O}{I} = \frac{6000}{15}$ gives the diameter of the object $O = \dfrac{0.15 \times 6000}{15} = 60$ mm., which is also the value of the prism-dioptry at 6 meters distance."

481. "Reference to the adjoining chart of letters shows that 60 mm. is the dimension of a letter which should be interpolated between *E* and *T*. In other words, all of the letters between *T* and *L*, when viewed at

VISUAL ANGLE 5′

Distance		Height	
60 M.	E	87.3	mm.
		60	mm.
36 M.	T	52.38	mm.
24 M.	D	34.92	mm.
18 M.	P	26.19	mm.
12 M.	F	17.46	mm.
9 M.	O	13.095	mm.
6 M.	L	8.73	mm.
1 M.	H	1.455	mm.
⅓ M.	V	0.485	mm.

a distance of 6 meters, produce separated chiasmal images for a manifest hyperphoria of 1^{Δ}; whereas, letters or objects which are larger than 60 mm. in height will produce vertically overlapped chiasmal images whose centers are uniformly 0.15 mm. apart. Separated chiasmal images also apply to type held at the reading distance whenever the letters are smaller than the prism-dioptral deviation between the visual axes in the object-space. For instance, in the accompanying chart the letter *v* is 0.485 mm. high at 1/3 M., whereas the prism-dioptral deviation at this distance is 1/3 of a centimeter, or 3.33 mm. This deviation, being considerably greater than the vertical dimension of the letter *v* in the object-space, will naturally cause visual confusion, through making a printed line of the same kind of type appear to be projected from the position of the line next following when the consecutive lines of type are 3.33 mm. apart. For this reason hyperphoric subjects of 1$^{\Delta}$, even with normal retinal perception in each eye, can not read very small type, nor without great difficulty even ordinary type, and frequently complain of a sense of uncertainty in following the lines of a printed page. The limits between which overlapped images are produced is made apparent in the following necessarily exaggerated diagrams, in which the dimensions of the various retinal images have been calculated for white square targets of different sizes placed at 6



Figs. 246, 247, 248.—Deviation Between the Visual Axes Equals 1 Prism. Dioptry. Foveal Displacement, Separation of the Chiasmal Image-Centers and the Image-Extension = d = 0.15 mm. for Donders' Reduced Eye.

Fig. 246.—Object 60 mm. sq. Image 0.15 mm. sq.

Fig. 247.—Object 87.3 mm. sq. Image 0.2185 mm. sq.

Fig. 248.—Object 120 mm. sq. Image 0.3 mm. sq.

meters distance. In Fig. 246 the retinal images are 0.15 mm. square, which, being equal to the foveal displacement corresponding to 1$^{\Delta}$, not only shows the chiasmal images to be peripherally in contact, but also that the chiasmal image for the left eye is extended below its horizon by an amount equal to the height of the chiasmal image for the right eye, thus producing a vertically elongated picture of the square target that is twice the height of the normal image conceived in orthoscopic binocular vision.''

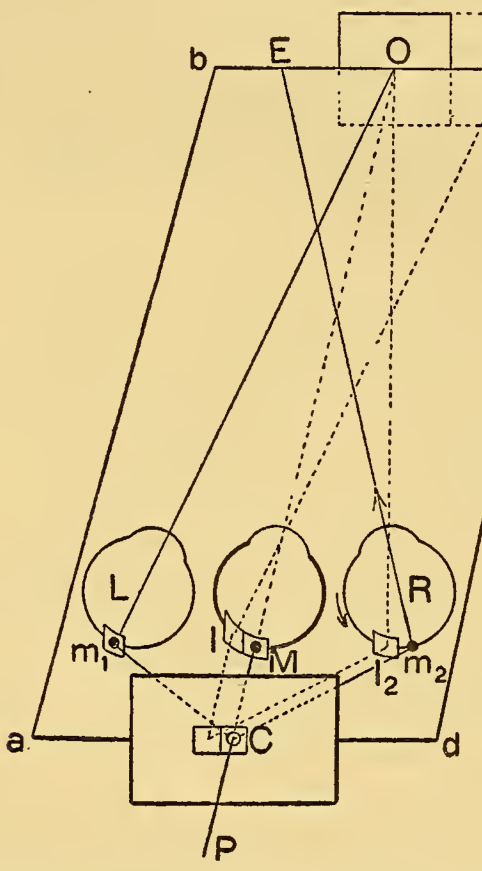
482. ''The retinal image in Fig. 247 is 0.2185 mm. square, and being larger than the foveal displacement, shows the chiasmal images to overlap. In fact, as the retinal images proportionately increase in size for larger objects placed at a fixed distance, so will the chiasmal images increase and invade each other vertically from their respective fixed centers. Fig. 248 represents a still larger image, 0.3 mm. square, demonstrating that the chiasmal images overlap each other by one-half.

Furthermore, in all of these diagrams it is made apparent that the foveal displacement, the separation of the chiasmal image-centers and the extension of the normal image are one and the same linear dimension. Therefore, it has been conclusively demonstrated that for each prism-dioptry of deviation between the visual axes there is a separation of the dual chiasmal image-centers equal to 0.15 mm.*; that the chiasmal images are separated when the retinal images are smaller than the foveal displacement, and that the images overlap when their retinal images are larger than the foveal displacement in the deviating eye. Recalling that the prism-dioptral deviation between the visual axes in the object-space bears the same proportion to the size of the object itself as the foveal displacement to the retinal image in the deviating eye, it is also to be understood that the chiasmal images are vertically separated when the size of the object viewed binocularly is less than the prism-dioptral deviation, and that they overlap when the object viewed is greater than the prism-dioptral deviation between the visual axes. In short, when the diameter of the object at 6 meters' distance is exactly equal to the prism-dioptral deviation between the visual axes, contiguous chiasmal images are formed whose line of contact is the boundary between separated and overlapped images.''

483. These images can be shown ''to serve a useful working hypothesis in making a lucid drawing to illustrate the phorias, and as a figure of speech when attempting to differentiate between the ocular images and the corresponding brain-images: also as a figure of speech, since it is just as imaginary as its correlative chiasmal image, which, at least figuratively, occupies a more definite location. Moreover, the chiasmal image is quite as conceivable as the all permanent ether through which light is supposed to be propagated. In order to demonstrate the purpose and need of at least one point of orientation, although two different ones will be here jointly applied, let it be supposed that the diagram, Fig. 249, represents a horizontal plane, *abcd,* in which the corresponding sections of the right eye, R, and the left eye, L, are located to view the object, *O,* upon the median line, *MO*. It is also assumed that the visual axis of the right eye, R, is faultily directed towards *E,* as in esophoria, so that its macula, m_2, is turned to the right: whereas, the macula, m_1, of the left eye, L, retains its normal position with respect to the object, *O;* and, therefore, also with respect to the center, *C,* of the chiasmal field and the macula, *M,* of the mean eye on the median line, and to which points of orientation the macular center, m_2, in the right eye is also projected. Consequently, the macular centers, m_1 and m_2, in both eyes have the same points of orientation, *C*

* Donders' reduced eye, whose first principal focal length F′ is 15 mm.

and M, in common, while the image-center, m_1, of the left eye alone is transmitted to these points. But the center of the image I_2, projected from O into the right eye, is situated on the left side of the macula, m_2, and is, therefore, transmitted with equal displacement so as to be located on the left side of the centers of orientation, C and M, in the chiasmal field and mean eye, respectively. Therefore, the displaced image, I_2, in the right eye, is transmitted to and located on the left side of the



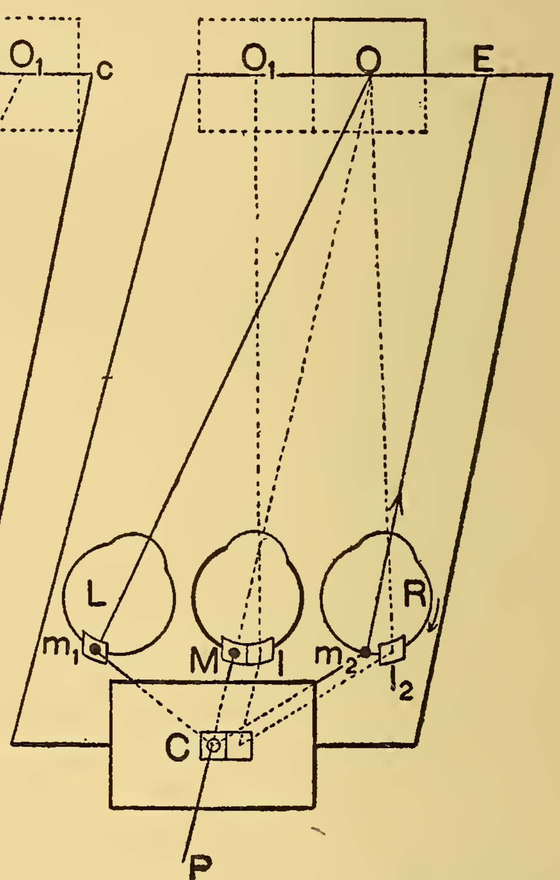


Fig. 249.—Homonymous Projection (After Prentice). Contiguous mean cyclopean image.

Fig. 250.—Heteronymous Projection (After Prentice). Derived from chiasmal images.

mean eye as the false cyclopean image I: and it is this image, belonging to the right eye, that is homonymously projected to O_1, on the right side of and at the same prism-dioptral distance, EO, from the object O. The points m_1, m_2, C and M are corresponding points with reference to the axis, PCM, of bilateral symmetry within the cranium, which coincides with the median line, MO, in the object-space and is directed to the supposed center, P, of image-perception in the brain."

484. In Fig. 250 "heteronymous projection of the ocular images is illustrated. Both diagrams show that the chiasmal and mean cyclopean

images, respectively, are contiguous, because the horizontal diameter of the object is made equal to the prism-dioptral deflection, *EO*, so that the real object, *O*, and the mentally conceived object O_1, are also in contact.''

''The figurative mean cyclopean images, *I* and *M*, are graphically projected from their corresponding chiasmal images, so that conjointly they make it possible to pictorially illustrate either homonymous or heteronymous diplopia in a manner not hitherto lucidly accomplished. In view of the indeterminate pyschophysical character of these images, and an effort made to picture them in a drawing, the delineator will at least need to assume that the center, *P*, of the so-called brain-image is the center of image-perception; that it is located in the horizontal plane, in juxtaposition to the center, *M*, of the centered cyclopean image of the object on the median line and, therefore, coincident with the center of the chiasmal image· said line, *PCM*, within the cranium, being considered the axis of visual orientation, or the directrix of bilateral symmetry of vision in the mind at least of the draftsman.''

APPENDICES

APPENDIX A

THE THEORY OF COLOR VISION AS PROPOSED BY MRS. LADD-FRANKLIN

The Development Theory of Color: This theory takes into account the fact of a gradual evolution of the color sense from a primitive condition of achromatic vision such as still exists in the periphery of the retina and in the eyes of the totally color blind. It assumes that the achromatic sensation (white—what we call in its lower intensities gray) is occasioned when terminations of nerve-fibres undergo excitation by means of a chemical substance dissociated out, under the influence of light, from a primitive light—sensitive material, which very likely is the only such material which occurs in the rods. Whether it is identical with the visual purple, or whether that acts simply as a sensitizer, it is impossible at the present time to say. This may for convenience be designated as the gray substance. This substance responds, in this form, non-specially to light from any portion of the spectrum (but most to the light wave length λ 5050). The cones, however, which are known to be structually more highly developed rods, contain a light sensitive substance which is of a more highly developed character in the sense that it is capable of responding specifically to the different rapidities of light wave motion—certain atomic groups within the molecule are fitted to being broken off by the action of light of certain definite periods of vibration. This development may be supposed to have taken place in two successive stages in accordance with what is known of the actual development of the color sense (bees, for instance, have yellow-blue vision only). The first of these stages consists in the formation of two groupings within the molecule, one of which is dissociated by the slower waves and gives a sensation of yellow, and one of which is dissociated by the more rapid waves and gives a sensation of blue. This stage continues to exist in the mid-periphery of the normal human retina, and it alone is the condition present in all the cones throughout the retina in the eyes of the red-green blind.

The next stage of development consists in the division of the yellow component into two fresh groupings in one of which the internal oscillations of electrons are of such a periodicity as to be affected by the longest visible waves, the red end of the spectrum, while the other group is dissociated by rays corresponding to the green of the spectrum, and gives rise to the sensation green. But if the red and green groupings are dissociated out at the same time (if red light and green light impinge upon the retina together) then we have substances which (being the exact constituents of the former "yellow" component) unite chemically to produce that component (just as an acid and a base, for instance, would, when present together, unite to produce a different substance, a salt). In this way is accounted for the fact that red and green are never sensed together but that they always mutually extinguish each other and are replaced by the sensation yellow. In the same way whenever the yellow nerve excitant is present together with the blue nerve excitant they form a chemical union which is then identical with that original nerve excitant whose effect was the primitive sensation white.

This theory of color may seem at first sight to be somewhat complicated, but it will appear upon examination to be no more complicated than the facts of color demand. In no other theory has there been devised a simple chain of chemical events which necessitates (parallels) the three most important and striking phenomena of color. These fundamental phenomena are so universally overlooked—some by the adherents of the Hering theory and some by the adherents of the Helmholtz theory—that it will be desirable to keep them before the eye in parallel columns.

(1)	(2)	(3)
The number of "adequate homogeneous, electro-magnetic vibration periods (wave-lengths) is THREE.	The number of homogeneous (non-blended), or unitary color-sensations is FIVE.	But the five distinct color-sensations are not independent variables, they are subject to the conditions: $R + G = Y$ $Y + B = W$ (and hence $R + G + B = W$).

The statement in column (3) amounts to saying that neither red and green, nor yellow and blue, ever occur in consciousness together: they are disappearing, or vanishing, or mutually extinguishing, color pairs: in place of them appear (for red and green) yellow, and (for yellow and blue) white. We have here, then, the whole situation in regard to color theories in a nut shell.

The Helmholtz theory is built up upon the phenomena (1), and is inconsistent with the phenomena (2) and (3). The Hering theory is built up upon the phenomena (2), explains, in a fashion, one part of phenomena (3)—namely that $Y + B = W$, but garbles the other parts, affirms that $R + G = W$, when reality $R + G = Y$. It is thus wholly consistent with (1), and with part of (3). There is evident necessity for a hypothesis which will take account of all of these seemingly inconsistent phenomena at once. The development hypothesis of color is an hypothesis, (1) in which the adequate electro-magnetic stimulants of the chemoreceptors of the retina are three; (2) in which the distinct, unitary, color sensations are five; and (3) in which red and green are yellow-constitutive, while yellow and blue are white—constitutive. It is surely worth while to make reasonable all these color phenomena, instead of being forced to deny part of them, as is done in both the Hering and the Helmholtz theories.

The conditions which must be fulfilled in an adequate color-theory are then these: yellow and white must be due to unitary chemical nerve-exciters, and nevertheless these nerve-exciters cannot be simply independent, disconnected, chemical substances, for they must satisfy the relations:

$$R + G = Y$$
$$Y + B = W$$
$$(\text{or, } R + G + B = W).$$

(From the essay on "Color Vision, Theories of," by C. Ladd-Franklin in *The American Encyclopedia of Ophthalmology*, Volume IV, pages 2499-2502. Reprinted by permission of the publishers, the Cleveland Press.)

APPENDIX B

REFRACTIVE DIFFERENCES IN FOVEAL AND PARAFOVEAL VISION

This important matter has been made the subject of experimental investigations by Ogata and Weymouth (*The American Journal of Ophthalmology*, Vol. I, page 631, 1918). They made use of a slender, illuminated triangle uniformly lighted: the apex of the triangle was sharp enough for a test of foveal vision and its tapering shape was found to be very well fitted for that of parafoveal vision. The conclusions to which the authors arrived indicate: (1) Small differences of refractions between foveal in the light and in the dark, and between parafoveal vision in the light and dark, are shown by 10% to 20%, of the reagents examined. (2) A definite refractive difference between foveal and parafoveal vision is shown by 40% to 45% of the reagents. The difference between the fovea and a spot 5° excentric (temporally) from it, amounts to from .33 D to .50 D, the parafoveal region being more myopic or less hyperopic than the fovea. (3) The observation that glasses suitable in bright light become less so in dim light accords with the facts established. (4) These facts have a practical application in the correction of refractive errors. Care should be taken in those cases where the person must distinguish signals in dim light (railway engineers, firemen, signalmen, ship's lookouts, etc.) to see that the glasses given are suitable for twilight vision, as in a certain percentage of cases the correction under ordinary conditions is distinctly different from that in dim lights. (5) A possible explanation is offered by the present facts for the differences found between the refraction as determined by the retinoscope and by subjective methods under otherwise similar conditions. (6) The differences found between foveal vision in the light and in the dark seem to be best explained by Jackson's theory of the influences of the widened pupil in the dark adapted eye in admitting the peripherally more strongly refracted rays. (7) The difference between foveal and parafoveal vision does not seem to us to be due to optical factors affecting the incident light in the two cases, but is more satisfactorily explained by the assumption that the membrana limitans externa bulges outwards (away from the vitreous) in one, two, three or all directions from the fovea. In the latter case the conditions are those which have been described by histologists as an external fovea.

APPENDIX C

"STEREOSCOPIC" COLORS OR "RETIRING" AND "ADVANCING" COLORS

Many have doubtless noticed that various colors in the same plane do not appear to be in the same plane. The phenomenon may be quite strikingly apparent when the images of words in various colors on lantern slides are projected by stereopticon. The writer of this monograph has observed, over a number of years, this rather striking apparent inequality in the distances of letters projected in clear focus on to a screen and has collected quite a little data. However, none of this has as yet been published. M. Luckiesh, of the Nela Research Laboratory of the General Electric Company, has published some facts and data in the *Journal of the Franklin Institute,* page 773, 1917, and in *The American Journal of Psychology,* Vol. XXIX, page 182, 1918. For experimental purposes he used two boxes of similar size and construction, carrying a plain letter X cut in one face and illuminated through red glass, while the companion box carried a plain letter E cut in a diaphragm illuminated by blue light. In a darkened room these two colored letters stood out in space: one (the blue E) was kept fixed in space at a given distance and the other (the red X) was shifted until the two appeared in the same plane. The distance which the red X had to be moved behind the blue E was designated as plus, (+), or in front of the blue E by minus, (—). The results for nine observers as obtained by Luckiesh are given in the accompanying table, in which the line "total distance in meters" signifies the distance from the eye of the fixed blue E; the numbers given under these respective distances indicate the positive or negative shifts of the red X in order that both colored letters might appear in the same plane.

Total Distance in Meters	2.5	4	5	6	7.25	
M. L.	2.8	16.0	24.6	39.8	57.7	cms.
H. K.	4.8	13.7	21.0	31.7	46.2	
F. G.	1.6	5.3	7.6	5.3	12.5	
P. H.	3.1	15.1	8.4	35.6	49.5	
L. M.	2.6	4.5	13.4	14.4	18.3	
E. K.	7.4	28.2	50.9	22.6	71.7	
H. P.	5.1	2.4	....	23.2	17.4	
L. C.	—0.4	— 1.9	1.4	8.9	— 2.7	
G. H.	—7.4	—18.5	—34.0	—37.6	—49.7	

It is seen that in most cases it was necessary to move the red X farther away than the blue E in order to make both appear in the same plane perpendicular to the line of sight, and that this distance generally increased with the distance of the test objects from the observer's eyes.

Interesting results were obtained by nearly closing the eyelids, (i. e. narrowing the palpebral fissure), for under such conditions the differently colored letters appeared to move into the same plane,—i. e. they appeared to be in the same plane as they were actually located and not experience the shifts recorded above. Furthermore, two very small artificial pupils placed before the eyes and moved closer together, that is, each moved slowly toward the nose, caused the letter E to be apparently moved forward very strikingly in the case of persons normally seeing the red X ahead of the blue E. Through such small apertures the two images appeared simultaneously in focus at all times, a condition which was, of course, not true with the natural pupils. The effect could not be observed with certainty with one eye only.

Luckiesh gives as a possible explanation the fact that the different refractive indices of the eye media for radiant energy may play a part in causing this effect. Chromatic aberration is, first hand, the most plausible cause and, in his own case, Luckiesh says that chromatic aberration would account for the effects observed.

The writer of this monograph, from his experimentation, believes that it is a combination effect of slight adjustment of accommodation, chromatic aberration and spherical aberration, particularly the two last named. Some of the Liberty loan posters in shop windows have been casually studied and such results as the following discovered. White, red and yellow letters (white and red on a horizontal line and yellow below) on a blue background, showed the *apparent* positions in the order white, yellow, red. Upon "fixing" the yellow letters, however, there appeared to be a shifting forward of these yellow letters into the same plane as that of the white. Red letters in a row immediately above the same sized blue letters on a white background showed that the blue was behind the red when the red letters were fixed and *vice versa* when the blue ones were fixed.

This matter deserves a thorough investigation.

APPENDIX D

THE EYE MOVEMENTS IN READING

That the eye movement during reading is not continuous was first observed by Dr. Javal in 1879. In company with Lamare he concluded that the eye moved over the line in a series of jerks interrupted by pauses and that no reading was done except during the pauses. By means of a microphone attached to the eyelid he could hear the faint sound made by the friction of the cornea upon the lid and estimated that the pauses came about once every ten letters.

In 1891 Dr. Lanholt published results, through the watching of the movements of the eye in a mirror, showing that 1.55 words were read on the average for each fixation.

In 1898 Professor Delabarre, of Brown University, secured tracings of eye movements. At Clark University, Huey carried forward Delabarre's ideas by attaching a light plaster of Paris cup to the cornea and recorded the eye movements by light reflected on to a device for recording time. Huey's results showed: (1) In reading relatively long lines of magazines, the average number of fixations is about 4.5 per line: (2) Doubling the distance from the eye to the page does not affect the number of fixations: (3) Minor modifications in the size of type do not affect the number of fixations: (4) The duration of the reading pauses, although extremely variable, averages about 0.19 second: (5) Fast reading entails fewer and shorter pauses, but not faster movements: (6) A short line, e. g., about 60 mm., makes possible fewer pauses, relatively, so that more is read with the single pause.

In 1908 Erdmann and Dodge made extended experiments at the University of Halle. Their chief conclusions are: (1) The number of pauses for reading is fairly constant for the same reader using the same material, but is affected decidedly by length of line, by difficulty of material read and by the personal equation of the reader: (2) The first fixation is not at the extreme left end of the line, but a short distance from this end, while the last fixation, similarly, is often not at the extreme right end, but a short distance from that end: (3) The fixations seem to be on words and usually on the middle of a word: (4) Of the entire time spent in reading, from 12/13 to 23/24 is devoted to the pauses, the remainder to the eye movement: (5) It is extremely doubtful if anything is perceived during the movement.

Further improvements in method have been made by Dodge. His device consisted in illuminating the cornea of the eye by an actinically powerful beam of light and in photographing the moving cornea through a narrow slit upon a very sensitive photographic plate arranged to fall vertically at a constant and controllable speed. This device entails no inconvenience to the reader, permits of extreme precision and enables the exact direction of the gaze to be subsequently located upon the printed page to within an error of one letter. With it Dodge was able to measure the speed of the forward movements and of the return movements, and to demonstrate that the former averaged only 23 and the latter only 40 thousandths of a second.

Detailed tests and experiments were later made by W. F. Dearborn at Columbia University. The chief conclusions reached are briefly stated as follows: (1) The speed of forward movements determined by Dodge is confirmed: (2) The duration of the fixation pause is of the order of one-fifth second: (3) For a line 10 cms. long there were, on the average, 7.45 fixations, so that 1.6 words were read per fixation: (4) Fixation is not absolute and constant, but only approximate: (5) The first and last fixations generally fall within the edges of the printed page: (6) Fixation is not in the middle of words but may be in practically any place in the word or in the spaces between words: (7) Fatigue causes a slower rate of reading and a gradual decrease in the velocity of eye movements.

INDEX OF SUBJECTS

The numbers refer to the pages

D

E

S

T

V

INDEX OF NAMES

The numbers refer to the pages